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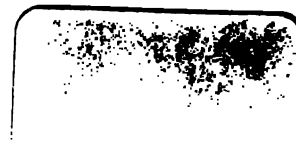
SOME APPLICATIONS OF THEORY TO THE PRACTICE OF CONSTRUCTION



SOME APPLICATIONS OF THEORY

TO THE

PRACTICE OF CONSTRUCTION,

WITH EXAMPLES;

BEING A PART OF THE COURSE OF INSTRUCTION

AT THE

SCHOOL OF MILITARY ENGINEERING,

CHATHAM.

BY

COLONEL HENRY WRAY, ROYAL ENGINEERS,

Instructor in Construction.

Entered at Stationers' Hall.

PRINTED AT THE SCHOOL OF MILITARY ENGINEERING, CHATHAM.

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186. e. 57.

PREFACE.

THE Papers of which this volume consists, have been prepared from time to time during the past six years from Notes of the Lectures which it has been my duty and pleasure to deliver to my Brother Officers at the School of Military Engineering.

In preparing them, I have endeavoured to make them a kind of index to the various works referred to, the greater number of which are to be obtained at most stations, even if they do not form a part of an Officer's own library, one of my objects being to enable an Officer who has to get out a design, to know where to turn for data, without which he must soon come to a stand still.

I have included in the Papers some few data, which I have gleaned personally from various sources, but as a rule I have referred the Officers to other works for such information as they require in applying Theory to Practice, the particular data required for each Example being commented on during the delivery of the Lectures.

As I proceed on the assumption that the Officers have acquired the necessary mathematical knowledge before joining the Corps, I do not, in delivering the Lectures, go into mathematical investigations, and I have excluded them from the Papers, wherever I could give a reference to any work, where they are sufficiently fully stated. There are only a few such investigations, which I have, on this principle, thought it desirable to give.

With the exception of the full manner in which, for the purpose of instruction, the Examples are treated, there is, of course, little that is new in the Papers, but I believe the method of determining the best section for cast-iron beams is a new application of an old principle, and the attempt to reduce to calculation the rivetting of grouped joints is, so far as I know, new, although the results must of necessity, from the nature of such joints, be only approximately true.

I have endeavoured, throughout, to acknowledge the sources from which my information has been derived, by repeated references to the authors drawn upon; and I wish to place on record the obligations I am under to the Members of the Civil Engineering Profession, to the Manufacturers of Engineering Material, and to many other Gentlemen, who in every instance, and with uniform kindness, have given my Brother Officers and myself permission to visit and examine in detail their various works, at the same time affording us the greatest facilities for studying them and putting at our disposal all the necessary information respecting them.

H.W.

School of Military Engineering,
Chatham, November, 1872.

LIST OF BOOKS, &c., REFERRED TO.

Humber's Iron Bridges—1861.	Abbreviation H.I.B.
Hurst's Architectural Surveyor's Hand Book—1866.	" H.A.S.H.
" Tredgold's Carpentry—1871.	" H.T.C.
Latham's Wrought Iron Bridges—1858.	" L.W.I.B.
Minutes of Proceedings of the Institution of Civil Engineers.	" M.P.I.C.E.
Molesworth's Pocket Book of Engineering Formulæ—1865.	" M.E.F.
Rankine's Manual of Civil Engineering—1862.	" R.C.E.
" Applied Mechanics—1864.	" R.A.M.
Stoney's Theory of Strains—1869.	" S.T.S.
Tredgold's Carpentry—1840.	" T.C.
Unwin's Wrought Iron Bridges and Roofs—1869.	" U.I.B.R.
Useful Rules and Tables, Rankine—1867.	" U.R.T.
Annales des Ponts et Chaussées, Tome 12, 1866.	No abbreviation.
Armstrong's Chimneys, Furnaces and Fireplaces—1866.	"
Buck's Oblique Arches—1857.	"
Dobson's Art of Building—1867.	"
Engineer's and Contractor's Book—1866.	"
Experimental Researches on the Strength and other properties of Cast Iron by Eaton Hodgkinson, F.R.S.—1846.	"
Fenwick's Mechanics of Construction—1861.	"
Fourniè's Traitè de la Stabilitè des Constructions—1864.	"
Handyside's Works in Iron—1868.	"
Hutton's Tracts, Vol. III,—1812.	"
Kirkaldy's Experiments on Wrought Iron and Steel—1867.	"
Moseley's Mechanical Principles of Engineering and Architecture—1855.	"
Philosophical Transactions—1856, 1857.	"
Pole's Iron as a Material of Construction—1872.	"
Reilly's Uniform Stress in Girder Work, Vol. XXIV. Minutes of Proceedings of the Institution of Civil Engineers—1864-65.	"
" Studies of Iron Girder Bridges, Vol. XXIX. Minutes of Proceedings of the Institution of Civil Engineers—1869-70.	"
Rouleaux's der Constructeur—1869.	"
Scheffler's Theorie der Gewölbe, Futtermauern und Eisernen Brücken—1857.	"
Sir H. James's Instructions for taking Meteorological Observations—1860.	"
The Engineer, weekly paper.	"
Unwin's Construction of Iron Bridges (lectures)—1871.	"
" Railway Construction (lectures)—1869-70.	"
Weisbach's Mechanics of Machinery and Engineering—1847, 1849.	"

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It is suggested that the errata be transferred to their proper pages, before studying the volume.

ERRATA AND ADDENDA.

Page 17, line 1, *for* total amount, *read* moment.

„ 17, lines 5 to 7, *for* $\int_{-y^o}^{+y^o}$ *read* $\int_{-y_o}^{+y_o}$

„ 43, line 13, *for* 13, *read* 12.

„ 43, last line, *for* p. 38, *read* p. 36.

„ 44, line 13, *for* $\frac{1}{8}$ W c^3, *read* $\frac{1}{8}$ W c^3.

„ 45, line 31, *for* ×, *read* +.

„ 46, line 17, *for* m, *read* n.

„ 47, line 7, *for* $\mathrm{E} \times 144 \times d_3$, *read* $\mathrm{E} \times 144 \times d^3$.

„ 52, line 3, *omit* vertical.

„ 56, line 14, *for* x_4, *read* x^4.

„ 56, line 16, *for* $\mathrm{W}l_3$, *read* $\mathrm{W}l^3$.

„ 78, line 8, *after* usual, *omit* the comma.

„ 89, line 36, *for* near together, *read* far apart.

„ 92, line 21, *after* possible, *add* uniformly.

„ 94, line 28, *for* 14·400, *read* 14,400.

„ 100, line 4, *for* p, *read* b.

„ 111, line 19, *for* x, *read* n.

„ 113, line 11, *for* $1\frac{\lambda}{16}$, *read* $1\frac{3}{16}$.

„ 117, line 35, *after* being, *add* so great.

„ 120, last line, *for* $\frac{2}{wc^2}$, *read* $\frac{wc^2}{2}$

„ 124, line 15, *after* web, *add* a comma.

„ 125, line 5, *for* $\frac{\mathrm{S}}{\mathrm{P}}$, *read* $\frac{\mathrm{P}}{\mathrm{S}}$

„ 127, line 17, *add*, If as near together as 4′ 6″, iron longitudinals are sometimes dispensed with.

„ 128, *Fig.* 20, the arrows should not be at the extreme edges of the angle irons, but about $\frac{3}{4}$ inch inside both at top and bottom.

„ 130, line 19, *for* $\frac{\cdot 50}{5}$, *read* $\frac{\cdot 05}{5}$

„ 131, line 4, *for* $\frac{16}{19}$, *read* $\frac{19}{16}$

„ 134, line 13, *for* 33 ft. tons, *read* 32·5 ft. tons.

„ 140, line 1, *for* 2, *read* 2·5.

„ 140, *Fig.* 36, the arrows should not be at the extreme edges of the flanges, but about $1\frac{1}{8}$ inch inside, both at the top and bottom.

„ 145, line 19, *for* 17 tons, *read* 15·4 tons, and see Appendix, p. 320.

„ 150, *Plate* III., *Built-up Wrought Iron Beams*, the scale of the rocker is about $\frac{1}{2}$ inch to 1 foot.

„ 162, line 15, *for* +(1), *read* (+1).

Page 165, line 25, *for* $w_1 \times \frac{8}{3}$, *read* $w_1 \times \frac{8}{3}$.
„ 166, line 13, *for* 60·08, *read* 60·80.
„ 167, line 4 from bottom, *before* =, *add* Stress in *b* A.
„ 186, line 6, *for* $\frac{5}{6}$W and $\frac{1}{6}$W, *read* $\frac{1}{6}$W and $\frac{5}{6}$W.
„ 187, line 13, *omit* latter.
„ 188, line 22, *for* B_2, *read* B_1.
„ 195, line 1, *for* 50°, 60°, and 70°, *read* 60°, 70°, and 80°.
„ 205, line 22, *for* F_3, *read* F_8.
„ 210, line 27, *for* 16, *read* 15.
„ 212, *Fig.* 22, *see* Note in Appendix, p. 320.
„ *Plate* II., *Roofs*, *Fig.* 3, *for* R_2 (the lower of the two reactions), *read* R_1.
„ „ „ *Fig.* 2, *for* R_5, *read* R_3.
„ 233, line 3, *for* C_1 D_1, *read* O_1 D_1.
„ 234, line 23, *for* eclipse, *read* ellipse.
„ 239, line 2, *for ad*, *read ab*.
„ 240, line 9, *for* n_1, *read n*.
„ 240, line 16, *for* satety, *read* safety.
„ 246, line 3, *for* 107,984, *read* 107,084.
„ 246, line 6 from bottom, *after* Example, *add* II.
„ 252, line 15, *for* R, *read* Z.
„ 256, line 7, *for* 15, *read* 16.
„ 265, last line, *after* resultant, *add* equal and opposite to the reaction.
„ 266, line 3, *for* it, *read* R_1.
„ 273, line 32, *omit* the square of.
„ 279, line 41, *for* latter, *read* batter.
„ 279, line 43, *before* vertical, *add* nearly, and *after* batter, *add* of any importance.
„ 281, lines 22 and 23, *before* overhanging, *for* a, *read* an.
„ 288, line 11, *for* the outer, *read* either.
„ 290, line 30, *after* feet, *add* from H, *and transfer bracket from after* foot, *to after* third.
„ 293, line 5 from bottom, *for Plate* III., *read Plate* II.
„ 302, line 29, *for* H, *read* P.
„ 302, line 2 from bottom, *for* −, *read* +.
„ 306, line 27, *after* pressure, *add* measured on the diagram.
„ 306, line 28, 3 × 1 × 1 signifies three times the distance, F 1, of the centre of pressure from the extrados multiplied by 1 foot in width of the arch (sce p. 233).
„ 307, lines 35 and 36, the two last verticals are required to give p_1 and p_2, p. 308. They are not shewn in *Plate* VII.
„ 308, line 34, *after* tons, *add* measured on the diagram.
„ 309, line 22, strictly, C D ought, in order to prevent tension at E, to be rather more than $\frac{1}{3}$rd C E; but how much more, depends upon the relative dimensions of the wall, and counterforts forming the abutment. In an abutment of ordinary proportions C D would equal about $\frac{4}{10}$ C E.
„ 309, line 32, at end of line, *omit* to.
Plate VI., *Arches*, the scale of *Fig.* 1 is $\frac{1}{8}$ inch to 10 feet.
„ VII., „ the scale is 5 feet to 1 inch.

STRESS IN BEAMS.

BEFORE an Engineer can design, in a scientific manner, a beam for the simplest purpose, he must be able to conceive a distinct mental picture of what his beam has got to do; *i.e.*, he must have, first a clear idea of the way in which the external forces act on the beam, what stresses these forces produce, and how these stresses are distributed over the different parts of the beam; second, a clear idea of the manner in which the material of the different parts of the beam resist these stresses.

And it is to be remembered that the Engineer has to make provision for the greatest stresses, that can possibly be produced in the beam, under any condition of the external forces or load, which it may be required to bear, and that to secure this is his first object. In the case of structures designed for loads of an invariable character, such as bressummers to carry fronts or cross walls of buildings, girders to carry aqueducts, &c., the stresses are always the same in amount. But in the case of bridge beams for roads or railways, the amount, and sometimes the nature of the stresses in the different parts of the beam varies as the load changes its position, and the Engineer has, first, to find that particular condition of the loading, which produces, in each part of the beam, the maximum of each kind of stress; second, to make each part of his beam strong enough to resist that maximum stress permanently.

When an Engineer has so designed his beam, that at every section it is (so far as the practical requirements of construction permit) equally strong to resist the maxima stresses of each kind, which can come upon that section, he has designed what will be called in these papers a beam of uniform strength. Such a beam is not a beam of uniform strength in the ordinary acceptation of the term, the maximum shearing stress being provided for in it, as well as the maxima direct stresses, whereas ordinarily, in the beam of uniform strength (Fenwick's *Solid of Equal Resistance*) the maxima direct stresses only are considered.

As the beam designed to meet both the direct and shearing stresses must be stronger and stiffer than the beam which takes into consideration the direct stresses only, any error that enters into the calculation by providing for each separately will be on the side of safety.

The whole subject of stress in beams is very thoroughly handled by Professor Rankine. His works, however, require long and careful study, and for the purposes of these papers it will be better to refer to

the lectures delivered at Chatham in February, 1861, by Mr. W. Cawthorne Unwin, in which the subject is very thoroughly treated* and to Stoney's *Theory of Strains* (Longman). In fact, except for the purpose of making these papers complete (so far as they go) in themselves, it would be unnecessary to do more than refer to the books above mentioned.

NATURE OF THE STRESSES IN A BEAM LOADED TRANSVERSELY.

The stresses induced in a beam loaded transversely are of three kinds, viz.:—

Tension.
Compression.
Shear.

The two first may be called the *direct stresses*, their action being to extend or compress the material in the direction in which the tension or compression is applied.

If a piece of wood or any material be broken across, say by supporting it at each end and loading it at the centre, it appears to break solely by the extension of the fibres on the under side, and the compression of the fibres on the upper side, these tensions and compressions being the direct stresses just spoken of.

But there have been other stresses at work, viz.: the shearing stresses, *i.e.*, stresses which have endeavoured to make the beam fail by the sliding of its parts on one another, both vertically and horizontally, somewhat as shown in *Figs.* 1 and 2.

Fig. 1.

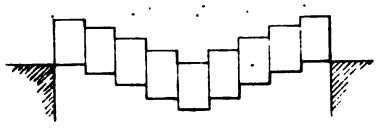


Fig. 2.

These stresses have all been acting at once on the piece of wood, and there is no Mathematical Theory, which takes notice of the effect of all of them at once.

If there were such a theory, it would probably result in a very complicated formula—too complicated in fact for ordinary use.

But there are Mathematical Theories, easy to understand, which take notice of these different stresses separately, and which enable the Engineer to design his beam in perfect confidence, that it will be sufficiently strong, and at the same time, that no waste of material of any importance has resulted.

And this economy of material and labour is the second object of the scientific Engineer.

The external forces, which the Engineer has to deal with in beams, are almost always vertical, or they may be treated as such without

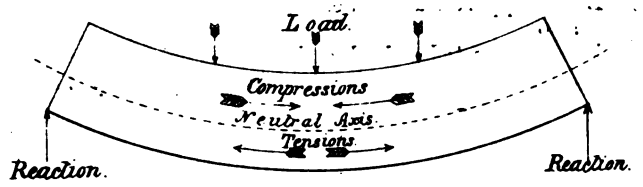
* These have since been enlarged and published in the form of a book.—*Iron Bridges and Roofs.* (*Spon.*)

serious error, and it will be sufficient for the purpose of this paper to deal with vertical forces only.

The Direct Stresses.

The direct stresses, consisting of tension on one side of the beam and compression on the other, are produced by the bending of the beam.

Fig. 3.

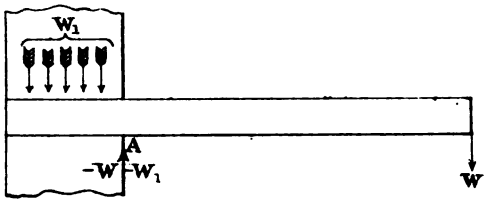


The bending of the beam is produced by the action of the external forces.

The external forces in beams fixed at one end, consist of the load, and the force necessary to fix the beam, with their reactions.

Thus, if *Fig.* 4 represents a beam embedded in a wall, and loaded with a weight W, W_1 being the weight of the wall just necessary to fix the beam, the external forces are W and W_1, and their reactions, — W and — W_1, acting, when equilibrium is just maintained, at A.

Fig. 4.



The external forces, in beams supported at both ends, consist of the load, and the reaction of the points of support.

Supposing the external forces to be all vertical, each acts with a leverage equal to the horizontal distance between its point of application, and the particular section of the beam under consideration.

If the moment of each external force about any vertical section, be found and these separate moments be added together, the total resultant moment of all the forces producing bending about that section is obtained, and this is called the "Moment of Flexure" or "Bending Moment" (sometimes "Moment of Strain") at that section. The first step is to determine its value.

The following cases are those usually occurring in practice:—

(1) Beam fixed at one end and loaded at the other.
(2) Beam fixed at one end and uniformly loaded.
(3) Beam fixed at one end and loaded, both at the end and uniformly.
(4) Beam fixed at one end and partially loaded with a uniformly distributed load.
(5) Beam fixed at one end, and loaded with any number of loads.
(6) Beam supported at both ends and loaded in centre.
(7) Beam supported at both ends and uniformly loaded.
(8) Beam supported at both ends and loaded in centre and uniformly also.
(9) Beam supported at both ends and loaded at any point.
(10) Beam supported at both ends and partially loaded with a uniformly distributed load.
(11) Beam supported at both ends and loaded with any number of loads.
(12) Beam supported at both ends and loaded with an even number of equal loads, symmetrically placed on each side of the centre.
(13) Beam fixed at both ends, or fixed at one end, and supported at the other, of which a few of the more simple cases are dealt with in a subsequent paper on "Fixed Beams."

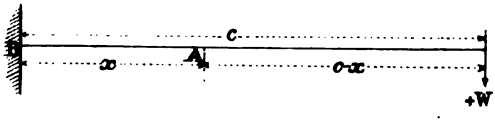
As all the forces are vertical, and the lever arms with which they act are horizontal, their moment about any section of the beam is simply the sum of the moment of all the forces acting on one side of that section, the forces acting downwards being taken as positive, while those acting upwards are taken as negative.

The weight of the beam in all ordinary structures may, with sufficient accuracy for the purposes of the Engineer, be taken as an uniformly distributed load. In this case the moment, at any section, of the weight of the beam, must be added to the moments of the load to get the total or "Resultant Moment."

Case 1.

Let A (*Fig.* 5) be any vertical section at a distance x from the fixed

Fig. 5.



end B of the beam, c the length of the beam, W the weight at the end. Then the only force acting to the right of A is the positive force W acting with the leverage $(c - x)$, so that the moment, M, about A is

$$M = W (c - x)$$

in foot-tons, inch-lbs., or any other convenient denomination.

The moment is a maximum, when $x = o$ and

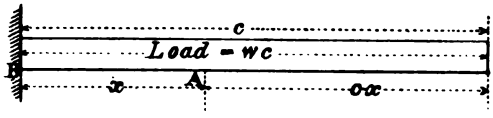
$$M = W c$$

i.e., at the point of fixing, B, and it vanishes at the end of the beam, where $x = c$.

Case 2.

Let A (*Fig.* 6) be any vertical section at a distance x from the fixed end of the beam, c the length of the beam, w the weight per unit of length.

Fig. 6.



Then the only force acting to the right of A is the positive force $w(c - x)$ and it acts with a leverage of $\frac{c - x}{2}$, so that the moment M about A is

$$M = w\frac{(c - x)^2}{2}$$

which is a maximum, when $x = o$, and $M = w\frac{c^2}{2}$

i.e., at the point of fixing, B, and it vanishes at the end of the beam, where $x = c$.

As wc is the total weight, it may be called W, and then

$$M = \frac{W c}{2}$$

or one half of its maximum value in case 1, the total weight being the same.

In case 3 the moment M will evidently be the sum of the moments in cases 1 and 2.

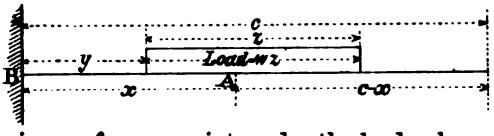
$$i.e., M = W(c - x) + w\frac{(c - x)^2}{2}$$

which again is a maximum, when $x = o$, *i.e.*, at the point of fixing, B, and vanishes at the end of the beam.

Case 4.

Let the partial load (*Fig.* 7) be uniformly distributed over a length z of the beam, let its inner end be at a distance y from the fixed end, B, of the beam, and let the weight per unit of length be w. Then if the section at A intersects the partial load, the only force to the right of the section is positive and equal to $w(z + y - x)$ acting with a leverage of $\frac{z + y - x}{2}$, so that $M = w\frac{(z + y - x)^2}{2}$

Fig. 7.



which is a maximum, for any point under the load, when $x = y$, *i.e.*, at the inner end of the load, where it equals $\frac{w z^2}{2}$, and which vanishes at the outer end of the load.

When A is to the right of the partial load, there is no moment due to the load; when A is to the left of the partial load, y is greater than x, and

$$M = w z \left(\frac{z}{2} + y - x \right)$$

which is a maximum, when $x = o$, *i.e.*, at the point of fixing, B.

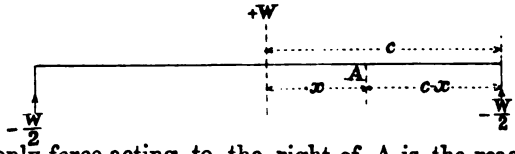
Case 5.

The moment is simply the sum of the moments of the different loads, and this is a maximum at the point of fixing, B, and vanishes at the outer end of the load.

Case 6.

Let A (*Fig.* 8) be any vertical section at a distance x from the centre of the beam, c half the length of the beam, W the weight at the centre.

Fig. 8.



Then the only force acting to the right of A is the reaction of the point of support $= -\frac{W}{2}$ and it acts with a leverage $= c - x$

Therefore the moment $M = -\frac{W}{2}(c - x)$

which is a maximum, when $x = o$, or at the centre of the beam, where $M = -\frac{W}{2}c$, and which vanishes at the point of support, where $x = c$.

By substituting $\frac{l}{2}$ for c, the moment M at the centre

$$= -\frac{W}{2} \times \frac{l}{2} = -W\frac{l}{4}$$

i.e., it is one half of the moment in case 2 for the same length of beam, acting in the opposite direction.

The same result is to be obtained by considering the forces on the left of A.

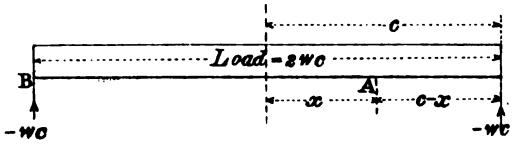
These forces are W positive or downwards, and $-\frac{W}{2}$ negative or upwards, and they act with leverages respectively equal to x and $c + x$

$$\therefore M = W x - \frac{W}{2}(c + x)$$
$$= -\frac{W}{2}(c - x)$$

Case 7.

Let A (*Fig.* 9) be any vertical section at a distance x from the centre of the beam, c half the length of the beam, w the weight per unit of length.

Fig. 9.



Then the forces acting to the right of A are the reaction of the point of support $= -wc$, acting with a leverage $(c - x)$, and the portion of the load lying between A and the point of support $= +w(c - x)$, acting with a leverage $\frac{(c - x)}{2}$

Then the moment $M = -wc(c - x) + \frac{w(c - x)^2}{2}$

$$= -w\frac{(c^2 - x^2)}{2}$$

which is a maximum, when $x = o$, or at the centre of the beam,

$$\text{where } M = -\frac{wc^2}{2}$$

and which vanishes at the point of support, where $x = c$.

This result might also be arrived at by considering the forces on the left of A.

By putting $\frac{l}{2}$ for c, the moment M at the centre

$$= -\frac{wl^2}{8} = -\frac{Wl}{8}$$

or one half the moment in case 6.

Case 8.

The moments will evidently be the sum of the moments in cases 6 and 7.

$$i.e., \mathrm{M} = -\frac{\mathrm{W}}{2}(c - x) - w\frac{(c^2 - x^2)}{2}$$

which again is a maximum, when $x = o$, or at the centre,

$$\text{where } \mathrm{M} = -\frac{\mathrm{W}\,c}{2} - \frac{w\,c^2}{2}$$

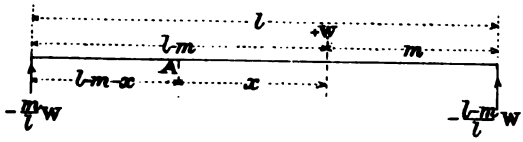
and which vanishes at the point of support, where $x = c$.

Case 9.

Let l = length of beam, m the distance from the load W to one point of support. Then $l - m$ is the distance from the load W to the other point of support.

The reactions are respectively $-\frac{l-m}{l}$ W, and $-\frac{m}{l}$ W.

Fig. 10.



Then at any vertical section, A, (*Fig.* 10) distant x on one side (say the left) of the point of application of the load, the only force to the left of A is the reaction of the point of support, and this acts upwards with a leverage equal to the distance between the load and the point of support, minus the distance from the load to the section in question, so that

$$\mathrm{M} = -\frac{m}{l}\mathrm{W} \times (l - m - x)$$

For any point distant x on the other side of the load

$$\mathrm{M} = -\frac{l-m}{l}\mathrm{W} \times (m - x)$$

In both these cases M is a maximum, when $x = o$, *i.e.*, at the point of application of the load,

$$\text{where } \mathrm{M} = -m\frac{(l-m)}{l}\mathrm{W}$$

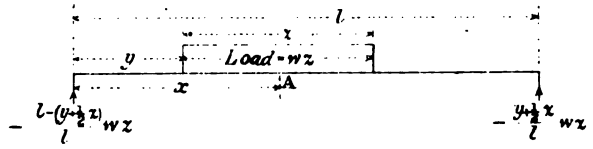
and it vanishes at the points of support, where $x = l - m$, or m, according as x is taken on one side or the other of W.

It is to be observed that the Moment of Flexure increases directly as W increases.

Case 10.

Let the partial load (*Fig.* 11) be uniformly distributed over a length z of the beam, and let one of its ends be at a distance y from the point of support nearest to it; let the load per unit of length be w, and l the length of the beam.

Fig. 11.



Then if the section A intersects the partial load, the forces acting on either side of the section are the upwards reaction of the point of support on that side, and the weight of the part of the load lying to that side of the section, acting downwards.

Taking, say, the forces on the left side of the section, they are the reaction $-\frac{l-(y+\frac{1}{2}z)}{l}wz$ acting with a leverage x, and the portion of the load $+(x-y)w$, acting with a leverage $\frac{(x-y)}{2}$, so that

$$M = -\frac{l-(y+\frac{1}{2}z)}{l}wzx + \frac{(x-y)^2}{2}w$$

the maximum value of which depends upon the relative values of y, z, and l.

It is a maximum when $x = y + z - \frac{(2y+z)}{2l}z$ and it vanishes at the points of support.

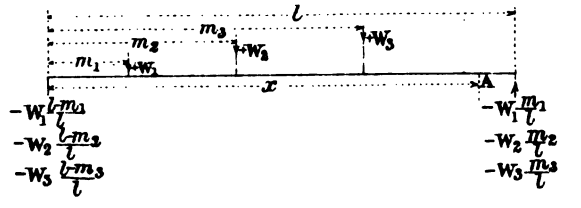
When x falls to the right or left of the partial load, the moment is that due to the reaction of the nearest point of support, multiplied by the horizontal distance of the section from that point.

The addition of any load at either end of z will produce a second moment of flexure of the same kind as the original load, and, consequently, any addition must increase the value of M everywhere.

Case 11.

Let W_1, W_2, W_3, ... (*Fig.* 12) be any number of loads on a beam, distant respectively from one point of support m_1, m_2, m_3, ...

Fig. 12.



Then the reactions at the left point of support are

$$-\frac{W_1(l-m_1)}{l}$$

$$-\frac{W_2(l-m_2)}{l}$$

$$-\frac{W_3(l-m_3)}{l}$$

and so on,

acting with regard to any section distant x from the point of support with a leverage x.

The loads lying between x and the point of support act in the opposite direction, *i.e.*, downwards with leverages.

$$x - m_1$$
$$x - m_2$$
$$x - m_3$$

and so on

so that $M = -\frac{W_1(l-m_1)}{l}x - \frac{W_2(l-m_2)}{l}x - \frac{W_3(l-m_3)}{l}x \ldots$

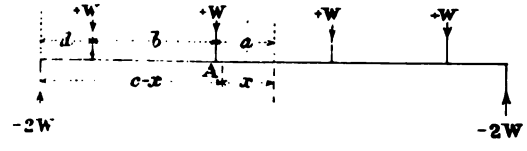
$+ W_1(x-m_1) + W_2(x-m_2) + W_3(x-m_3) \ldots\ldots\ldots\ldots$

The maximum value of M depends upon the relative values of $= W_1, W_2, W_3 \ldots\ldots$and their positions, and it vanishes at the points of support.

Case 12.

Let the number of loads be even, and let them be all equal and symmetrically placed on the beam, as in the case of a cross girder of a railway bridge. (*Fig.* 13).

Fig. 13.



Let the distance from the centre to the nearest load on either side be a, the distance between that load and the next b, and the distance from the last load to the point of support, d.

Then at any section distant x from the centre, lying between the two central loads,

$$M = -2W(d+b+a-x) + W(b+a-x) + W(a-x)$$
$$= -2Wd - Wb.$$

a constant quantity, which shews that the Moment of Flexure between the two central loads is the same at all sections.

In the case of a cross-girder for a single line of way, the loads are only two in number, b vanishes, and each reaction $= -W$, whence $M = -Wd$.

The cases above given, or modifications of them, include all cases likely to arise in practice, except those of beams fixed at one end, and

supported at the other; or fixed at both ends; or supported on more than two supports.

Of the first two classes, some of the more simple cases are dealt with in the paper on "Fixed Beams," and the third class, which is that of "Continuous Beams," is dealt with in *Humber's Iron Bridges*, p. 22, *Stoney's Theory of Strains*, vol. I., p. 185, and *Rankine's Civil Engineering*, p. 287.

GRAPHIC REPRESENTATION OF MOMENTS OF FLEXURE.

It is very convenient, in designing girders, to represent the maxima Moments of Flexure graphically. The application of this practically is shewn in the Example of a plate girder. It is also fully explained in *Unwin's Iron Bridges and Roofs*, p. 57.

In the meantime it is only necessary to explain what is meant by graphic representation.

Taking the case of the uniformly loaded beam, case 7;

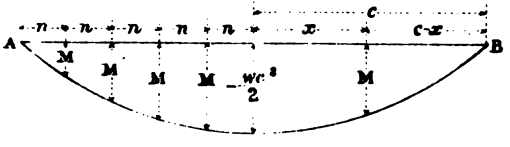
At any point distant x from the centre

$$M = - \frac{w (c^2 - x^2)}{2}$$

Calculate the values of M for values of x, a given number (n) of feet apart, beginning at the centre, where $x = o$.

Set off on any scale (*Fig.* 14) the length of the beam A B, and divide it into parts n feet long on each side of the centre.

Fig. 14.



Set up at these points perpendiculars to A B, and on any scale of foot-tons, inch-tons, or other convenient units, lay off the value of the moments calculated above.

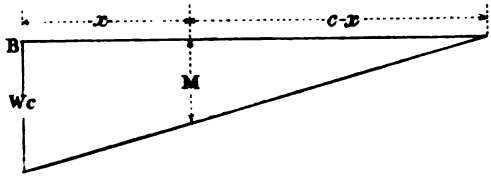
Join their ends.

If the number of parts into which A B is divided be sufficiently numerous, the line joining the ends of the perpendiculars is a curve, commonly called "The Curve of Moments;" and at any section in the beam the Moment of Flexure is represented by the perpendicular at that point.

In this particular case $M = - \frac{w (c^2 - x^2)}{2}$ which is the equation to a parabola when the origin of co-ordinates is at a point in the axis distant $\frac{w c^2}{2}$ from the vertex, and the line joining the ends of the different values of M is a parabola.

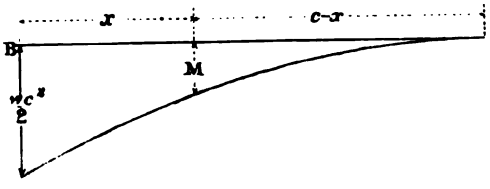
In case 1, (*Fig.* 15), as $M = W (c - x)$, which is the equation to a straight line, the line joining the ends of the different values of M is a straight line.

Fig. 15.



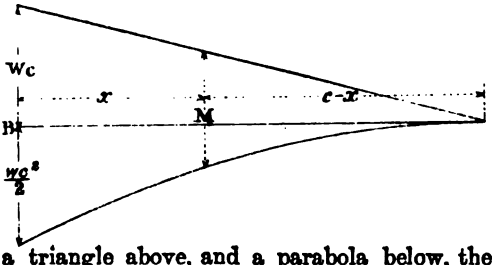
In case 2, (*Fig.* 16), $M = w\frac{(c-x)^2}{2}$ which is the equation to a parabola when the origin of co-ordinates is at a point distant c from the apex of the curve in a line touching the apex of the curve.

Fig. 16.



On the same principles, in case 3, the moments may be represented

Fig. 17.

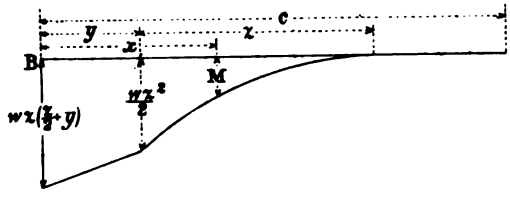


graphically by a triangle above, and a parabola below, the ordinates of the triangle representing the moments of flexure of the load at the end, those of the parabola representing the moments of flexure of the uniform load, and the sum of the two at any section representing the whole moment at that section.

Similarly,

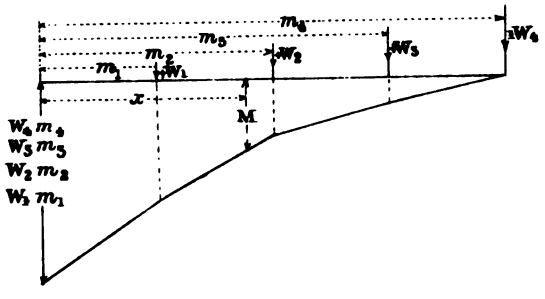
In case 4, (*Fig.* 18), the moment of the load is represented by a parabolic curve under the load, and an inclined straight line between the load and the point of fixing.

Fig. 18.



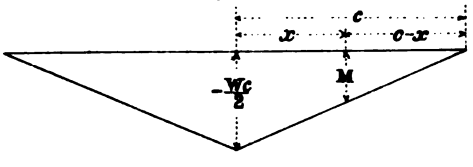
The representation of case 5 depends upon the proportion of the loads to one another. It would be somewhat as shown in *Fig.* 19.

Fig. 19.



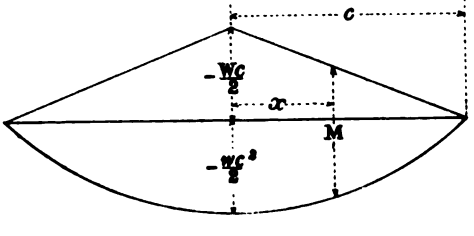
In case 6 the representation is as in *Fig.* 20.

Fig. 20.



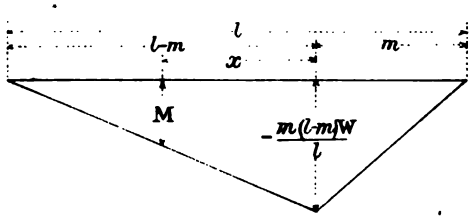
Case 7 has already been dealt with.
In case 8 the representation is as in *Fig.* 21.

Fig. 21.



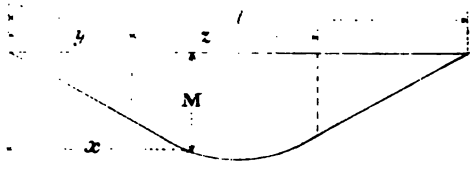
In case 9 the representation is as in *Fig.* 22.

Fig. 22.



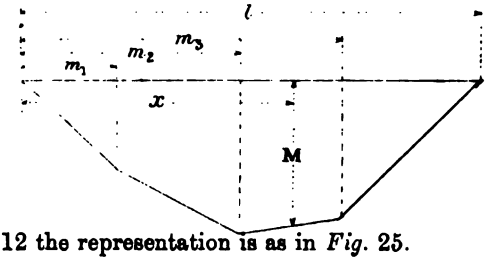
In case 10 the representation is as in *Fig.* 23.

Fig. 23.



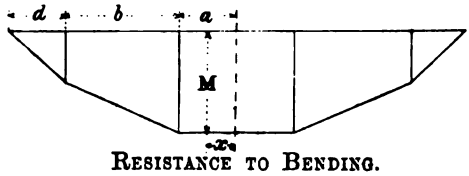
In case 11 the representation is as in *Fig.* 24.

Fig. 24.



In case 12 the representation is as in *Fig.* 25.

Fig. 25.



Resistance to Bending.

The forces which resist bending at any vertical transverse section of the beam are the internal forces excited in the different particles or fibres of the beam.

These forces, so long as the bending is slight, as in practice it always is, act sensibly in a horizontal direction, and the resistance exerted by each particle or fibre acts with a leverage equal to the vertical distance between the particle or fibre, and the horizontal axis about which the direct stresses tend to make the section rotate.

The moment of resistance of each fibre is, therefore, its direct resistance multiplied by the leverage with which it acts, and the sum of all these moments constitute the Moment of Resistance of the section.

When a piece of any material is bent by any forces, the fibres on one side are shortened or compressed, the fibres on the other are lengthened or extended.

The effect of this when the forces act vertically, is, that no transverse section, except one originally vertical, is any longer vertical, every other transverse section being now more or less inclined.*

Then in every transverse section of the piece, there is manifestly some line of fibres where there is no direct stress at all.

This line of fibres is called the neutral axis of the section, and it is about this axis, that the rotation above described takes place.

To produce and maintain the extension and compression of the fibres, which is the immediate cause of this rotation, direct forces have been and continue to be applied, which forces, when equilibrium is established, are equal to the resistance excited in the fibres.

If, then, the direct force or stress applied to each fibre is known, the resistance which that fibre exerts against further extension or compression is known also.

Now it has been determined experimentally, that so long as the elastic limit is not exceeded, the quantity of extension or compression produced in the materials ordinarily used by the Engineer is directly proportional to the force applied to produce it, and further, that the quantity of extension produced in any material by a given force is equal to the quantity of compression produced in the same material by an equal force, *i.e.*, if one ton per square inch produces an extension of $\frac{1}{10}$ inch in a bar, two tons per square inch will produce an extension of $\frac{2}{10}$ inch, and so on; and further, one ton per square inch will produce a compression of $\frac{1}{10}$ inch on the same bar, two tons per square inch will produce a compression of $\frac{2}{10}$ inch, and so on.

It has also been found by experiment (*R.C.E.*, p. 250) that in most cases which occur in practice, the longitudinal (*i.e.*, the direct) stress of the layers of a beam may, without material error, be assumed to be uniformly varying, its intensity being simply proportional to the distance of the layer from the neutral surface, *i.e.*, a fibre at an inch distant from the neutral axis is subjected to only half the stress, and exerts only half the resistance, which a fibre at two inches distance is subjected to, and exerts, and so on.

On these data it is easy to shew that the neutral axis passes through the centre of gravity of the section of the beam.

* In beams, fixed at one end, the section which remains vertical is at the point of fixing; in beams supported at both ends, and symmetrically loaded, it is at the centre of the beam, while in beams unsymmetrically loaded, its position depends upon that of the load.

The investigation of this question is given very clearly in *Fenwick's Mechanics of Construction*, p. 27.*

The Moment of Resistance.

Taking it as proved, that, within the limits of elasticity, the neutral axis passes through the centre of gravity of the section, let *Figs.* 26 and 27 represent the section and elevation of a beam (it may be of any section, provided the law of the variation of its section be known), and let O O be the neutral axis. Let Y Y represent a layer of fibres, of which the breadth is z, and thickness $d\,y$. Let the distance of this layer from the neutral axis be y, and the distance of the lower side of the beam from the neutral axis y_0. Then the area of the layer is $z\,d\,y$, and the moment of resistance of this layer of fibres round the neutral axis is $y\,z\,d\,y$, multiplied by some factor representing the actual intensity of the resistance of the material of the layer, which has to be determined by experiment.

Fig. 26.

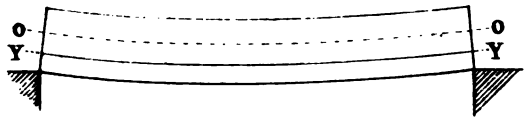
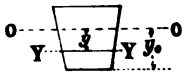


Fig. 27.



In the particular case under consideration, the outer fibres on the lower side being farthest from the neutral axis are subjected to a greater stress than any others, and these fibres (which are in tension) must not be stretched beyond their elastic limit.

Let the intensity of the stress, which will stretch the material as far as it can safely be stretched, be f_0, *i.e.*, f_0 lbs., cwts., or tons per square inch, or square foot, or other convenient unit of section.

Then because the resistances of the fibres are as their distances from the neutral axis, the intensity of the stress on the fibres at the distance y from the neutral axis is to the intensity of the stress on the fibres at the distance y_0 from the neutral axis as y is to y_0.

If f be the intensity of the stress at the distance y,

$$f : f_0 :: y : y_0$$

$$\text{and } f = \frac{f_0\, y}{y_0}$$

which is the factor previously referred to.

---

* Experiments made for the War Department by Mr. Kirkaldy have shewn, that in armour plate iron, the quantity of extension produced, by a given force, when the force is considerable but still within the elastic limit, is considerably greater than the quantity of compression produced in the same material by an equal force; and the same result will probably be given by all ductile irons.

As further experiments on other qualities of iron, and other materials are made with the improved testing machine, and more accurate data in regard to their extension and compression under equal forces obtained, it will be necessary for scientific construction, in dealing with materials which are not extended or compressed equally by equal forces, to modify the theory of the Resistance to Bending, as in such cases the neutral axis does not pass through the centre of gravity of the section, but lies nearer to that side of the section on which the alteration of form is least.

Then the total amount of this particular layer of fibres is

$$\frac{f_o\, y}{y_o}\, y\, z\, dy$$

Now this is true for every layer of fibres of the whole section, and therefore the Moment of Resistance of the whole section is

$$\int_{-y^o}^{+y^o} \frac{f_o\, y}{y_o}\, y\, z\, dy \quad \text{or}$$

$$\text{Moment of Resistance} = \frac{f_o}{y_o} \int_{-y^o}^{+y^o} y^2\, z\, dy$$

Now $\int_{-y^o}^{+y^o} y^2\, z\, dy$ is the Moment of Inertia of the section about an axis passing through its centre of gravity, and it is usually represented by the letter I, the value of which, for any particular section, is given in all the text books, and the result is the well known formula,

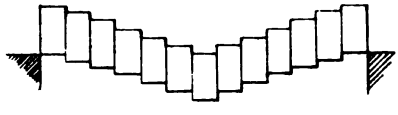
$$\text{Moment of Resistance} = \frac{f_o\, I}{y_o}$$

Having now determined both the Moment of Flexure at any section of a beam and the Moment of Resistance, and knowing that when equilibrium obtains, they are equal, it is only necessary to equate them, and by assuming some of the quantities involved, the remainder can be ascertained, *i.e.*, given the load, span and form of beam, its dimensions, so that the elastic limit may be nowhere exceeded, can be determined; or given the dimensions and load, the maximum intensity of stress in the beam can be arrived at.

The Shearing Stresses.

When a load is put upon a beam, in addition to the direct stresses, a tendency is produced at every vertical section for the part of the beam on one side of that section to slide past the part of the beam on the other side, and the beam tends to give way in the manner shown in *Fig.* 28.

Fig. 28.



This is the vertical shearing force. Its existence may be proved by means of the small instrument shown in *Figs.* 29 and 30.

Fig. 29.

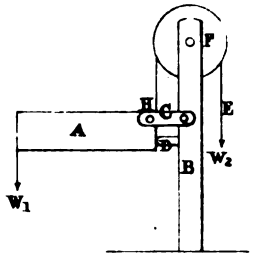
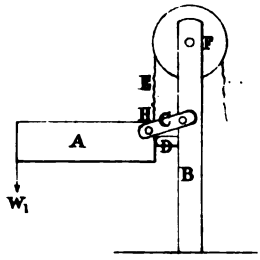


Fig. 30.



A represents a beam, which, instead of being fixed into the upright B, is connected with it by means of two loops, C, (one on each side of A), working easily on pins at each end.

The lower part of A rests against a stud D. The weights W_1 and W_2 are equal, W_1 being suspended at the outer lower end of A, and W_2 to a cord E passing over the pulley F, and attached to the inner upper end of A.

The loops, like the upper fibres of the beam, are in tension, the stud, like the lower fibres of the beam, is in compression, and so long as W_2 is attached to the line, the equilibrium is maintained as shown in *Fig.* 29.

If W_2 be removed, the beam A falls as nearly vertically, and as far, as the loops will allow it to fall, and assumes the position shown in *Fig.* 30, demonstrating the necessity of a vertical force at H to prevent this vertical sliding or shearing taking place.

The effect of the bending action is to produce also at every horizontal section a similar shearing stress, the existence of which it is much easier to demonstrate.

Before considering the horizontal shearing stress, it is better perhaps to ascertain the value of the vertical shearing streess.

The Vertical Shearing Stress.

For this, two general and very simple rules may be laid down, viz.,—

(1). In the case of beams fixed at one end; the shearing stress at any section is the total weight lying between that section and the unsupported end of the beam.

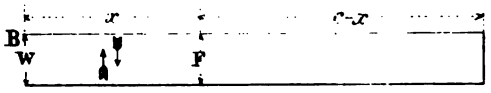
(2). In the case of beams supported at each end; the shearing stress at any section is the difference between the reaction at either point of support, and the total weight lying between that point of support and the section in question.

Thus in the cases previously taken;

Case 1. Shearing Stress, $F = W$,

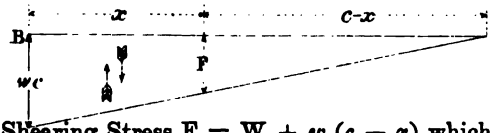
which is the same at every section of the beam, and may be represented graphically by a rectangle (*Fig.* 31,) *i.e.*, the shearing stress at every section in the beam is the same and equal to W.

Fig. 31.



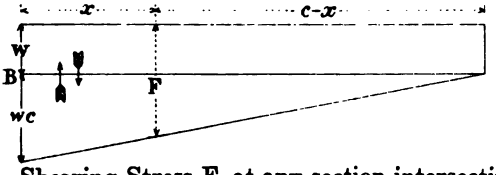
Case 2. Shearing Stress $F = w(c - x)$, which is a maximum, when $x = o$, or at the point of support B, and may be represented graphically as shown in *Fig.* 32.

Fig. 32.



Case 3. Shearing Stress $F = W + w(c - x)$ which, as before, is a maximum at the point of support B, and which may be represented graphically as shown in *Fig.* 33.

Fig. 33.

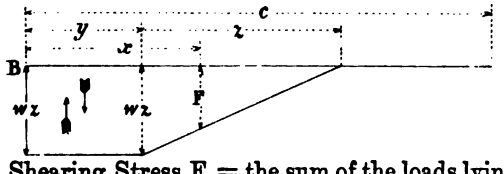


Case 4. Shearing Stress F, at any section intersecting the load,

$$= w(z + y - x)$$

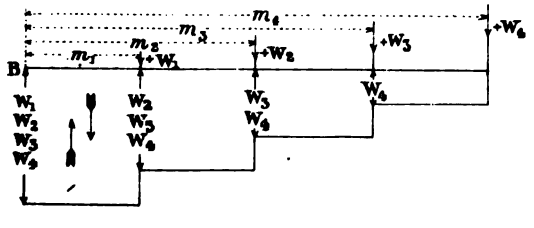
which has its greatest value when $y - x = o$, *i.e.*, when $y = x$, or at the inner end of the load and which may be represented graphically as shown in *Fig.* 34.

Fig. 34.



Case 5. Shearing Stress F = the sum of the loads lying between the section and the unsupported end of the beam, which is a maximum at the point of support B, and may be represented graphically, as shown in *Fig.* 35.

Fig. 35.

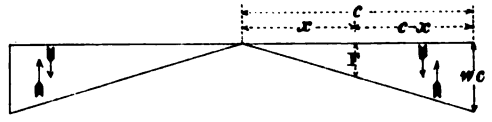


Case 6. Shearing Stress $F = \frac{W}{2}$ everywhere, except at the point of application of the load, where it is nothing. It may be represented graphically, as shown in *Fig.* 36, the arrows indicating the direction in which shearing tends to take place.

Fig. 36.

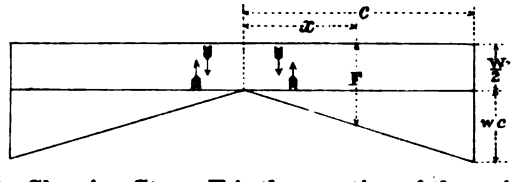
Case 7. Shearing Stress $F = w\,c - w\,(c - x)$ which is a maximum when $x = c$, *i.e.* at the points of support, and nothing at the centre, and which may be represented graphically as shown in *Fig.* 37.

Fig. 37.



Case 8. The Shearing Stress F is the sum of (6) and (7), and may be represented as shown in *Fig* 38.

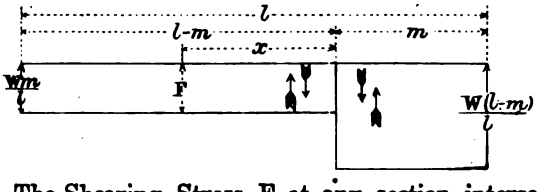
Fig. 38.



Case 9. The Shearing Stress F is the reaction of the point of support on that side of the load.

The Graphic Representation is shown in *Fig.* 39, there being no shearing stress at the load itself, where the shearing stress changes direction, as indicated by the arrows.

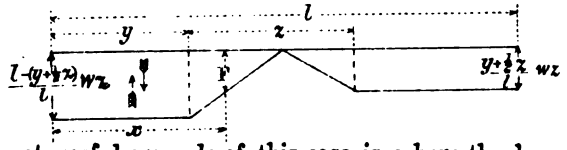
Fig. 39.



Case 10. The Shearing Stress F at any section, intersecting the partial load, $= -\frac{l - (y + \frac{1}{2}z)}{l} w z + w (x - y)$, which increases from some section under the load to each end of the load, the position of the

section where there is no shearing stress, and the rate of increase, depending upon the relative values of y, z and l. It may be represented graphically as shown in *Fig.* 40.

Fig. 40.



The most useful example of this case is, where the beam is divided into two parts and the longer part is fully loaded, the other part not being loaded at all.

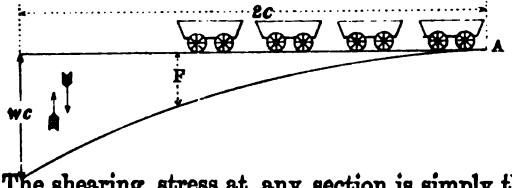
In this case $y = l - z$ and the Shearing Stress F then equals $\frac{w z^2}{2l} - w \left\{ x - (l - z) \right\}$ which is a maximum when $x - (l - z) = o$ or when the section is at the end of the load.

The maximum stress F at any section under these circumstances $= \frac{w z^2}{2 l}$ and it will be represented by the ordinate of a parabola.

If a train of carriages be supposed to come on to a bridge at the end A (*Fig.* 41) and gradually cover it, and if the train be considered as having a uniform weight per unit of length, the maximum stress at each section during this gradual loading may be represented as shown in *Fig.* 41, the extreme ordinate being half the total weight of the train.

This case is more fully explained in a subsequent paper on *Built-up Wrought Iron Beams.*

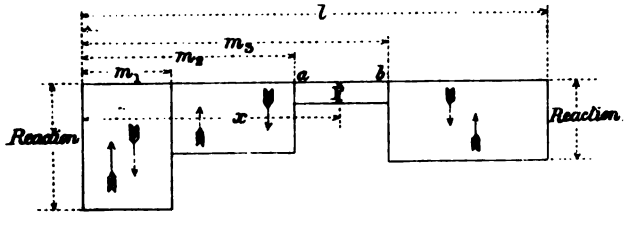
Fig. 41.



Case 11. The shearing stress at any section is simply the reaction at either point of support, minus the loads lying between the section and that point of support.

It may be represented graphically as in *Fig.* 42, but its graphic representation is not of any practical value.

Fig. 42.

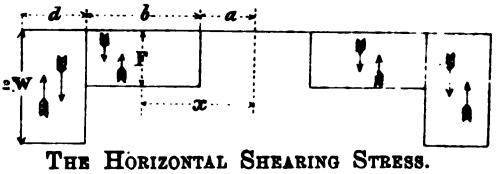


The direction, ↑↓ or ↓↑ in which the shearing stress in one space, in this case $a\ b$, acts, depends upon the relative value and position of the loads, and it may be determined as follows:—Consider the reactions as negative, and the loads as positive; beginning at either point of support deduct the loads in succession from the reaction at that point of support, and the change of direction in the shearing stress will take place at that load, where the difference changes sign.

Case 12. The Shearing Stress F at any section is the reaction at either point of support, minus the load lying between that point of support and the section.

It may be represented as shown in *Fig.* 43, there being no shearing stress between the two central loads.

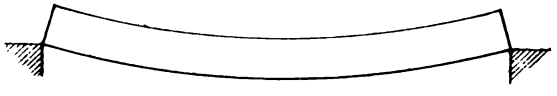
Fig. 43.



THE HORIZONTAL SHEARING STRESS.

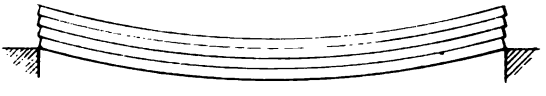
If a single piece of wood be loaded, it bends, *Fig.* 44, and nothing of

Fig. 44.



this shearing force is seen, but if this piece of wood be cut into a number of layers and loaded as before, each bends separately and the whole assumes the form shown in *Fig.* 45.

Fig. 45.



Now it is clear that there was some force exerted in the uncut piece of wood, which prevented the sliding which has taken place in the cut piece of wood, and this force can be measured by the force which, applied at the ends of the cut piece of wood, would be required to force its parts into the position of that which is uncut.

The measurement of this force is effected in this way.

Professor Rankine has shown (*A. M*, p. 88), that a shearing stress cannot exist on any plane in a body, without another shearing stress of equal intensity acting on a different plane, so that if, at any point A (*Fig.* 46), of a beam, there is a vertical shearing stress, and a horizontal

Fig. 46.

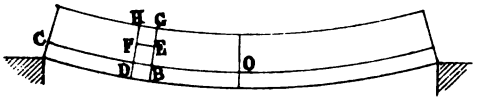
shearing stress they must be of equal intensity. Then if one is measurable, the other is known.

In Article 309, p. 338 *A. M.*, he shows how to get this stress in the following manner:—

Let H F D (*Fig.* 47) be a vertical transverse section of a beam of uniform section, G E B another very near to it.

Then if the Moment of Flexure at H F D differs from the Moment of Flexure at G E B (which it generally does), there must be a corresponding difference in the amount of the direct stress on two corresponding parts of the planes of section such as G E and H F.

Fig. 47.



In this particular case the direct stress is a thrust and is greatest at G E, *i.e.*, towards the centre of the beam.

The difference of the direct stresses at G E and F H constitutes a horizontal force acting on the solid H F E G, and in order to maintain the equilibrium of that solid, the amount of shearing stress on the plane F E must be equal and opposite to the horizontal force. The amount of the difference being divided by the area of the plane F E gives the intensity of the shearing stress.

DISTRIBUTION OF SHEARING STRESS.

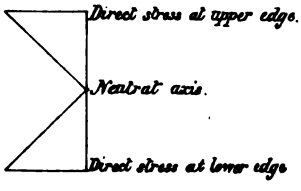
Now it is obvious from the above that the shearing stress is greatest at the neutral axis, and nothing at the upper and lower edges of the beam, but this may be demonstrated in the following manner.

The direct stresses at any section are nothing at the neutral axis and increase uniformly to the edges of the beam.

Therefore the direct stresses on any vertical layer may be represented by a pair of triangles *Fig.* 48.

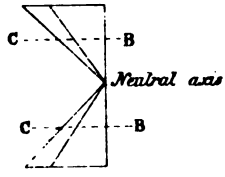
When the Moment of Flexure differs at different sections, which it does in almost every case, the direct stresses differ, and therefore at any other section the direct stresses on any vertical layer, may be represented by a pair of triangles, larger or smaller than the first pair.

Fig. 48.



Then the total difference between the direct stresses at the two sections is represented by the total difference of the four triangles *Fig.* 49, and the difference between the direct stresses on the part of the section outside of any line B C, at any other distance from the neutral axis is represented by the difference of the trapeziums outside of that line, which difference manifestly decreases as B C approaches the edge of the beam, and finally vanishes.

Fig. 49.



This is not a uniformly varying stress. It decreases slowly at first and rapidly as it reaches the outer edges.

The rate of the decrease of the shearing stress on any vertical layer of the beam may be investigated as follows:—

Let A B O, *Fig.* 50, be the triangle representing the direct stresses on one side of the neutral axis at a vertical layer M N, one unit thick, of a beam, *Fig.* 51, and let C B O be the triangle representing the direct stresses on the same side of the neutral axis at the section Q R, one unit thick in the same beam.

Fig. 50.

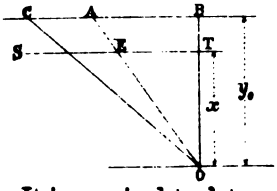
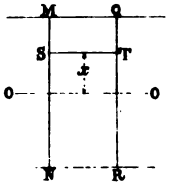


Fig. 51.



It is required to determine the value of the shearing stress on any plane S T, one unit thick, distant x from the neutral axis O O, the direct stresses on the two vertical sections M N and Q R, being given.

Let O B $= y_o$

Then area of trapezium C S T B

$$= \frac{\text{C B} + \text{S T}}{2} (y_o - x)$$

and the area of trapezium A E T B

$$= \frac{\text{A B} + \text{E T}}{2} (y_o - x)$$

and the difference of these two, viz.,

$$\frac{\text{C A} + \text{S E}}{2} (y_o - x) \qquad (1)$$

is the shearing stress on any plane, S T, distant x from the neutral axis.

Now C A : S E :: O B : O T :: y_o : x

$$\text{whence } S E = \frac{C A x}{y_0}$$

Substituting this in (1), the shearing stress on S T

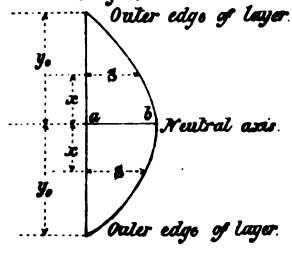
$$= \frac{C A + C A \frac{x}{y_0}}{2} (y_0 - x)$$

$$= \frac{C A}{2} \times \frac{y_0 + x}{y_0} (y_0 - x)$$

$$= \frac{C A}{2} \frac{y_0^2 - x^2}{y_0}$$

As this is the equation to a parabola, when the origin of co-ordinates is at a point in the axis, whose distance from the apex represents the shearing stress on U V, *i.e.*, the area of the triangle C A O, the length of the ordinate at that point being y_0, the distribution of the shearing stress over a vertical layer may be represented graphically as in *Fig.* 52, in which $a\,b$ represents the shearing stress on U V, *i.e.*, at the neutral axis, S represents the shearing stress on S T distant x from the neutral axis, and the area of the parabola represents the total shearing stress on the layer.

Fig. 52.



The complete investigation of this question is given in *Rankine's Applied Mechanics*, p. 339.

Resistance to Shearing.

It is sufficient, here, to say that the resistance to shearing in any material is taken as directly proportional to the surface subjected to the action of the shearing stress.

In proportioning the web of a wrought iron plate girder, it is, however, necessary specially to consider the mode in which the shearing stresses are conveyed through the web to the points of support.

The investigation of this is given by Mr. Unwin (*Iron Bridges and Roofs*, p. 29,) and the application of it is given in a subsequent paper on *Built-up Wrought Iron Beams.*

Results.

1. In all cases, the Moment of Flexure at every point is a maximum, when the maximum load, which the beam is intended to bear, is imposed upon it.

2. In beams fixed at one end, the Moment of Flexure is always a maximum at the point of fixing.

3. In beams supported at both ends, and symmetrically loaded, part of the load being applied at the centre, the Moment of Flexure is a maximum at the centre of the beam.

Where the load is symmetrical, no part of it being applied at the centre, the Moment of Flexure is constant between the two weights nearest to the centre. (Case 12.)

4. In beams supported at both ends and loaded unsymmetrically with a single weight, the Moment of Flexure is a maximum at the point of application of that weight.

5. In beams supported at both ends and loaded unsymmetrically, either with a partial uniformly distributed load, or with several loads, the point, where the Moment of Flexure is a maximum, depends upon the amount and position of the load.

6. In beams fixed at one end, the Moment of Flexure vanishes at the outer end of the beam.

7. In beams supported at both ends, the Moment of Flexure vanishes at the points of support.

8. Within the elastic limit, the extension or compression of any of the ordinary materials used by the Engineer, under any direct stress, is directly proportional to that stress, and under equal stresses, the extension is equal to the compression.

9. Within the elastic limit, the direct stress on the fibres of a bent beam increases from the neutral axis, where there is no direct stress, at a uniform rate to the edges of the beam, the rate of increase of the stress on the fibres in tension being the same as the rate of increase of the stress on the fibres in compression.

10. Within the elastic limit, the neutral axis passes through the centre of gravity of each vertical section of a bent beam.

11. The expression for the Moment of Resistance of any section of beam is $\frac{f_o I}{y_o}$, where f_o is the safe stress per unit of surface, or intensity of stress, on the outer fibres of the section, on that side of the neutral axis, where the elastic limit is first reached; y_o is the distance between the neutral axis and the outer fibre of the section on that same side, and I is the Moment of Inertia of the section about a horizontal axis passing through the centre of gravity of the section.

12. The vertical shearing stress, at any section of a beam fixed at one end, is the total weight lying between that section and the unsupported end of the beam.

13. The vertical shearing stress at any section of a beam, supported at both ends, is the difference between the reaction at either point of support, and the total weight lying between that point of support and the section in question.

14. In all beams, however fixed or supported, the shearing stress is nowhere greater than at the points of fixing or support.

15. In beams fixed at one end, the shearing stress vanishes at the outer end of the beam.

16. In beams, supported at both ends, and symmetrically loaded, part of the load being applied at the centre, the shearing stress vanishes and changes direction at the centre of the beam.

17. In the same beams, when the load is symmetrical, no part of it being applied at the centre, there is no shearing stress between the two central loads. Case 12.

18. In the same beams subjected to a load, having a uniform weight per unit of length, rolling on from one end, the maximum shearing stress at any section is produced, when the longer part into which the section divides the beam is fully loaded, and the shorter part is not loaded at all.

19. The horizontal shearing stress on a vertical layer of any section, and consequently the vertical shearing stress, is greatest at the neutral axis. They decrease slowly at first and afterwards more rapidly to the outer edges of the section.

20. Except in very large structures, the shearing stress may be treated as uniformly distributed over the vertical section.

21. The resistance to shearing stress is taken as directly proportional to the surface subjected to the action of that force.

DEFLECTION OF BEAMS.

All beams used by the Engineer must be stiff enough as well as strong enough for the work they have to do.

In the Paper on *Stress in Beams* it has been shewn how to determine the dimensions necessary for strength; in the present Paper it is proposed to shew how the dimensions necessary for stiffness may be ascertained.

The investigation of the formula for deflection requires a knowledge of the Calculus, but the principles on which the investigation is carried out can be understood without such knowledge.

Very complete formulæ for deflection are given by Professor Rankine in his *Applied Mechanics*, p. 322 and 328, in his *Civil Engineering*, p. 273, and in his *Useful Rules and Tables*, p. 222, and his formulæ provide for all cases which usually arise in practice.

There is, however, one case of unfrequent occurrence, not given by Professor Rankine, but given by Professor Moseley, viz., the case of a beam of uniform section loaded at one point, not the centre. This is to be found in *Moseley's Principles of Engineering and Architecture*, p°. 509 to 513, Edition 1855, and the result is given at the end of this Paper.

As regards the principles on which deflection is calculated:

When a beam is loaded transversely, stresses of compression and tension are induced, as well as shearing stresses. All these stresses produce deflection of the beam. The deflection, however, due to the shearing stresses is so small, that it may be practically neglected. This is shewn in *Applied Mechanics*, p. 324, Art. 312.

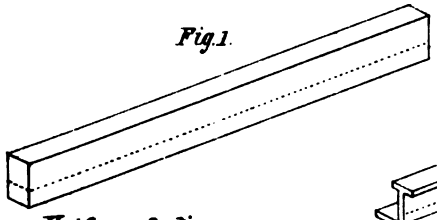
It remains then only to determine the deflection caused by the direct stresses, *i.e.*, by the tensions and compressions.

The deflection is measured by the deflection of the neutral line of the beam—that neutral line being a line passing through the neutral axis of every section of the beam.

When the beam deflects, this line, if originally straight, becomes curved, or, if originally curved, alters its curve.

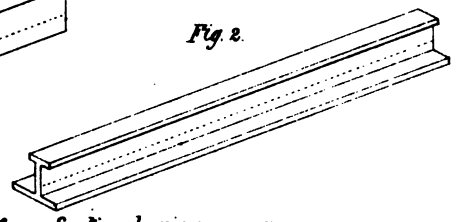
Figs. 1 to 6 show the position of the neutral line in beams of uniform section, where it is horizontal—in beams of uniform strength and uniform depth, where it is also horizontal—in beams of uniform strength and uniform breadth, where it is a curve.

Fig. 1.



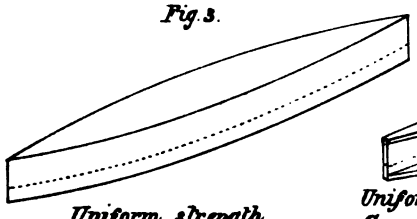
Uniform Section.

Fig. 2.



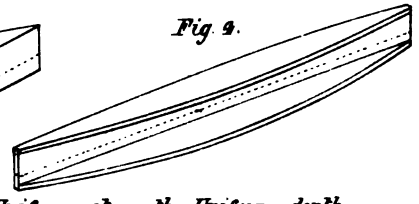
Uniform Section having a cross section of equal strength.

Fig. 3.



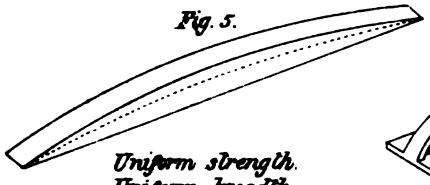
Uniform strength. Uniform depth.

Fig. 4.



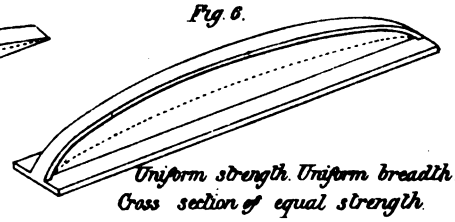
Uniform strength. Uniform depth. Cross section of equal strength.

Fig. 5.



Uniform strength. Uniform breadth.

Fig. 6.



Uniform strength. Uniform breadth. Cross section of equal strength.

If the nature of the deflection curve can be ascertained, by referring it to a pair of rectangular co-ordinates x and v, *i.e.*, if the equation to the curve can be determined, the deflection at any point, and consequently at the point where it is greatest, can be calculated.

Let A D in *Figs.* 7 and 8 represent the neutral line of a bent beam, *Fig.* 7 representing that of a beam fixed at one end, and loaded in any manner whatever, *Fig.* 8 representing that of a beam supported at both ends, and loaded symmetrically on either side of the centre, but otherwise in any manner whatever. These are the cases of most usual occurrence.

Fig. 7.

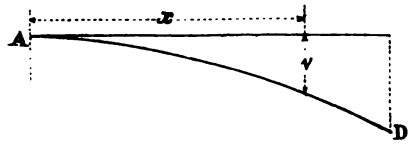
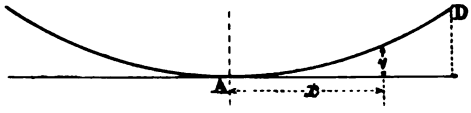


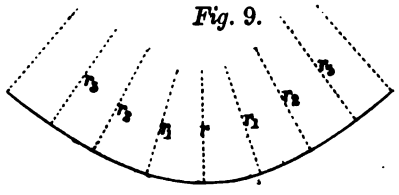
Fig. 8.



Then it has been shown *Stress in Beams*, p. 4, *et seq.*, that the section where the greatest direct stresses occur is in each case at A.

Let the dotted lines (*Fig.* 9) represent the radii of curvature of the curve at the different points on the supposition that the curve is other than a circle.

Fig. 9.



Then it is shown in all the text books, that at any section of a bent beam, the curvature, which is the reciprocal of the radius of curvature, is

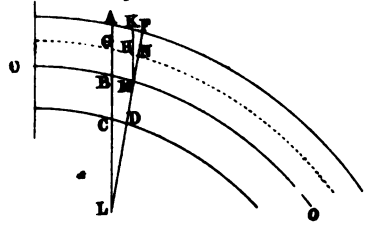
$$\frac{1}{r} = \frac{M}{EI}$$

r being the radius of curvature at the same point in the beam, where M is the moment of flexure, and I the moment of inertia.

This is obtained in the following way:

Let A C, D F be two sections of a bent beam very near together, and let O O be the neutral axis of the beam; then, as the sections are very close together, L M will be the common radius of curvature.

Fig. 10.



Also the fibre B M not only is the same length as it was originally, before the beam was bent, but it is the same length that all the other fibres A F, G N, and C D were.

From M draw M H K parallel to A C.

Then H N will be the elongation which has been produced upon the fibre G H, at a distance M N (y) from the neutral axis.

Now by similar triangles

$$\text{L M} : \text{B M} :: \text{M N} : \text{N H}. \qquad (1)$$

or, in words, the radius of curvature (r) is to the original length B M of the fibres, as the distance from the neutral axis (y) is to the elongation of the fibre at that distance.

Now it is known that the force per unit of section required to produce a given elongation is directly as that elongation, and the force per square inch of section required to produce on B M an elongation of B M in length is the modulus of elasticity, E, of the material.

Therefore, calling the direct stress per square inch at the fibre G H p,

$p : \text{E} :: \text{N H} : \text{B M}$ (all dimensions being in inches, p and E in lbs.), and substituting this in (1)

$$r : \text{E} : y : p$$

$$r = \frac{\text{E}\,y}{p} \qquad (2)$$

Now the direct stress at any extreme layer is to be obtained from the equation

$$\text{M} = \frac{f_0\,\text{I}}{y_0} \text{ (\textit{Stress in Beams}, p. 17.)}$$

where f_0 is the stress in lbs. per square inch at the extreme fibre, and y_0 its distance from the neutral axis (also in inches),

$$i.e., f_0 = \frac{\text{M}\,y_0}{\text{I}}$$

$$\text{and as } p : y : f_0 : y_0, p = \frac{f_0\,y}{y_0} = \frac{\text{M}\,y}{\text{I}}$$

Substituting this value of p in (2)

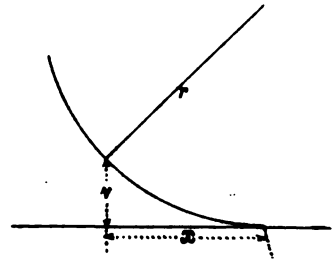
$$r = \frac{\text{E}\,y}{\frac{\text{M}\,y}{\text{I}}} = \frac{\text{E I}}{\text{M}} \qquad \text{or } \frac{1}{r} = \frac{\text{M}}{\text{E I}}$$

There is a way of obtaining the deflection from this equation by graphic construction, which is described in *Rankine's Applied Mechanics*, p. 326, and in *Rankine's Civil Engineering*, p. 269; but it is far more troublesome and not nearly so accurate as the method by calculation, and therefore it is not recommended.

Now the radius of curvature of any curve is known by the differential calculus in terms of rectangular co-ordinates, x and v, to be

$$r = \frac{\left\{1 + \left(\frac{dv}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2v}{dx^2}}$$

Fig. 11



and as in the case of beams in practice the deflection is always small, the value of the square of the tangent to the curve, $\left(\frac{dv}{dx}\right)^2$ is at all points so small as to be inappreciable, and may be neglected, so that

$$r = \frac{1}{\frac{d^2v}{dx^2}} \text{ and } \frac{1}{r} = \frac{d^2v}{dx^2}$$

and the equation now becomes

$$\frac{d^2v}{dx^2} = \frac{M}{EI} \qquad (3)$$

which is the general equation to any deflection curve, the loading, form of beam, and modulus of elasticity of the material being known.

The principle on which the deflection of all beams, fixed at one end and loaded in any way whatever, or supported at both ends and loaded symmetrically in any way whatever, is determined, is the same.

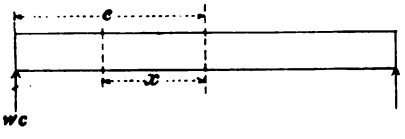
The integration of the equation, however, differs for different forms of beams, *i.e.*,—

1st. For beams of uniform section.
2nd. Do. of uniform strength and uniform depth.
3rd. Do. of uniform strength and uniform breadth.

Beams of Uniform Section.

In beams of uniform section, the moment of inertia, I, is constant, and the general equation (3) can be integrated at once by inserting the value of M for any point distant, x, from the origin of co-ordinates, which value contains the variable x only—*i.e.*, for example, in a beam supported at both ends and loaded with a uniform load.

Fig. 12.



$$M = w\frac{(c^2 - x^2)}{2}$$

$$\text{and } \frac{d^2v}{dx^2} = \frac{w}{2EI}(c^2 - x^2)$$

an equation easily integrated.

Beams of Uniform Strength and Uniform Depth; and Beams of Uniform Strength and Uniform Breadth.

In these beams the cross section is variable, and the moment of inertia, I, is different at every section of the beam.

The term $\frac{M}{EI}$ therefore contains two variables, viz., M the moment of flexure, and I the moment of inertia.

It is necessary, therefore, in these cases, before the general equation can be integrated, to find an equivalent value for I, which shall make the general equation true for every section in the beam.

Let M_0 be the moment of flexure and I_0 be the moment of inertia of the beam at the section of greatest stress, and let M be the moment of flexure and I be the moment of inertia at any other section.

Then (*Stress in Beams*, p. 17)

$$M_0 = \frac{f_0}{y_0} I_0, \text{ and } M = \frac{f_0}{y} I$$

$$i.e., M_0 : M :: \frac{f_0 I_0}{y_0} : \frac{f_0 I}{y}$$

f_0 being the limiting intensity of stress at the outside of the beam, which in beams of this kind is the same at every section, y_0 being the distance from the neutral axis to that outside the beam, where the limiting intensity of stress is attained at the section of greatest stress, and y being the distance from the neutral axis to that outside of the beam, where the limiting intensity of stress is attained, at any other section.

Then from this proportion,

$$I = \frac{M \times I_0 \times y}{M_0 \times y_0}$$

and the general equation (3) becomes

$$\frac{d^2 v}{dx^2} = \frac{M_0 y_0}{E I_0 y} \tag{4}$$

IN BEAMS OF UNIFORM STRENGTH AND UNIFORM DEPTH,

the neutral line is horizontal, $y_0 = y$, and equation (4) becomes

$$\frac{d^2 v}{dx^2} = \frac{M_0}{E I_0} \tag{5}$$

an equation still more easily integrated than that in the first case, and which shows that the radius of curvature is uniform, and, consequently, that under any load whatever (provided that it is symmetrical on each side of the centre) the deflection curve is an arc of a circle.

IN BEAMS OF UNIFORM STRENGTH AND UNIFORM BREADTH,

using the same notation as before, and calling h_0 and h the total depths of the beam at the section of greatest stress, and at any other section respectively,

$$M_0 = \frac{f_0 I_0}{y_0} \text{ and } M = \frac{f_0 I}{y} \text{ as before,}$$

where f_0 is constant. Therefore,

$$\frac{M_0 y_0}{I_0} = \frac{M y}{I} \tag{6}$$

$$\text{and } \frac{y_0}{y} = \frac{M I_0}{M_0 I}$$

Also in beams of this form

$$h_0 : h :: y_0 : y$$

$$\text{and } I_0 : I :: n' b h_0^3 : n' b h^3 *$$

where b is the constant breadth and n' a constant factor,

$$\therefore I_0 : I : y_0^3 : y^3$$

$$\frac{I_0}{I} = \frac{y_0^3}{y^3}$$

Substituting this in (6),

$$\frac{y_0}{y} = \frac{M}{M_0} \times \frac{y_0^3}{y^3}$$

$$\text{whence } \frac{y_0}{y} = \sqrt{\frac{M_0}{M}} \qquad (7)$$

Substituting this again in (4)

$$\frac{d^2 v}{d x^2} = \frac{M_0 \sqrt{M_0}}{E I_0 \sqrt{M}} \qquad (8)$$

an equation easily integrated by substituting the values of M_0, the moment of flexure at the section of greatest stress, and of M, the moment of flexure at any other section, E and I_0 being constant

Beams under a known Maximum Intensity of Stress.

When beams have been designed to bear any given maximum intensity of stress, the equation may be put in a somewhat different form.

Let f_0 be the limiting intensity of stress as before, and y_0 the distance from the neutral axis to the outside of the beam on that side, where the limiting intensity of stress is attained, at the section of greatest stress. Let f be the actual intensity of stress at the outside of the beam at any other section, and y the distance from the neutral axis to the outside at that section; both being on the same side of the neutral axis as f_0 and y_0.

Then as before

$$M_0 = \frac{f_0 I_0}{y_0} \text{ at the section of greatest stress,}$$

$$\text{and } M = \frac{f I}{y} \text{ at any other section.}$$

$$\text{whence } f = \frac{f_0 M y I_0}{M_0 I y_0}$$

From (3)

$$\frac{d^2 v}{d x^2} = \frac{M}{E I} = \frac{f}{E y}$$

* It is to be observed that this is strictly true in the case of solid beams having triangular, trapezoidal, or rectangular sections, as well as in the case of hollow rectangular beams or I shaped beams, in which the thickness of the material (d) in the vertical web or webs is constant and the external and internal heights, h, h_1, bear a constant ratio to one another,

It is not strictly true for the ordinary sections of either cast or wrought iron beams; but the error introduced into the result, by regarding it as true, is very small, and on the side of safety.

Substituting the value of f, as above obtained

$$\frac{d^2v}{dx^2} = \frac{f_0}{E y_0} \frac{M I_0}{M_0 I} \tag{9}$$

IN BEAMS OF UNIFORM SECTION,

I is uniform and the equation becomes

$$\frac{d^2v}{dx^2} = \frac{f_0}{E y_0} \frac{M}{M_0} \tag{10}$$

IN BEAMS OF UNIFORM STRENGTH AND UNIFORM DEPTH,

as $M = \frac{f_0 I}{y_0}$, and f_0 and y_0 are constant, $\frac{M}{I}$ is constant at every section, $\frac{M I_0}{M_0 I}$ = unity, and the equation becomes $\frac{d^2v}{dx^2} = \frac{f_0}{E y_0}$ (11)

which shows, as before, that in beams of this kind the curvature is uniform and the deflection curve an arc of a circle.

IN BEAMS OF UNIFORM STRENGTH AND UNIFORM BREADTH,

it has already been shown (6) that

$$\frac{y_0}{y} = \frac{M I_0}{M_0 I}$$

It has also been shown (7) that in these beams (the breadth being constant)

$$\frac{y_0}{y} = \sqrt{\frac{M_0}{M}}$$

$$\therefore \frac{M I_0}{M_0 I} = \sqrt{\frac{M_0}{M}}$$

Substituting this in (9)

$$\frac{d^2v}{dx^2} = \frac{f_0}{E y_0} \sqrt{\frac{M_0}{M}} \tag{12}$$

an equation in which both M_0 and M can be expressed in terms of x; the other factors being constant, and which is easily integrated.

BEAMS WITH CROSS SECTIONS OF EQUAL STRENGTH.

These beams are those which are of such a form that the limiting stress is reached on each outside of the beam at the same instant.

As an example, take a cast iron double-T girder. Then, if the iron be of such a nature that a stress of 7·5 tons per square inch can be put on it in compression with safety, while a stress of only 1·5 tons per square inch can be put on it in tension, the metal, for a cross section of equal strength, should be arranged so that the extreme fibres in compression are five times as far from the neutral axis as the extreme fibres in tension.

The stress on the fibres being directly as their distance from the neutral axis, it is evident that the outside fibres on each side reach their limiting stress at the same moment.

In this case calling f_a the limiting stress on one side, f_b the limiting stress on the other, y_a the distance on one side, y_b on the other.

$$f_a : f_b : y_a : y_b$$
$$f_a : f_a + f_b : y_a : y_a + y_b$$
$$\frac{f_a}{y_a} = \frac{f_a + f_b}{y_a + y_b} = \frac{f_a + f_b}{h}$$ at any section, where h is the total depth at that section.

At the section of greatest stress, $\frac{f_a}{y_a} = \frac{f_b}{y_b}$ becomes $\frac{f_o}{y_o}$ and h becomes h_o, whence $\frac{f_o}{y_o} = \frac{f_a + f_b}{h_o}$, which can be substituted for $\frac{f_o}{y_o}$ in equations 10, 11, and 12.

It is by the integration of the foregoing equations, for beams under different loads, that Professor Rankine obtains the formula and tables given in p. 332 and 329 *Applied Mechanics*, and p. 273 *Civil Engineering*, and in *Useful Rules and Tables*, p. 222.

These results may be tabulated, as follows:

For deflection under any load

$$V = \frac{n''' W c^3}{E I} \qquad (13)$$

where V is the maximum deflection, n''' is the factor given in the following table, W the total load, c the total length in beams fixed at one end, and half the length in beams supported at each end, I the moment of inertia at the section of greatest stress, *i.e.*, the point of fixing in beams fixed at one end, and the centre of the beam in beams supported at both, all dimensions being in inches.

Uniform Cross Section.	n''' Factor for Deflection.
I. Fixed at one end, loaded at the other,.....	$\frac{1}{3}$
II. Fixed at one end, loaded uniformly,.....	$\frac{1}{8}$
III. Supported at both ends, loaded in the middle,.....	$\frac{1}{6}$
IV. Supported at both ends, uniformly loaded,.....	$\frac{5}{48}$
Uniform Strength and Uniform Depth.	
V. Fixed at one end, loaded at the other,.....	$\frac{1}{2}$
VI. Fixed at one end, loaded uniformly,.....	$\frac{1}{4}$
VII. Supported at both ends, loaded in the middle,.....	$\frac{1}{4}$
VIII. Supported at both ends, loaded uniformly.....	$\frac{1}{8}$

UNIFORM STRENGTH AND UNIFORM BREADTH.

IX. Fixed at one end, loaded at the other,..... $\frac{2}{3}$

X. Fixed at one end, loaded uniformly,..... $\frac{1}{2}$

XI. Supported at both ends, loaded in the middle,..... $\frac{1}{3}$

XII. Supported at both ends, uniformly loaded,..... 0·1427

For deflection under Proof load, or any load giving a known limiting stress,

$$V = \frac{n'' f_o c^2}{E y_o} \text{ for all beams;} \qquad (14)$$

$$V = \frac{n'' (f_a + f_b) c^2}{E h_o} \text{ for beams with cross sections of equal strength;} \qquad (15)$$

where n'' is the factor given in the following table, f_o the limiting stress on the weakest side of the beam, y_0 the distance of the neutral axis from the outside of the beam on that weakest side at the section of greatest stress, f_a, f_b, the limiting stresses in compression and tension, and h_0 the total depth at the section of greatest stress.

UNIFORM CROSS SECTION.	n'' Factor for Deflection.
I. Constant Moment of Flexure,.....	$\frac{1}{2}$
II. Fixed at one end, loaded at the other,.....	$\frac{1}{3}$
III. Fixed at one end, loaded uniformly,.....	$\frac{1}{4}$
IV. Supported at both ends, loaded in the middle,.....	$\frac{1}{8}$
V. Supported at both ends, loaded uniformly.....	$\frac{5}{12}$
UNIFORM STRENGTH, UNIFORM DEPTH.	
VI. Supported and loaded in any manner,.....	$\frac{1}{2}$
UNIFORM STRENGTH, UNIFORM BREADTH.	
VII. Fixed at one end, loaded at the other,.....	$\frac{2}{3}$
VIII. Fixed at one end, loaded uniformly,.....	1
IX. Supported at both ends, loaded in the middle,.....	$\frac{2}{3}$
X. Supported at both ends, loaded uniformly,.....	0·5708.

The deflections given by these formulæ are, in the cases of beams of uniform strength supported at both ends, somewhat greater than would be given by actual experiment, because, in these beams, it is always necessary to add more material to the point of support than the theory assumes to be present; such additional material being necessary to resist the shearing stresses.

In the case referred to at p. 28, *i.e.*, the deflection of a beam of uniform section supported at both ends, and loaded with a single load not at the centre, the deflection is given by two equations:

$$v = \frac{P_1}{E\,I}\left\{\frac{1}{2}a_1x^2 - \frac{1}{6}x^3\right\} + x\frac{P_3a_1a_2(a_2-a_1)}{3\,E\,I\,a} \qquad (16)$$

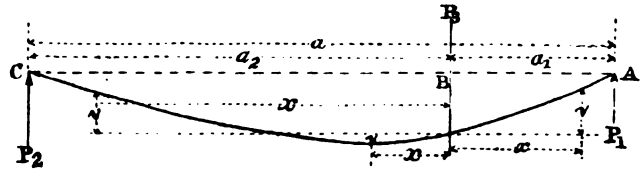
$$\text{and } v = \frac{P_2}{E\,I}\left\{\frac{1}{2}a_2x^2 - \frac{1}{6}x^3\right\} - x\frac{P_3a_1a_2(a_2-a_1)}{3\,E\,I\,a} \qquad (17)$$

(16) being applicable to the part AB (*Fig.* 13) of the beam, and (17) being applicable to the part BC (*Fig.* 13) of the beam; the origin of co-ordinates in both cases being at the point of application of the load P_3, and the co-ordinates being as shewn in *Fig.* 13.

The deflection at the point of application of the load $= \frac{P_3(a_1a_2)^2}{3\,E\,I\,a}$

and the maximum deflection below the line AC is at a point between B and C, whose distance from $B = a_2 - \sqrt{\frac{1}{3}a_2(a_2-2a_1)}$

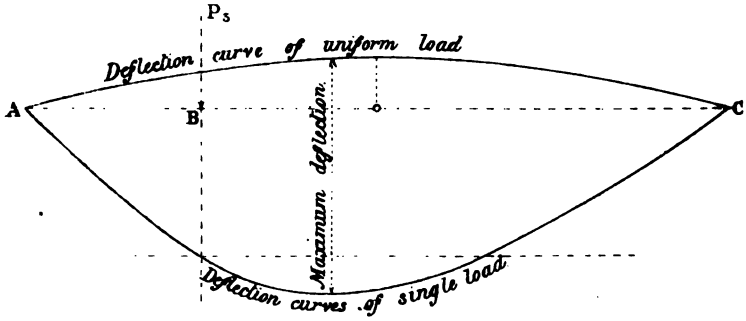
Fig. 13.



Where a beam is under more than one load, such for instance as a single load as well as a uniformly distributed load, the deflection at any point is given by adding together the deflections at that point due to each load; and when the point of maximum deflection caused by each of the loads is the same, the total maximum deflection is given by adding the maximum deflection due to each load together.

When the points of maximum deflection due to two loads on one beam considered separately do not coincide, as for instance, in a beam loaded with a uniform load as well as a single load not at the centre, in which case the point of maximum deflection for the uniform load is at the centre, while that for the single load is between the centre and the load, the position of the point of maximum deflection varies with the relation of the two loads to one another, and can be most easily ascertained by laying down the deflection curves of each with ordinates to an exaggerated scale (*Fig.* 14), and measuring the actually longest joint ordinate.

Fig. 14.


P5
Deflection curve of uniform load
A
B
C
Maximum deflection.
Deflection curves of single load

TIMBER BEAMS.

It has already been stated that all beams for permanent structures, of whatever material, must be both sufficiently strong and sufficiently stiff.

Sufficient strength consists in the beam being able to sustain permanently the stresses induced by the load for which it is intended, the elastic limit not being exceeded in any fibre.

Sufficient stiffness consists in the beam being able to sustain permanently these same stresses without deflecting more than a certain proportion of its length, this proportion being an arbitrary factor deduced from experience.

In timber beams supported or fixed at both ends this is usually $\frac{1}{360}$ in. per foot run of length, $\frac{1}{4\,8\,0}$, (*Tredgold's Carpentry, p.* 48) which would be equivalent to $\frac{1}{1\,4\,0}$ in beams fixed at one end.

There is nothing which destroys the appearance of a room more than excessive deflection of the ceiling, and every one, in these days of cheap houses, has felt the inconvenience and discomfort of having everything in a room set shaking by a person walking across an insufficiently stiff floor.

Timber beams, in practice, are always either square, rectangular, or circular in section, generally one of the two first forms in all works except those of a temporary character, such as scaffolding, and military bridges.

It is seldom worth while, for the purpose of economy, to cut timber beams, intended to carry heavy weights, to any but square or rectangular sections; and these forms have the advantage of allowing the timber to be cut as nearly as possible in a direction parallel to the longitudinal fibres of the wood.

As the strength of beams of similar forms, equal lengths, and under equal loads, increases as the breadth and as the square of the depth, while their stiffness increases as the breadth and as the cube of the depth, and their resistance to shearing as their sectional area, it is plain that the deeper a beam can be made, provided its sectional area is large enough to resist the shearing forces, and provided it is broad enough to have sufficient stiffness laterally, and to allow of its being properly secured, the smaller its sectional area will be and the less the quantity of material in it. In many cases, however, it is more economical not to have deep beams, as for instance, in a house with a double floor, where every inch added to the depth of the beam entails an additional inch of height to the walls of the building, and where, therefore, it may be cheaper to employ broader and shallower, or more numerous girders.

In determining the dimensions of a timber beam for any purpose it is

convenient first, to fix its breadth, then to calculate its depth for strength and stiffness, and to adopt the greater of the two values thus obtained.

It may be here said, that it is customary to omit altogether the consideration of the shearing stress in timber beams, an omission which may be made with perfect safety in all beams, of any material, having a rectangular or circular section.

As the direct stresses increase from nothing at the neutral axis uniformly to the outer edges of the beam, there is always near the neutral axis a mass of material which has little or no direct stress to bear, and this material is precisely in the position where the shearing stress is greatest, so that, provided the whole sectional area is sufficient to bear the shearing stress, which in these beams it always is, there can be no tendency to fail by shearing, and the consideration of this stress may be omitted.

In calculating the dimensions, necessary for strength, of a timber beam, the most scientific method would be to determine on which side of the beam the fibres would soonest reach their elastic limit, substitute the value of the safe stress per unit of section, of the nature (compression or tension) which that side could safely be called upon to bear, and the value of y_0 for that side (*Stress in Beams*, p. 17) in the expression for the Moment of Resistance, *i.e.*,

$$\frac{f_0 \, I}{y_0}$$

and equate this with the Moment of Flexure.

It is, however, customary with timber beams of rectangular or circular sections, instead of this, to substitute for f_0, which is the safe direct stress per unit of surface in tension or compression, according as the material is stronger under compression or tension, a factor called the Modulus of Rupture, which has been derived from experiments, and is given in *Useful Rules and Tables*, p. 199, dividing it by a factor of safety proper to the material, which in timber is 5 for a dead load and 10 for a live load (*Useful Rules and Tables*, p. 205); and to take y_0 as $\frac{1}{2}$ the depth of the beam.

This substitution of the Modulus of Rupture, cannot be so accurate as the more scientific method, because it proceeds on certain assumptions, which are certainly not true; but the result is accurate enough for the purposes of the Engineer in dealing with such a material as timber.

EXAMPLE I.

A fir beam (red pine) of rectangular section and average quality, is built into a wall at one end, and uniformly loaded with a weight of 192 lbs. per foot run; dead load. The length of the projecting part of the beam is 6 feet, its width is 4 inches.

* The principle is fully stated here; but in timber, the fibres on the compression side invariably reach their elastic limit first.

Determine;

1st, its depth at every foot, for uniform strength, allowing a proper factor of safety, and neglecting its own weight.

2nd, its deflection, when it is cut to this form.

3rd, its deflection, supposing its section throughout to be the same as the section at the wall.

4th, the depth necessary to limit the deflection to $\frac{1}{240}$ of the length, the section being uniform throughout.

Remarks.

This is the case of a beam for a balcony or gallery.

The equal distribution of the load on all the beams would be ensured by framing their ends together, and the weight of the beam is included in the total weight. The load is considered as dead, as it is calculated for a number of persons so packed on the balcony that they could not come on suddenly, or move when on. The breadth of four inches is taken as sufficient for lateral stiffness.

It illustrates, by showing how the deflection is increased, the disadvantage of cutting the beam to the longitudinal section proper for "uniform strength."

Preliminaries.

Let d inches = depth at any point distant x inches from fixed end A of beam.

„ b „ = breadth = 4 inches.

„ c „ = length = 72 inches.

„ w lbs. = weight on each inch in length of the beam = $\frac{192}{12}$ lbs. = 16 lbs.

„ f_o lbs. = Modulus of Rupture per square inch of section for fir (Red pine or Scotch fir) of average quality (*Useful Rules and Tables*, p. 199) = 8,400 lbs.

„ E lbs. = Modulus of Elasticity for the same class of fir (*Useful Rules and Tables*, p. 196) = 1,680,000 lbs.

„ 5 = Factor of safety proper for timber with a dead load (*Useful Rules and Tables*, p. 205.)

1. The Moment of Flexure, at any section of the beam distant x from A, $= \frac{w\,(c-x)^2}{2}$ (*Stress in Beams*, p. 5) (1)

The Moment of Resistance at any point of the beam $= \frac{\frac{f_o}{5}\,I}{y_o}$ (2)

where I = Moment of Inertia of the section of the beam about a horizontal axis passing through its Centre of Gravity, *i.e.*, (in a rectangular section) $\frac{1}{12}\,b\,d^3$ (*Useful Rules and Tables*, p. 218.)

$y_o = \frac{1}{2}\,d$

5 = Factor of Safety.

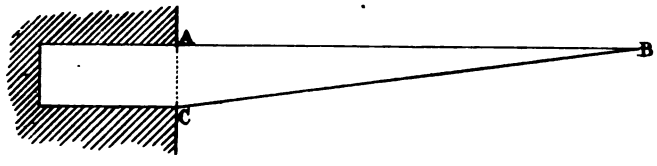
When equilibrium obtains (1) and (2) are equal

$$i.e., \quad \frac{w\,(c-x)^2}{2} = \frac{\frac{f_o}{5}\,I}{y_o}$$

$$\text{and} \quad \frac{w\,(c-x)^2}{2} = \frac{\frac{f_o}{5}\,b\,d^2}{6} \qquad (3)$$

in which equation all the factors are constant except x and d; and therefore $c - x$ is directly proportional to d, the equation to the underside of the longitudinal section of the beam being the equation to a straight line.

Fig. 1.



Substituting in equation (3) successive values of x,

when x =	60 inches,	d =	1·014 inches
x =	48 „	d =	2·028 „
x =	36 „	d =	3·042 „
x =	24 „	d =	4·056 „
x =	13 „	d =	5·070 „
x =	0 „	d =	6·085 „

It is evident that the same results would be obtained by setting down A C = the depth at the wall = 6·085 inches and joining B C.

2. The formula for the total deflection of the beam, which (if it be cut to the triangular section shown in *Fig.* 1 is a beam of uniform strength and uniform breadth) is given in *Deflection of Beams*, p. 36, and is

$$V = \frac{n'''\,W\,c^3}{E\,I}$$

where $n''' = \frac{1}{2}$

W lbs. = total load on beam = 192 × 6 = 1152 lbs.

c inches = total length = 72 inches.

E lbs. = Modulus of Elasticity (as before stated).

I = Moment of Inertia = $\frac{1}{12}\,b\,d^3$

$= \frac{1}{12} \times 4 \times (\text{depth at wall})^3$

$= \frac{1}{12} \times 4\,(6{\cdot}08)^3$

whence V = 1·8960 inches,

a deflection which would be inconveniently large, the maximum allowance being $\frac{1}{240} \times 72'' = {\cdot}3$ inch.

3. The formula for the total deflection of the beam, if it be not cut into the form for uniform strength is still (*Deflection of Beams* p. 38)

$$V = \frac{n''' W c_3}{E I}$$

but the value of n''' is now $\frac{1}{8}$, instead of $\frac{1}{2}$, all the other factors being the same as before, whence the deflection would be $\frac{1{\cdot}8960}{4} = {\cdot}474$ inches.

This deflection is still too large, if the deflection be limited, as is usual to $\frac{1}{10}$ inch per foot run, (p. 40), and it would be necessary, either to widen or deepen the beam. The latter would be the more economical application of the material.

4. In order that the beam may be stiff enough, the section remaining uniform throughout, it is necessary to find the depth which will give the maximum deflection, *i.e.*, $\frac{1}{10}$ inch per foot run.

$$= \tfrac{6}{10} \text{ inch} = V$$

Then as before $V = \frac{n''' W c^3}{E I}$

$$\text{or } \tfrac{6}{10} = \frac{\frac{1}{8} \cdot W c^3}{\frac{1}{12} E \cdot b d^3}$$

whence $d^3 = 319{\cdot}925$

and $d = 6{\cdot}834$ ins., which depth being greater than the depth necessary for strength would be adopted.

EXAMPLE II.

A beam of English oak of first-rate quality and of rectangular section, 6 inches wide, has a clear bearing of 12 feet, is supported at both ends, and weighted with a single weight of 5000 lbs. at a distance of 3 feet from one end, and with a uniformly distributed load, viz., the weight of the beam and ceiling, of 1,600 lbs.

Determine,

1st, the depth necessary for strength.

2nd, the depth necessary for stiffness.

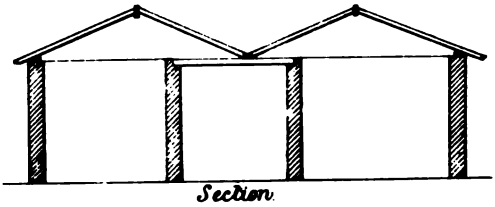
Remarks.

This is a case not frequently occurring in practice. It does, however, occur, as in a passage between two rooms of unequal width, the rooms and passage being roofed by a double roof only (*Fig.* 2).

The breadth of 6 inches is taken as convenient for fixing the plates and ceiling joists.

The beam is considered as supported, not fixed, because it is desirable, whenever it is possible, to leave a free access of air all round the ends of timber beams to prevent decay, and it cannot therefore be secured in such a manner as to be considered "fixed."

Fig. 2.



Section

The load is that due to the length of the roof between two of the oak beams.

Preliminaries.

Let d inches = depth at any section distant x from one end A of the beam.

b „ = breadth = 6 inches.

l „ = length = 144 inches.

m = the distance of the single weight to one point of support = 36″

n = the distance of the single weight to the other point of support = 108″

W lbs. = total load at C = 5,000 lbs.

w lbs. = weight on each inch run of the length of the beam due to the uniformly distributed load = $\frac{1000}{144}$ lbs.

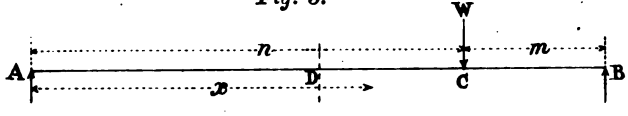
f_0 lbs. = Modulus of Rupture per square inch of section for English Oak of first class quality (*Useful Rules and Tables*, p. 199) = 13,600 lbs.

E lbs. = Modulus of Elasticity for the same class of oak (*Useful Rules and Tables*, p. 196) = 1,750,000 lbs.

5 = Factor of safety proper for timber with a dead load (*Useful Rules and Tables*, p. 205).

(1) The section of greatest stress of the single load W is at C, while that of the uniformly distributed load is at the centre D, (*Stress in Beams*, p. 8 and 7.).

Fig. 3.



If the single load W is (as in the present case) large in comparison with the uniformly distributed load, $w\,l$, the section of greatest stress is still at C; but if it is small, the section of greatest stress is somewhere between C and D.

This is shown as follows:

At any section x between A and C

$$\text{Moment of Flexure} = -\frac{W\,m}{l}x - \frac{w\,l}{2}x + \frac{w\,x^2}{2} \qquad (1)$$

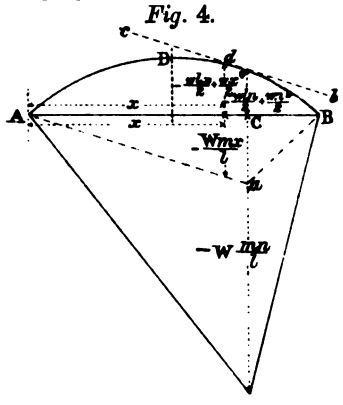
which, by differentiation, is found to be a maximum, when $x = \frac{l}{2} \times \frac{W\,m}{w\,l}$

If $\frac{W\,m}{w\,l}$ exceeds C D in length, the section of greatest stress is at C; and if it does not, *i.e.*, if $w\,l$ is large in comparison with $W\,m$, the section of greatest stress is in C D, because however small $\frac{W\,m}{w\,l}$ may be, it must always have some positive value.

In the present case $\frac{W\,m}{w\,l} = 9{\cdot}37$ feet, while C D = 3 feet, and the section of the greatest stress is at C.

This may be shown graphically, as in *Fig.* 4,

Fig. 4.



where the ordinates of the parabola represent the moment of flexure of the uniformly distributed load, and the ordinates of the triangle, those of the single load (*Stress in Beams*, p. 11 and 14), a mere inspection of which shows the maximum Moment of Flexure to be at C.

The case where W is small in comparison with $w\,l$ is shown by the dotted lines in *Fig.* 4, where the maximum value is obtained by drawing a line $b\,c$ parallel to A a, tangential at d to the curve of Moments of the uniformly distributed load.

As W becomes smaller, $b\,c$ becomes more and more nearly horizontal, and d approaches nearer and nearer to D, never reaching it, so long as W has any value at all.

Equating (1) with the value of the Moment of Resistance, as in *Example* I., and recollecting that the — signs in (1) refer only to the direction in which bending takes place.

$$\frac{W\,m}{l}\,x + \frac{w\,l\,x}{2} - \frac{w\,x^2}{2} = \frac{f_o\,b\,d^2}{30}$$

Substituting the values given above, and making $x = m$,

$$d^2 = 57{\cdot}57$$
$$d = 7{\cdot}6 \text{ nearly.}$$

(2) The depth necessary for stiffness.

As before stated (*Deflection of Beams*, p. 38) the easiest way to ascertain the maximum deflection due to two loads, where the points of maximum deflection do not coincide, is to lay down the deflection curve for each, and measure off the maximum deflection.

The equation to the deflection curve of the uniform load is obtained by integrating equation (3) (*Deflection of Beams*, p. 33).

$$i.e.\ \frac{d^2\,v}{d\,x^2} = \frac{M}{E\,I} = \frac{w\,(c^2 - x^2)}{2\,E\,I}$$

between the limits of o and x, whence

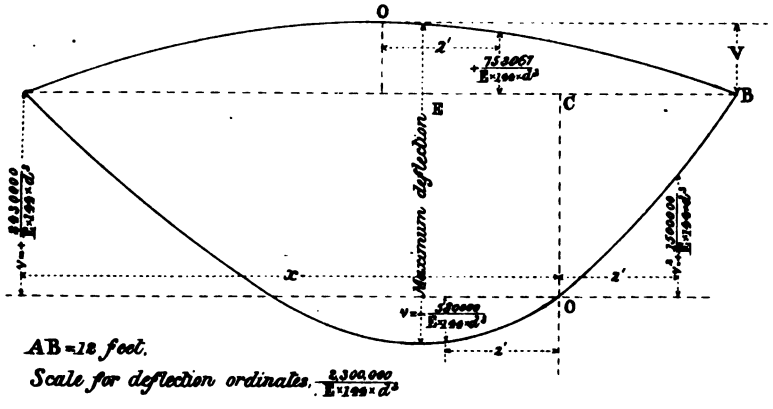
$$v = \frac{w}{2\,E\,I}\left\{\frac{c^2\,x^2}{2} - \frac{x^4}{12}\right\}$$

The equations to the deflection curve of the single load are given at p. 38 *Deflection of Beams.*

Calculating the deflections in terms of $\frac{1}{E\,d^3}$, taking for simplicity all dimensions in feet, at distances of 1 foot apart, and laying them down to a scale of parts (*Fig.* 5) it is found that the maximum deflection in this case is about 9″ from the centre of the beam, and by measurement

$$V = \frac{3888000}{E \times 144 \times d_3} = \frac{243}{15750\,d^3}$$

Fig. 5.



The maximum allowable value for V, in order that the ceiling may not crack (p. 40) is $\frac{1}{240}$ of B E = $\frac{5}{240}$ foot.

$$\text{Whence } \frac{5}{240} = \frac{243}{15750\,d_3}$$

$$d = \cdot 90 \text{ foot} = 10\cdot 8 \text{ inches}$$

and as this dimension is greater than that necessary for strength, it would be adopted.

This Example shews very clearly the necessity of considering stiffness as well as strength.

EXAMPLE III.

A rectangular Memel fir girder, average quality, 18 feet bearing, is to carry a floor and ceiling in a barrack. Its breadth is 12 inches, and its centre is 10 feet from the centre of the next girder on either side. It is "supported only" at each end. The weight that such a floor has to support may be taken at $1\frac{1}{4}$ cwt. per foot super., including its own weight and the weight of the ceiling.

Determine the depth necessary for strength and stiffness, the greatest allowable deflection being $\frac{1}{40}$ inch per foot run of span.

Remarks.

This is the case of an ordinary girder for a double floor. Its width is determined by the ordinary size of Memel logs in the market, *i.e.*, about 12 or 13 inches square. The bearing, 18 feet, is about the limit for which untrussed timber beams would now be used.

The weight of the floor and ceiling is taken from the ordinary practice, and includes the maximum weight that could in any case come upon the floor, supposing the room to be crowded with men, in which case the load would come on slowly, and would not move when on.

The distance, 10 feet, from centre to centre of the girders is taken as about the usual distance from centre to centre of the piers between the windows of any ordinary house. The girders should rest on the piers, not on the arches over the windows. The beam is considered as supported, not fixed, for the reasons given in Example 2.

Preliminaries.

Let d inches = depth.

b " = breadth = 12 inches.

c " = half clear span of girder = 108 inches.

w lbs. = weight of each inch run of the length of the girder

$$= \frac{10 \times 1 \times 1\frac{1}{4} \times 112}{12} = \frac{1400}{12} \text{ lbs.}$$

f_o lbs. = Modulus of Rupture per square inch of section of Memel Fir (Red Pine) average quality (*Useful Rules and Tables*, p. 199) = 8,400 lbs.

E lbs. = Modulus of Elasticity for the same class of fir (*Useful Rules and Tables*, p. 196) = 1,680,000 lbs.

5 = Factor of Safety proper for timber with a dead load (*Useful Rules and Tables*, p. 205).

Then the Moment of Flexure at any section distant x from the centre of a uniformly loaded beam $= w\,c\,(c - x) - \frac{w(c-x)^2}{2} = \frac{w}{2}(c^2 - x^2)$ which is a maximum when $x = o$, *i.e.*, at the centre, when Moment of Flexure $= \frac{w\,c^2}{2}$ (*Stress in Beams*, p. 7.) (1).

The Moment of Resistance at any point of the girder

$$\frac{\frac{f_o}{5} I}{y_o} \quad (\textit{Stress in Beams}, \text{ p. } 17). \tag{2}$$

where I, y_o, and 5 have the same meaning as in Examples 1 and 2.

Then, when equilibrium obtains (1) and (2) are equal, and

$$\frac{w\,c^2}{2} = \frac{\frac{f_o}{5} I}{y_o} = \frac{f_o\,b\,d^2}{30} \tag{3}$$

whence $d^2 = 202{\cdot}5$

and $d = 14{\cdot}23$

which is the depth necessary for strength.

(2). The maximum deflection allowable being taken at $\frac{1}{4\,0}$ inch per foot run of girder, (*Tredgold's Carpentry*, p. 48), the total deflection is $\frac{1}{4}\frac{8}{0} = \frac{9}{2\,0}$ inch.

The case being that of a uniformly loaded beam of uniform section supported at both ends, (*Deflection of Beams*, p. 36).

$$V = \frac{n''' W c^3}{E I} \qquad (4)$$

where $n''' = \frac{5}{4\,8}$, $c = 108''$, $I = \frac{1}{1\,2} \times 12 \times d^3$

ft. ft. cwt. lbs. lbs.

$$W = \text{total load} = 10 \times 18 \times 1\tfrac{1}{4} \times 112 = 25200$$

$$\text{whence } \frac{9}{2\,0} = \frac{5 \times 25200 \times 108^3}{48 \times 1680000 \times d^3}$$

$$d^3 = 4374$$

$$d = 16{\cdot}35 \text{ inches,}$$

which is the depth necessary for stiffness, and as it is greater than the depth necessary for strength, it would be adopted.

If there were any difficulty in getting timber of this depth, or if for any other reason so great a depth were objectionable, the depth might be reduced; 1st, by putting the girders closer together and so decreasing the weight on each: 2nd, by putting two girders alongside of one another, and so decreasing the weight on each by one half; 3rd, by trussing the girders; 4th, by using cast or wrought iron.

FIXED BEAMS.

UNDER this head it is proposed to deal only with the simpler cases, which occur in ordinary practice, *i.e.*, where the beams are of uniform section throughout, symmetrical above and below the neutral axis, and under a dead load.

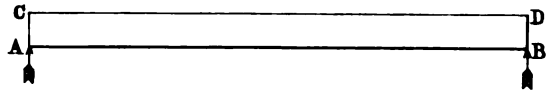
The subject of fixed beams, in the form of continuous girders subjected to a rolling load is one of considerable difficulty, and is beyond the scope of these elementary papers. It may, however, be stated, that little advantage accrues from making "small" girders, intended to support passing loads, continuous; and that even where such girders are large enough to allow of considerable saving of material, they should not be used, where the foundations of the piers are insecure, as a subsidence of any of the points of supports will cause a change of stress, the amount of which it is impossible to foresee, and which may seriously injure the structure or perhaps render it dangerous (*Stoney, Theory of Strains*, p. 168, Vol. 1).

Mr. Stoney fixes the span, at which it becomes worth while to consider the question of continuity, at about 150 feet for each opening.

DEFINITION OF A "FIXED BEAM."

Let A B (*Fig.* 1) be a beam, rectangular in elevation and supposed to be without weight, supported on two points of support.

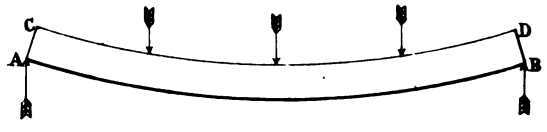
Fig. 1.



Then the ends A C, B D are vertical.

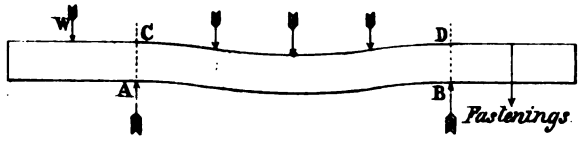
If a load be placed on the beam, it assumes the form shewn in *Fig.* 2, the ends A C, B D inclining inwards.

Fig. 2.



If the beam be now supposed to be lengthened at each end, and either weighted by the weight W or held down by fastenings (*Fig.* 3), so that the ends A C, B D, when the load is imposed are prevented assuming the inclined position shown in *Fig.* 2, and are compelled to retain their original vertical position, they are said to be "fixed," and the bending of the beam takes place as shown in *Fig.* 3.

Fig. 3.



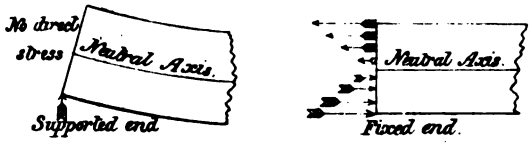
It is obvious, that if the load on the beam be large, and the weight W insufficient, or if the fastening be too weak, the ends will be able to assume an inclined position, the inclination being somewhere between the vertical, and the inclination which they would have had, if supported only. Such a beam is said to be "imperfectly fixed."

The Direct Stresses in Fixed Beams.

When a beam is merely supported at the ends, and loaded, it assumes the form shown in *Fig.* 2, and there is then no direct stress at the points of support.

When it is fixed at the ends, the ends remain vertical, and it is evident that in this latter case direct stresses must be acting at the ends with sufficient intensity to retain them in this position. These stresses may be represented as shown in *Fig.* 4.

Fig. 4.



The beam is now concave downwards at the points of support, and there is a direct stress of tension above the neutral axis, and a direct stress of compression below the neutral axis at these points.

At the centre the beam is still concave upwards, and the stresses are the same in character as before, though different in amount, *i.e.*, they are still in compression above, and in tension below the neutral axis.

It is evident, that in every fibre between the points of support and the centre of the beam, the nature of the stress changes somewhere, those fibres which are in tension at the supports being in compression at the centre, and those which are in compression at the supports being in tension at the centre.

The points where this change takes place are called the points of contrary flexure, contra flexure, or inflection; and at them there is no direct stress, but only a vertical shearing stress.

The cases most useful to the Engineer, in ordinary constructions, are the following, all the beams being of uniform section throughout their length, and symmetrical above and below the neutral axis.

(1). Beam fixed at one end, supported at the other, loaded at the centre.

(2). Beam fixed at one end, supported at the other, loaded with a uniformly distributed load.

(3). Beam fixed at one end, supported at the other, loaded both with a load at the centre, and with a uniformly distributed load.

(4). Beam fixed at both ends, loaded at the centre.

(5). Beam fixed at both ends, loaded with a uniformly distributed load.

(6). Beam fixed at both ends, loaded both with a load at the centre and with a uniformly distributed load.

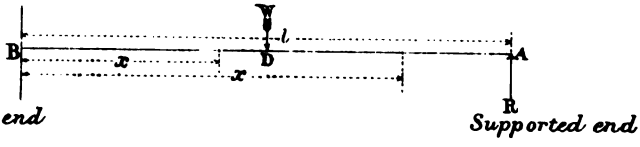
(7). Beam supported on more than two points, equidistant from each other, and in the same straight line, loaded with a uniformly distributed load.

(8). Beam fixed at both ends, and supported on one or more intermediate points, all the points of support being equidistant, and in the same straight line, loaded with a uniformly distributed load.

CASE 1. The investigation of this case in its most general form is given in *Moseley's Engineering and Architecture*, p. 524, article 377, edition 1855.

Reducing the general investigation to suit this particular case, it stands as follows:

Fig. 5.



Let M = Moment of Flexure at any section.
" B be the origin of coordinates.
" l = length of beam.

$$\text{Then } \frac{1}{r} = \frac{d^2v}{dx^2} = \frac{M}{EI} \text{ and } M = EI\frac{d^2v}{dx^2}$$

$$\text{For any point between B and D, } EI\frac{d^2v}{dx^2} = -R(l-x) + W\left(\frac{l}{2} - x\right) \quad (1)$$

$$\text{and for any point between D and A, } EI\frac{d^2v}{dx^2} = -R(l-x) \quad (2)$$

Integrating (1) between the limits o and x.

$$EI\frac{dv}{dx} = \frac{W}{2}(lx - x^2) - R\left(lx - \frac{x^2}{2}\right) + \text{constant} \quad (3)$$

When $x = o$, $\frac{dv}{dx} = o$ $\therefore$ constant $= o$.

Integrating again

$$\text{E I } v = \frac{\text{W}}{2}\left(\frac{l x^2}{2} - \frac{x^3}{3}\right) - \text{R}\left(\frac{l x^2}{2} - \frac{x^3}{6}\right) + \text{constant} \qquad (4)$$

when $x = o$; $v = o$ $\therefore$ constant $= o$.

Substituting $\frac{l}{2}$ for x in these equations, representing v at D, by D_1, and calling γ the inclination to the horizon of the tangent at D.

$$\text{E I tan. } \gamma = \frac{\text{W } l^2}{8} - \frac{3\,\text{R } l^2}{8} \qquad (5)$$

$$\text{and E I D}_1 = \frac{\text{W } l^3}{24} - \frac{5\,\text{R } l^3}{48} \qquad (6)$$

Integrating (2) between the limits of $\frac{l}{2}$ and x.

$$\text{E I } \frac{dv}{dx} = -\text{R}\left(lx - \frac{x^2}{2} - \frac{l^2}{2} + \frac{l^2}{8}\right) + \text{constant} \qquad (7)$$

$$\text{When } x = \frac{l}{2}, \text{ constant} = \text{E I } \frac{dv}{dx} = \text{E I tan. } \gamma$$

Eliminating tan. γ between (7) and (5)

$$\text{E I } \frac{dv}{dx} = -\text{R}\left(lx - \frac{x^2}{2}\right) + \frac{\text{W } l^2}{8} \qquad (8)$$

Integrating again between the limits of $\frac{l}{2}$ and x.

$$\text{E I } v = -\text{R}\left(\frac{l x^2}{2} - \frac{x^3}{6} - \frac{l^3}{8} + \frac{l^3}{48}\right) + \frac{\text{W } l^2 x}{8} + \text{constant}$$

When $x = \frac{l}{2}$, constant $= \text{E I } v - \frac{\text{W } l^3}{16}$, and v in this case, (*i.e.*, when $x = \frac{l}{2}$) $= \text{D}_1$

Substituting the value of D_1 from (6), constant $= -\frac{\text{W } l^3}{48} - \frac{5\,\text{R } l^3}{48}$

$$\therefore \text{E I } v = -\text{R}\left(\frac{l x^2}{2} - \frac{x^3}{6}\right) - \frac{\text{W } l^3}{48} + \frac{\text{W } l^2 x}{8} \qquad (9)$$

When $x = l$, $v = o$, and

$$-\text{R}\left(\frac{l^3}{2} - \frac{l^3}{6}\right) - \frac{\text{W } l^3}{48} + \frac{\text{W } l^3}{8} = o$$

$$\text{whence R} = \tfrac{5}{16}\,\text{W}. \qquad (10)$$

Substituting this in (1) $\text{M} = -\tfrac{5}{16}(l - x)\,\text{W} + \left(\frac{l}{2} - x\right)\text{W}$

$$= \tfrac{3}{16}\,\text{W } l - \tfrac{11}{16}\,\text{W} x$$

When $x = o$, M is a maximum, and $= \tfrac{3}{16}\,\text{W } l$ (11)

When $\text{M} = o$, *i.e.*, at the point of contraflexure, $x = \tfrac{3}{11}\,l$ (12)

Substituting the value of R from (10) in (2).

$$M = -\tfrac{5}{16}(l-x).W$$

When x is as small as possible, *i.e.*, between D and A, when $x = \frac{l}{2}$, M is a maximum and $= -\tfrac{5}{32} W l$ (13)

When $x = l$, *i.e.*, at A, $M = o$

Substituting the value of R in (9)

$$EIv = -\frac{5}{16} W \left\{ \frac{l x^2}{2} - \frac{x^3}{6} \right\} - \frac{W l^3}{48} + \frac{W l^2 x}{8} \qquad (14)$$

which is the equation to the deflection curve between D and A.

By differentiation, v is found to be a maximum, when

$$x = l - l\sqrt{\tfrac{1}{5}} = l(1 - \sqrt{\tfrac{1}{5}})$$

Substituting this value of x in (14).

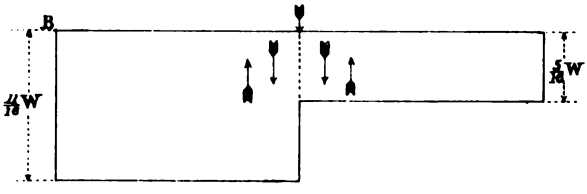
$$V = \frac{W l^3}{EI} \frac{1}{48\sqrt{5}}$$

or substituting $2c$ for l; $V = \dfrac{W c_3}{EI} \dfrac{1}{6\sqrt{5}}$

The equation to the deflection curve between B and D is obtained by substituting for R its value, *i.e.*, $\frac{5}{16}$ W in (4).

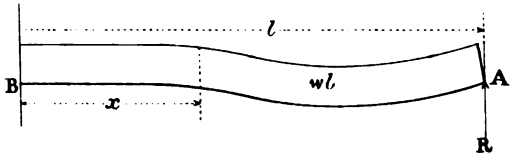
The shearing stress may be represented graphically as shewn in *Fig.* 6.

Fig. 6.



CASE 2.

Fig. 7.



Proceeding in this case on similar principles to case 1, the following results are obtained:

$$R = \frac{3}{8} w l$$

$$\text{Moment of flexure at B} = \frac{w l^2}{8}$$

Maximum negative moment of flexure $= -\frac{9}{128} w l^2$, when $x = \frac{5}{8} l$.

Point of contra flexure, where $x = \frac{l}{4}$.

Equation to the deflection curve

$$v = \frac{1}{E I}\left\{-\frac{3 w l}{8}\left(\frac{l x^2}{2} - \frac{x^3}{6}\right) + \frac{w}{2}\left(\frac{l^2 x^2}{2} - \frac{l x^3}{3} + \frac{x^4}{12}\right)\right\}$$

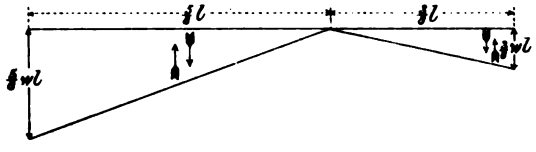
which makes v a maximum, when $x = \cdot 578\ l$, and

$$V = \frac{W l^3}{E I}\ \frac{\cdot 2614}{48}$$

or, substituting $2c$ for l, $V = \frac{W c^3}{E I}\ \frac{2 \cdot 09}{48}$

Shearing stress.

Fig. 8.



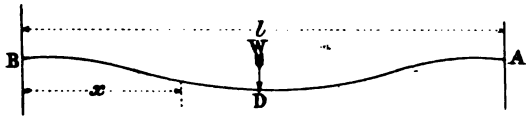
CASE 3.

It will be seen, on reference to cases 1 and 2, that the section of greatest stress in both is at the point of fixing. The moment of flexure at this point decides the dimensions or strength of the beam, and equals the sum of the moments of flexure in cases 1 and 2.

To determine the maximum deflection, the simplest way is to lay down the deflection curves, exaggerating the ordinates, and to measure the deflection as described in *Deflection of Beams*, p. 38.

The shearing stress may be obtained by combining *Figs.* 6 and 8.

CASE 4. *Fig.* 9.



Moment of flexure at B (and A) $= \frac{W l}{8}$

Maximum negative moment of flexure, at D, $= -\frac{W l}{8}$

Points of contra flexure, where $x = \frac{l}{4}$ and $\frac{3\,l}{4}$,

Equation to the deflection curve between B and D,

$$v = \frac{W}{E\,I}\left\{\frac{l\,x^2}{16} - \frac{x^3}{12}\right\}$$

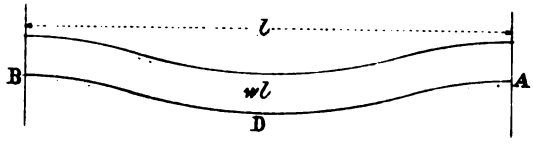
which makes v a maximum, when $v = \frac{l}{2}$ and

$$V = \frac{W\,l^3}{E\,I}\,\frac{1}{192}$$

or, substituting $2\,c$ for l, $V = \frac{W\,c^3}{E\,I}\,\frac{1}{24}$

The shearing stresses are the same as in a beam supported at both ends (*Stress in Beams*, p. 20).

CASE 5. *Fig.* 10.



Moment of Flexure at B (and A) $= \frac{w\,l^2}{12}$

Maximum negative moment of Flexure, at D, $= -\frac{w\,l^2}{24}$

Points of contra flexure, where $x = \cdot 211\,l$ and $\cdot 789\,l$

Equation to the deflection curve between B and D,

$$v = \frac{w}{24\,E\,I}\left\{l^2\,x^2 - 2\,l\,x^3 + x_4\right\}$$

which makes v a maximum, when $x = \frac{l}{2}$ and

$$V = \frac{w\,l^4}{E\,I}\,\frac{1}{384} = \frac{W\,l_3}{E\,I}\,\frac{1}{384}$$

or substituting $2\,c$ for l, $V = \frac{W\,c^3}{E\,I}\,\frac{1}{48}$

The shearing stresses are the same as in a beam supported at both ends. (*Stress in Beams*, p. 20).

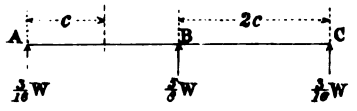
CASE 6. The sections of greatest stress, both in Case 4 and Case 5, being at the points of fixing, the Moment of Flexure at these points decides the dimensions or strength of the beam, and equals the sum of the Moments of Flexure in Cases 4 and 5.

The maximum deflection is at the centre in both 4 and 5, and the whole deflection is the sum of the two.

The shearing stresses are given by adding the shearing stresses in 4 and 5 together.

CASE 7. The results in this case are shewn on *Figures* 11 to 14, W being the total load, and w the load per unit of length.

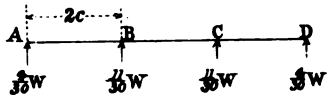
Fig. 11.



M at A and C = 0

M at B = $\frac{w\,c^2}{2}$

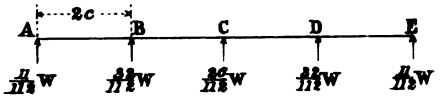
Fig. 12.



M at A and D = 0

M at B and C = $\frac{4}{5}\,w\,c^2$

Fig. 13.

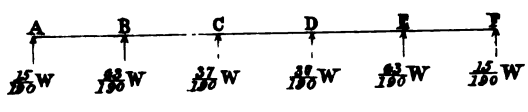


M at A and E = 0

M at B and D = $\frac{3}{4}\,w\,c^2$

M at C = $\frac{1}{4}\,w\,c^2$

Fig. 14.



M at A and F = 0

M at B and E = $\frac{8}{15}\,w\,c^2$

M at C and D = $\frac{6}{15}\,w\,c^2$

This case is useful in designing principal rafters of iron roofs, when the deflection and shearing stresses are not required to be known, and they are therefore not given.

CASE 8. In this case each length of the beam is in every respect in the condition of a beam fixed at both ends.

RESULTS OF FIXING ONE OR BOTH ENDS OF BEAMS OF UNIFORM SECTION* SUPPORTED AT BOTH ENDS.

(1) Load at centre, one end fixed; the strength is increased in the proportion of 4 to 3; the stiffness in the proportion of 1 to ·447; the shearing stress is increased at the fixed end, and reduced at the supported end.

(2) Uniformly distributed load, one end fixed; the strength is not increased; the stiffness is increased in the proportion of 5 to 2·09; the shearing stress is increased at the fixed end, and reduced at the supported end.

(3) Load at centre, as well as a uniformly distributed load, one end fixed; the strength and stiffness are both increased in some proportion depending upon the ratio of the two loads to one another, and the shearing stresses are also altered, the alteration depending upon the same ratio.

(4) Load at centre, both ends fixed; the strength is increased in the proportion of 2 to 1; the stiffness in the proportion of 4 to 1; the shearing stresses are not altered.

(5) Uniformly distributed load, both ends fixed; the strength is increased in the proportion of 3 to 2; the stiffness in the proportion of 5 to 1; the shearing stresses are not altered.

(6) Load at centre, as well as a uniformly distributed load, both ends fixed; the strength and stiffness are both increased in some proportion depending upon the ratio of the two loads to one another; the shearing stresses are not altered.

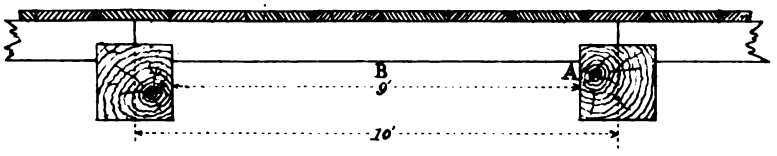
EXAMPLE I.

The bridging joists resting on the girders of Example III., *Timber Beams*, are of fir, average quality, $2\frac{1}{2}$ inches wide and 1 foot apart from centre to centre. They are firmly nailed at the ends, and, being also kept down by the floor boards, may be considered, while the wood is sound, as "fixed" at the ends.

Determine;

1st. The dimensions necessary for strength, the section of greatest stress being at A in *Fig.* 15.

Fig. 15.



* These results are given only when the beam is symmetrical above and below the neutral axis, as in square, circular and double T beams having equal flanges. For other beams, such as single T or double T beams with unequal flanges, the increase of strength and stiffness (if any) depends upon the section, and the resistance of the material in compression and tension. It may happen that a beam of this kind is weakened by fixing it, as in cast iron beams having cross sections of equal strength.

2nd. The deflection at B.

3rd. Compare the dimensions thus obtained with the dimensions which would be necessary for strength and stiffness, if the joists were considered as "supported only."

Remarks.

This is the case of an ordinary bridging joist resting on two girders. Its breadth is the minimum on which the heading joint of the floor boarding can be properly made. (These joists are, however, frequently made 2″ broad, and *Tredgold* gives 2″ as the minimum).

These joists are so fixed and weighted that they might be considered as "fixed," but in ordinary practice they are not so considered.

Preliminaries.

Let d inches = depth in inches.

b „ = breadth = 2·5 inches.

c „ = half clear span of joist in inches = 54 inches.

w „ = weight on each inch run of the length of the joist $= \frac{1\frac{1}{4} \times 112}{12} = \frac{140}{12}$ lbs.

f_o lbs. = Modulus of Rupture per square inch of section of fir, (Scotch) average quality (*Useful Rules and Tables*, p. 199) = 8400 lbs.

E lbs. = Modulus of Elasticity for the same class of fir, (*Useful Rules and Tables*, p. 196) = 1,680,000 lbs.

5 = Factor of Safety proper for timber with a dead load (*Useful Rules and Tables*, p. 205).

Then (p. 56) the Moment of Flexure is a maximum at the points of support, and

$$= \frac{w\, l^2}{12} \qquad (1)$$

The Moment of Resistance at any section of the joist

$$= \frac{\frac{f_o}{5} \mathrm{I}}{y_o} \qquad (\textit{Stress in Beams}, \text{ p. } 17). \qquad (2)$$

where I, y_o, and 5 have the same meaning, as in previous Examples.

Then when equilibrium obtains, (1) and (2) are equal, and

$$\frac{w\, l^2}{12} = \frac{\frac{f_o}{5}}{y_o} = \frac{f_o\, b\, d^2}{30}$$

whence $d^2 = 16{\cdot}2$

$d = 4{\cdot}02$ inches.

(2). The deflection is given by the formula in p. 56, viz.:

$$\mathrm{V} = \frac{w\, l^4}{\mathrm{E\,I}} \frac{1}{384}$$

whence V = ·184 inch.

which, taking $\frac{1}{40}$″ per foot run as the maximum deflection to be allowed, is sufficiently stiff.

(3). If the joist be considered as "supported only," as in Example III., *Timber Beams*,

$$\frac{w\,c^3}{12} = \frac{f_o\,b\,d^2}{30} \text{ (*Stress in Beams*, p. 7 and 17).} \qquad (3)$$

whence $d = 7$ inches nearly, for strength.

And for stiffness

$$V = \frac{n'''\,W\,c^3}{E\,I} \text{ (*Deflection of Beams*, p. 36).} \qquad (4)$$

$$i.e., \frac{5}{16} = \frac{5}{48}\,\frac{9 \times 1\frac{1}{4} \times 112 \times 54^3}{\frac{1}{12} \times 2{\cdot}5 \times d^3 \times 1680000}$$

whence $d = 6{\cdot}4$ inches,

so that the depth of 7 inches would be adopted.

$7'' \times 2\frac{1}{2}''$ are the dimensions given by *Tredgold* for floor joists, 1 foot apart, and 8 feet bearing.

General Remarks. Timber beams, such as floor girders built into walls, cannot properly be calculated as "fixed" beams; for even if they are originally built solidly into the walls (which, however, is bad construction, as ventilating spaces ought to be left round them) they are sure to shrink, and become "imperfectly fixed" to some extent.

Common floor joists, common rafters, bridge or jetty stringers, nailed firmly at the ends, are, so long as the timber and fastenings hold, "fixed" beams, though the dimensions in ordinary use for them are for "supported" beams only.

The principal rafters of timber king and queen post roofs are, from the manner in which they are secured, "fixed" beams.

The principal rafters of iron roofs are often secured by a single bolt at the foot, in which case the lowest length approaches the condition of a beam fixed at one end, and supported at the other. Where they are secured by two bolts at the foot, they are "fixed" beams.

Wrought iron beams for use as floor girders should rest upon a stone template, and should have another template resting above and on their ends. This construction ensures their acting as "fixed" beams.

Cast iron beams are not suited for use as "fixed" beams; for if they are of the ordinary section, the upper and smaller flange is thus brought into tension at the points of fixing; and it is not economical to make the flanges of the same size, merely for the purpose of fixing them.

CAST IRON BEAMS.

Cast Iron is, for many reasons, inferior to Wrought Iron as a material for beams, but it possesses one advantage which makes it valuable to the Engineer, especially for beams of comparatively small dimensions; viz.: that it can be run into moulds of *almost* any form, and the material can thus be arranged to meet in the most advantageous manner and with the least possible waste, the stresses which the different parts of the beam have to resist.

It has already been seen (*Stress in Beams*, p. 15) that the direct stresses, in any section of a beam under transverse stress, increase uniformly on both sides of the neutral axis, to the upper and lower edges of the section, while the shearing stress increases (*Stress in Beams*, p. 23), but not at a uniform rate, from the upper and lower edges of the section to the neutral axis.

As the resistances to the direct stresses are directly as the stresses, it follows that when the material at the edges of the section is exerting as great a resistance as it can exert without its elasticity being impaired, the material lying near the neutral axis is hardly called upon to exert any resistance at all.

In order then that the quantity of material required to resist the direct stresses may be as small as possible, it must be placed where each part of it will exert, as nearly as is possible, its greatest safe resistance, and this is manifestly at the upper and lower edges of the section.

As there is no shearing stress at the upper and lower edges of the section, the material thus collected near the edges is under very little shearing stress, and, in fact, the intensity of the shearing stress upon it is so small that it may be neglected.

If, then, the material required to resist the direct stresses be collected at the upper and lower edges of the section, so as to form two flanges or tables, and if they be connected by means of a vertical layer or web, capable of resisting the whole of the shearing stress, the whole of the stresses are provided for, and the section assumes the form known as the double T shaped section.

The extent, however, to which the metal can be removed from near the neutral axis, and collected in the flanges, is limited by the conditions necessary for obtaining a sound casting. These conditions are: 1st: That there shall be no sudden variation in the thickness of the metal, any such variation causing the thinner parts of the metal to cool sooner than the thicker parts, inducing internal stresses in the metal before the

load comes on, and sometimes loss of shape. It is best to have no variation whatever, for although a beam designed in this way may take a little more metal, it is more to be depended upon. 2nd: That the thickness of the different parts shall be such as to allow the fluid metal to fill all parts of the mould before it has time to part with any material part of its heat and fluidity.

The thicknesses of the different parts of a flanged beam necessary to ensure these conditions can only be determined by the experience of the founder, as they must depend to some extent upon the distance which the metal has to run, in order to completely fill the mould, but the proportions may be approximately stated as follows: for flanges of 2 feet in width, it is not wise to have a less thickness than $1\frac{1}{2}$ inches, and for flanges 18 inches wide, not less than 1 inch, while for narrower flanges a somewhat less thickness may be used, although it is not a good plan to fine down the metal too much.

The thickness of the web should be very nearly (better quite) the same as that of the flanges.

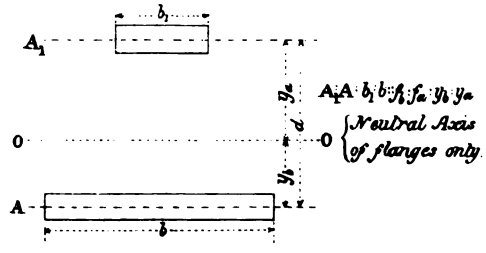
The proportion, which the areas of the flanges of the section bear to each other depends, to some extent, upon the elastic limit of the iron in tension and compression, *i.e.*, more metal should be put into the tension flange than into the compression flange, the effect of which is to bring the centre of gravity, and therefore the neutral axis, nearer to the tension side of the beam, while the metal in compression is thus placed at a greater distance from the neutral axis, and consequently in a position to exert the greater resistance of which it is capable.

It is shewn further on, that if the area of section, the thicknesses of flanges and web, and the total depth of the section be given, there is one proportion for the widths of the two flanges, which gives the Moment of Resistance of the section a maximum value, or in other words, arranges the metal in such a way as to get from it the greatest possible resistance to fracture by cross breaking.

Before, however, explaining how this proportion of the widths of the flanges is arrived at, it may be better to shew on what principles such beams are usually dealt with by Engineers.

The usual method is to neglect the resistance of the web to the direct stresses, and to suppose that it is removed, the area of the flanges as well as their breadths being made inversely proportional to their working resistances; thus

Fig. 1.



and the supposition is also made that each fibre of each flange exerts the same resistance as the extreme fibre of that flange.

On these suppositions the resistance of the flanges are equal, and constitute a couple, the Moment of which is equal to the total resistance of either flange, multiplied by the distance between the centres of resistance of the flanges, *i.e.*, $A_1 \times f_a \times d = A \times f_b \times d =$ Moment of Resistance of section, which being equated with the Moment of Flexure at any section gives the dimensions of the flanges required.

In practice it is usual to take d as the whole depth of beam at the particular section under consideration.

This formula overrates the strength of such beams, for it proceeds on two assumptions neither of which is correct. 1st; That the neutral axis passes through the centre of gravity of the two flanges only, and that consequently the working resistance in the extremes fibres of each is excited nearly at the same time, whereas the neutral axis in all beams having webs of the ordinary thickness is really much higher up in the section, and the working resistance of the extreme fibre of the tension flange is excited long before that of the extreme fibre of the compression flange is nearly called out. 2nd; That the full working resistance of which the metal is capable is excited in all the fibres of both flanges, whereas it is excited in the extreme fibres only.

Beams designed by this formula are not, as is frequently supposed, beams having cross sections of equal strength, but are always weakest on the tension side, and the formula can only be admitted for use by taking a low working resistance in tension.*

The foregoing formula is the same as that given in most books of Engineering formulæ,

$$\text{viz., } W = \frac{2\,ad}{l}$$

where W is the weight in tons, which, applied at the centre, will break a beam supported at both ends, l is the clear span of the beam in feet, a the area of the bottom flange in square inches, and d the total depth in inches.

This formula is derived from the experiments of Mr. Hodgkinson, full details of which are given in *Experimental Researches on the Strength and other properties of Cast Iron, by Eaton Hodgkinson, F.R.S.*, p. 444.

The conclusions therein given are, that "when the length, depth, and top flange in different cast iron beams with very large flanges are the same, and the thickness of the vertical part between the flanges is "small" and invariable, the strength is nearly in proportion to the size of the bottom flange; and that in beams which vary only in depth, every other dimension being the same, the strength is nearly as the depth.

Whence in different beams whose length is the same, the strength must be nearly as their depth, multiplied by the area of a middle section

* A cross section of equal strength is one in which the material is so arranged that when the greatest resistance of which the material is capable in tension is excited in the extreme fibre on the tension side of the section, the greatest resistance of which the material is capable in compression is excited in the extreme fibre on the compression side of the section.

of their bottom flange, and when the length is different, the strength will be as this product divided by the length,

$$\therefore W = \frac{cad}{l}$$

where W = the breaking weight at the centre of a beam supported at both ends, a = area of a section of the bottom flange at the centre, d = depth of the beam at the centre, l = the distance between the supports, and c = a constant determined by experiment.

The value of this constant was found to = 26·8 tons for beams cast erect, and 25·7 tons for beams cast on their side.

Neglecting the decimals and taking 26 tons as the mean value of c,

$$W = \frac{26\,ad}{l}$$

where W is in tons and a, d and l are in inches.

Putting l into feet the formula becomes

$$W = \frac{2{\cdot}166\,a\,d}{l}$$

and omitting the decimals it becomes

$$W = \frac{2\,ad}{l}$$

It is to be observed, with regard to this formula that Hodgkinson's experiments were made with beams having much thinner webs, than it is customary, or wise to use in practice. In his beams the quantity of metal in the web was smaller, relatively to the quantity in the flanges, than it is in ordinary beams, and the Centre of Gravity, of the two flanges only, did not differ so much from that of the whole section, as it does in ordinary beams, in which the thickness of the web is nearly (better quite) the same as that of the flanges, and the formula gives results which are inaccurate in proportion to the thickness of the web.

The custom in using this formula is to determine the area of the tension flange, and to make the area of the compression flange a certain fixed proportion, generally $\frac{1}{6}$, of the area of the tension flange.

Some Engineers recommend a larger fraction, and Humber (*Iron Bridges*, p. 85) recommends $\frac{1}{4}$; and this no doubt has the advantage of stiffening the compression flange against lateral bending.

It is also recommended by some writers, that where the load is on the top, the proportion should be $\frac{1}{3}$, apparently with the object of giving room for the proper putting on of the load.

One objection to the formula in this form is that it only determines the section of a beam at the centre, and does not enable the Engineer to design a beam of uniform strength.

The determination of the proportion of the width of the two flanges, which gives the best arrangement of the metal, is as follows:

It has already been shewn (*Stress in Beams*, p. 17) that the moment of resistance of the section of a beam is $\frac{f_o}{y_o} I$, where f_o is the working resistance of the extreme fibre of the section on that side, where the working resistance is soonest reached, y_o is the distance from the neu-

tral axis to the extreme fibre on that same side, and I is the moment of inertia of the section about a horizontal axis passing through its centre of gravity.

If f_a, f_b be taken as the working resistance in compression and tension respectively, y_a, y_b as the distances of the extreme compression and tension fibres from the neutral axis respectively, then the moment of resistance of the section is either $\frac{f_a}{y_a}$ I, or $\frac{f_b}{y_b}$ I, according as one or the other has the smaller value, and when they are equal, the section is called a cross section of equal strength.

In this case $\frac{f_a}{y_a}$ I $= \frac{f_b}{y_b}$ I, whence $f_a : f_b :: y_a : y_b$, *i.e.*, the distances of the extreme fibres from the neutral axis are directly as the working resistances.

The strongest form of section for a given quantity of metal, given depth of section and given thickness of web and flanges, will be that in which the smaller of the two expressions $\frac{f_a}{y_a}$ I or $\frac{f_b}{y_b}$ I, has its maximum value.

Thus supposing a section (*Fig.* 2) to be assumed (or calculated by the rough method already given), let it be required to determine, whether, and how the metal may be redistributed so as to obtain greater strength.

In order to simplify the investigation it is considered best to deal with a dimensioned section rather than with one described by symbols, and for the same reason to take the web and flanges of uniform thickness.

There is no difficulty, though a little more trouble, in making a similar investigation for a section of varying thickness, provided the web is treated as a rectangle having a width equal to the mean thickness of the web.

Let *Fig.* 2 be the section in question.

Let x be the width of the part to be taken from the top flange, and added to the bottom flange.

Fig. 2.

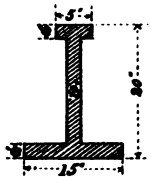
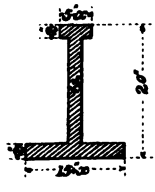


Fig. 3.



Then the width of the top flange becomes (*Fig.* 3) $5 - x$ and the width of the bottom flange (*Fig.* 3) $15 + x$.

Then $y_b = 7{\cdot}5 - \frac{x}{2}$

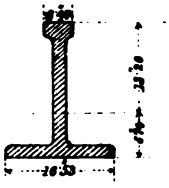
$$I = 3485{\cdot}9 - 180x - 18x^2$$

$$\frac{I}{y_b} = \frac{3485{\cdot}9 - 180x - 18x^2}{7{\cdot}5 - \frac{x}{2}}, \text{ which is a maximum.}$$

By differentiation $x = 1{\cdot}53$.

Whence the best distribution of the metal is as shewn in *Fig.* 4,

Fig. 4.



and it is seen that the stress on the extreme fibre of the top flange is not quite double that on the extreme fibre of the bottom flange. So that although the metal in the top flange does not exert nearly the resistance of which it is capable, any further transfer of the metal, with a view to increasing the stress on that in the top flange, will have the effect of reducing the whole strength of the section.

Substituting in the expression $\frac{f_b\, I}{y_b}$ the actual maximum value of $\frac{I}{y_b}$, and equating this with the moment of flexure for the case for which the beam is to be designed, the value of f_b is determined, thus

$$\frac{f_b\, I}{y_b} = \text{Moment of Flexure}$$

$$\text{whence } f_b = \frac{\text{Moment of Flexure} \times y_b}{I}$$

and if the value f_b is greater than the proposed working resistance, another section with more metal in it must be assumed and the best arrangement of the metal determined, the operation being repeated until a section suitable for the particular case is arrived at. It is of course essential that when the extreme tension fibre is exerting the required resistance per square inch, the stress on the extreme compression fibre does not exceed its safe working resistance. The resistances exerted being directly as the distances from the neutral axis, it is at once seen whether this is the case. For instance in the section just examined, if the load be such as to make $f_b = 1{\cdot}5$ tons per square inch, f_a will be 2·93 tons, which is safe for any cast iron whatever.

The ratio of the elastic limits in tension and compression in different kinds of cast iron varies very much, being in the softer kinds of cast iron as high as 1 to 3, while in the harder kinds it falls as low as 1 to 6 or 7.

The iron best suited for girders, especially where they are to be subjected to a rolling load or shocks of any kind, as in railway bridges,

is of an intermediate kind, and it is customary for the Engineer to specify that an iron of this kind shall be supplied by the ironfounder, and to take an average value for the working resistance of such iron to different kinds of stress.

The average value of the resistance to each of the three kinds of stress which such cast iron can safely be called upon to exert, may be taken (*H. l. B.*, p. 71,) to be as follows:

Working resistance in	tension	1·5	tons per sq. inch.
,, ,,	compression	8·0	,,
,, ,,	shearing	2·33	,,

The Modulus of Elasticity may be taken at 16,000,000 lbs.

There are two other forms of cast iron beams in use, viz., the trough girder (*Fig.* 5), and the single T (*Fig.* 6).

The first is used on railways to receive a longitudinal sleeper to which the rails or chairs are fastened; the second is used in small spans, generally to form the abutments for brick or concrete arches as shown in *Fig.* 6.

Fig. 5.

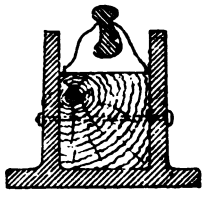


Fig. 6.

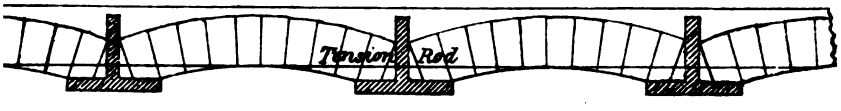


Fig. 7.

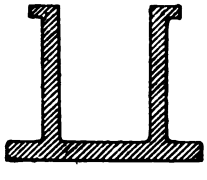
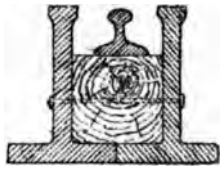


Fig. 8.



Trough girders are frequently cast with an upper flange, in which case it is more convenient for the subsequent replacing of the timber to cast the flange on the outside, (*Fig.* 7), and frequently often in two parts, (twin girders, *Fig.* 8), afterwards bolted together. They may be reduced in depth to the ends, so as to become approximately beams of uniform strength and breadth.

When there are two flanges, the calculations are in every respect the same as for an ordinary T girder.

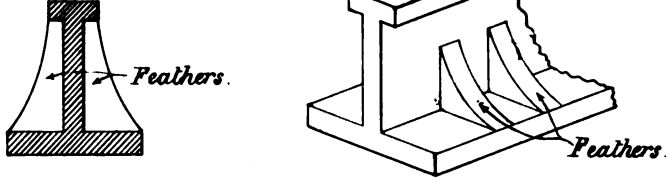
When there is no compression flange (and this applies equally to the single T girders for small spans) the outer fibres of the web have to resist the maximum compression, and it is advantageous to have them numerous, both in order that the metal may work to advantage, and to give lateral stiffness to the parts under compression. The total width of the web at the top should not be less than $\frac{1}{5}$ of the width of the bottom flange (it would be better to make it $\frac{1}{4}$) in which case the area of the tension flange may be calculated in the same manner as the tension flange in the double T girder, the area of the web being sufficient to resist the shearing stress. If this proportion entails a very thick vertical web, it may be cheaper to design the girder with an upper flange.

Cast iron beams and cantilevers are sometimes made with open webs but there is an objection to open webs, that they afford more than ordinary facility for the formation of air bubbles in the process of casting, and are specially liable to unequal cooling, and it is therefore better not to use them, where heavy, and more especially heavy rolling loads are to be carried.

In calculating the dimensions of such beams and cantilevers, the flanges are calculated in the ordinary way, and there should be at every vertical section ample metal left in the web to resist the direct shear.

The objection to open webs above mentioned applies equally to projections on the web of the girder, technically called "feathers."

Fig. 9.



Cast iron girders should never be "fixed" at the ends. If they are "fixed," the upper flange is in tension at the points of fixing (*Fixed Beams*, p. 60), and the greatest stress on the section is produced precisely where there is the least material to meet it.

EXAMPLE I.

It is required to design a pair of cast-iron double T girders to carry a single line of railway over a road 18 ft. wide.

The weight of the rolling load on the bridge is to be taken at $1\frac{1}{2}$ tons per foot run. The approximate weight of the structure resting on and including the girders may for first calculations be taken at 4 cwt.* per foot run.

The relative dimensions of the upper and lower flanges are to be inversely as the working resistances of the iron in compression and

* This weight, 4 cwt. per foot run, is not sufficient, as will be seen by running out the weight of the bridge as designed.

tension, and in the calculations the resistance of the web to the direct stresses is to be neglected.

The depth of the girders at the centre is to be $\frac{1}{11}$ of the span, say 22 inches, and it is to diminish to the ends, the dimensions of the flanges being the same throughout.

The working resistance of soft grey cast iron in tension is to be taken at 1·5 tons, its resistance in compression at 8 tons, its resistance to shearing at 2·33 tons, per square inch.

Determine:

1st, The sectional area of the flanges at the centre of the girder.

2nd, The central section.

3rd, The sectional area of the web at the piers necessary to resist shearing.

4th, The longitudinal section and plan of the girder, allowing 1 ft. at each end beyond the centre of bearing.

5th, The deflection.

Remarks.

This is the case of a railway bridge over a country road, the platform being supposed to be carried on the lower flanges of the girders.

Cast iron girders are not very often used for carrying lines of railway over much greater spans than that given in this example, but this depends upon the opinion of the Engineer. Some Engineers have not long since used them up to nearly 40 ft long, while others restrict their use on main lines to 12 ft. openings.

It is desirable, in calculating all iron girders, to consider the span to be the distance from centre to centre of bearings, and not merely to be the distance between the piers.

If these girders, therefore, have a bearing of 2 ft. in length on each pier, the clear span to be calculated for will be 20 ft.

The rolling load per foot run is greater on short bridges than on long bridges, because the bridge has to bear at some time during the passage of a railway train, the whole weight of an engine, whereas in long bridges, the load is made up of carriages and engines. The position of the load in which the greatest direct stresses are produced on the girders is when the driving wheels of the locomotive are at the centre of the girders, and if the effect of the weights on the other wheels be reduced to an equivalent central load, and then to an uniformly distributed load, the weight per ft. run will be found to be about $1\frac{1}{2}$ tons,* and the longer the bridge the less the greatest load per foot run will be found to be.

The weight of the structure in short bridges may generally be obtained with sufficient accuracy from the books of Engineering formulæ, (*M. E. F.* p. 12 and 13) and the girder may be designed from these data. It is always advisable, however, after designing the girder on these data to calculate the weight of the structure as proposed, for the purpose of comparison with the assumed weight, a second calculation for the girder

* Mr. Unwin, in Iron Bridges and Roofs, page 45, recommends 2 tons to be taken as the rolling loads for spans of 25 ft.; $1\frac{3}{4}$ tons for spans of 30 ft.; $1\frac{1}{2}$ tons for spans of 40 ft.; and $1\frac{1}{4}$ tons for spans of 60, for each line of way.

being made if any material difference is found between the assumed and calculated weights.

The depth of the girder is determined by convenience, and is as great as the depth of such girders usually is. It diminishes to the ends for uniform strength against the direct stresses.

Preliminaries.

Let d = depth in inches = 22 inches.
A = area of bottom flange in square inches.
f_b tons = working resistance per sq. inch in tension = 1·5 tons.
A_1 = area of top flange in square inches.
f_a tons = working resistance per sq. in. in compression = 8·0 tons.
A_2 = area of web in square inches.
2·33 tons = working resistance to shearing per square inch.
c = half clear span in inches = 120 inches.
w tons = weight on each inch in length of the girder.
E lbs. = Modulus of Elasticity for cast iron = 16,000,000 lbs.

As the Board of Trade regulations compel a double factor of safety for the moving load on cast iron bridges, it is multiplied by 2 to reduce it to dead load.

1. Rolling load on the bridge = $20 \times 1\frac{1}{2} = 30$ tons $= 60$ tons dead load.
   Weight of structure $= 20 \times \cdot 2 = 4$ tons $= 4$ tons dead load.

Total dead load = 64 tons.
Total dead load on each girder = 32 tons.

Whence $w = \frac{32}{240} = \cdot 133$ tons per inch run.

The Moment of Flexure is greatest at every section, when the girder is fully loaded, and at any section distant x from the centre of the girder is $M = \frac{w(c^2 - x^2)}{2}$ (*Stress in Beams*, p. 7.) ∴ (1)

The Moment of Resistance $= f_b \times A \times d$ (2) $= f_a \times A_1 \times d$ (3).

Then, when equilibrium obtains, (1) and (2), as well as (1) and (3) are equal, and for the bottom flange

$$\frac{w(c^2 - x^2)}{2} = f_b \times A \times d$$

Substituting in this equation the values w, c, f_b, and d, and making $x = o$,

$A = 29$ square inches at the centre.

Equating (1) and (3) and substituting in a similar manner, $A_1 = 5{\cdot}44$ square inches at the centre.

2. The central section may now be determined.

If the bottom flange be made 17 inches wide, which will give sufficient lateral stability, the thickness will be 1·7 inches, and if the top flange be made 4 inches wide, the thickness will be 1·36 inches, say 1·4 inches. The thickness of the web at the centre of the girder may therefore be 1·7 inches at the bottom and 1·4 inches at the top. The

arrisses being rounded off and the angles filled up, the section given in *Fig.* 10 is obtained.

Fig. 10.

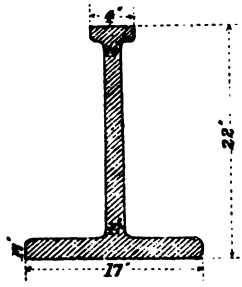
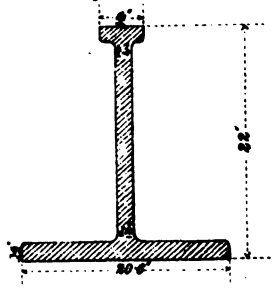


Fig. 11.



This agrees with ordinary practice, but it would be better to reduce the thickness of the lower flange a little, increasing its width, and making the thickness of all the parts the same, as in *Fig.* 11, which gives a stronger section without increasing the quantity of metal.

3. As the shearing stress is a maximum and the area of the web is a minimum at the piers, it is evidently only necessary to calculate the dimensions of the web at the pier.

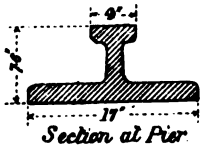
The maximum shearing stress at the pier $= \frac{1}{2}$ total load $= 16$ tons.

The resistance of the iron to shearing $= 2{\cdot}33$ tons per square inch, therefore $A_3 \times 2{\cdot}33$ tons $= 16$ tons, whence $A_3 = 6{\cdot}9$ inches nearly.

The mean thickness of the web is $\frac{1{\cdot}7+1{\cdot}4}{2} = 1{\cdot}55$ inches.

Whence the height of web $= \frac{6{\cdot}9}{1{\cdot}55} = 4{\cdot}5$ inches nearly, and the total depth of the beam at the pier $= 7{\cdot}6$. (*Fig.* 12.) The depth at the pier is often made more than this in order to allow of the planking forming the platform to be put on without cutting.

Fig. 12.

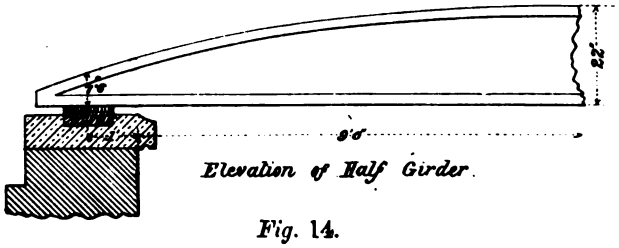


Section at Pier

4. The elevation of the upper side of the beam may now be drawn with sufficient accuracy, by setting up the total depth of the beam at the centre and at the piers, and describing an arc through these points, carrying the curve on to the end of the bearing (*Fig.* 13), and the plan will be as shewn in *Fig.* 14.

5. Owing to the inaccuracy of the formula, the deflection of the girder cannot be accurately ascertained without, in the first instance, determining the value of the actual working resistance excited in the extreme fibre of the lower flange, when the girder is fully loaded.

Fig. 13.



Elevation of Half Girder.

Fig. 14.

Plan of Half Girder.

The deflection, on the assumption that the value of the working resistance is no more than it was intended to be, viz. 1·5 tons per sq. inch, is given by the formula, (p. 37 *Deflection of Beams*,), viz.:

$$V = \frac{n'' f_o c^2}{E y_o}$$

where $n'' = \cdot 5708$, $f_o = 1 \cdot 5$ tons, $c = 120$ inches, $y_o = 7 \cdot 4$ inches, whence $V = \cdot 233$ inch.

It is shewn further on that it is rather more than this.

To determine accurately the strength of this girder—

$$\text{Moment of Flexture at centre} = \frac{w c^2}{2} \qquad (1)$$

$$\text{Moment of Resistance} = \frac{f_a}{y_a} I \text{ or } \frac{f_b}{y_b} I \qquad (2)$$

according as the extreme fibre on the compression or tension side of the section reaches its limit of resistance first.

To determine this, find the Centre of Gravity of the section, and draw a horizontal line through it. This line is the neutral axis of the section.

It is found to be 7·36 inches from the lower edge of the section.

Then, as the resistances of the fibres are as their distances from the neutral axis, the resistance of the extreme upper fibre is to the resistance of the extreme lower fibre as 14·6 : 7·4, nearly as 2 to 1; so that when the lower fibre has reached the limit of its resistance, *i.e.*, 1·5 tons

per square inch, the upper fibre will only be exerting a resistance of 3 tons per square inch, whereas it is capable of exerting a resistance of 8 tons per square inch.

Therefore the lower side is the weaker side of the section, and decides the strength of the beam.

$$\text{Then } \frac{w\,c^2}{2} = \frac{f_b}{y_b} \text{ I}$$

$$\frac{w \times 120^2}{2} = \frac{1{\cdot}5}{7{\cdot}4} \times \text{I}$$

Finding I by Rule III., p. 154 *Useful Rules and Tables*,

$$w = \frac{1{\cdot}5 \times 3714 \times 2}{120 \times 120 \times 7{\cdot}4} = {\cdot}104 \text{ tons per inch run}$$

a result which shows that in this case the approximate method overrates by 28 per cent. the true strength of the girder, or, in other words, that the working resistance, which the extreme fibre of the lower flange is called upon to exert, is nearly 2 tons, instead of 1·5 tons per square inch, the deflection being increased similarly from ·233 inches to ·311 inches.

Probably, for all cases where cast iron is now used, the approximate formula is near enough, taking care to use a low working resistance in tension.

Should, however, an Engineer think it worth while in any case to design his girder more accurately, he will find it most convenient to calculate his cross section first by the rough method, and then to ascertain its real strength and deflection as above, adding to or deducting from the different parts and re-calculating, recollecting that one of the first conditions for a sound casting is that there must be no sudden change of thickness of metal, and better no change at all.

It is unnecessary to give an Example of a beam of uniform section, as in that case it is only necessary to design the section of greatest stress, and to see that the quantity of metal in the web is sufficient to resist the shearing stress at the piers.

Cast iron beams are sometimes made of approximately uniform strength and uniform depth, in which case they assume the forms shewn in *Figs.* 15 to 17.

The same principles hold good for them, as in Example I.

Fig. 15.

Fig. 16.

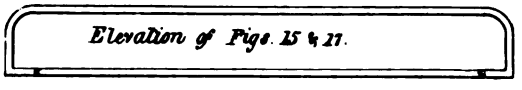
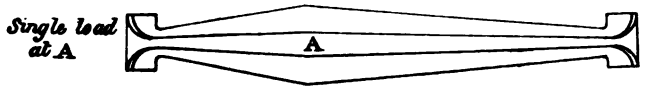


Fig. 17.



EXAMPLE II.

A cast iron cantilever is built into a wall at one end and uniformly loaded with a weight of 192 lbs. per foot run, dead load. The length of the projecting part is 6 ft. The section is to be single T or double T, as may be found necessary, the upper (tension) flange is to be 3 inches wide, so as to allow room for the loading, and the depth at the wall is to be 7 inches.

Determine:

1st, the longitudinal section for uniform strength, and the section at the wall, taking the working tensile resistance of cast iron at 1·5 tons, the working resistance to compression at 8 tons, the working resistance to shearing at 2·33 tons, per square inch, the proportion of the upper and lower flanges inversely as the working resistance, and neglecting the strength of the web.

2nd, If the web is large enough to resist the shearing stress.

3rd, The deflection.

Remarks.

This is the case of an ordinary balcony beam.

The load is considered dead, as it is supposed to be calculated for the greatest number of people that could come on the balcony, in which case they would come on by degrees, and would not move when on.

It illustrates the case of single T beams, in which the thickness of the web is $\frac{1}{3}$ of the width of the tension flange, a compression flange being unnecessary.

Preliminaries.

Let d = depth in inches = 7 inches.

A = area of tension (*i.e.* top) flange in square inches.

f_b tons = working resistance per sq. inch in tension = 1·5 tons.

A_1 = area of compression (*i.e.* bottom) flange in sq. inches.

f_a = working resistance per sq. inch in compression = 8 tons.

A_2 = area of web in square inches.

2·33 tons = working resistance to shearing per square inch.

c = length in inches = 72 inches.

w lbs. = weight on each inch in length of cantilever = $\frac{192 \text{ lbs.}}{12} = 16$ lbs.

E lbs. = Modulus of Elasticity for cast iron = 16,000,000 lbs.

(1) Then Moment of Flexure at any section distant x from the fixed end of the beam is

$$\frac{w\,(c-x)^2}{2} \quad (\textit{Stress in Beams}, \text{ p. } 5) \qquad (1)$$

Moment of Resistance, at any section, is

$$f_b \times A \times d\,(2) = f_a \times A_1 \times d \qquad (3)$$

When equilibrium obtains (1) and (2) as well as (1) and (3) are equal, and for the top flange

$$\frac{w\,(c-x)^2}{2} = f_b \times A \times d \qquad (4)$$

which is the equation to a parabola, whose apex is at the end of the

beam, the origin of co-ordinates being at the wall end of the beam, and the ordinate at that point equal to the depth of the beam.

Whence the underside of the longitudinal section of the beam should be a parabola.

Substituting in (4) the actual value of the factors, the area of the upper flange at the wall, where the direct stress is a maximum, is obtained, thus,

$$\frac{w c^2}{2} = f_b \times A \times d$$

$$\frac{16 \times 72^2}{2} = 1{\cdot}5 \times 2240 \times A \times 7$$

whence A = 1·76 square inches,

and, as the breadth is given as 3 inches, the depth or thickness of this flange will be ·6 inch nearly.

This thickness determines that of the web.

The area of the bottom flange will be to that of the top flange as 1·5 : 8 = ·33 square inches.

It should not be less in breadth than $\frac{1}{6}$ of the breadth of the top flange, *i.e.*, 6 inch wide, which is the width of the web, so that no compression flange is necessary.

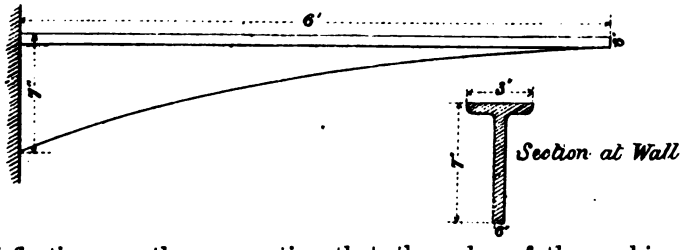
lbs. ft. lbs.

(2) The shearing stress at the wall is 192 × 6 = 1152.

The area of the web A_2 is 6·4″ × ·6″ = 3·84″, which is capable of exerting a working resistance, per square inch, of 2·33 tons, so that there is no danger of shearing.

(3) As in Example I., the deflection, owing to the inaccuracy of the formula on which the cantilever has been designed, cannot be accurately ascertained without, in the first instance, determining the value of the actual working resistance excited in the extreme fibre of the upper flange.

Fig. 18.



The deflection, on the assumption that the value of the working resistance is no more than it was intended to be, viz., 1·5 tons per sq. inch, is given by the formula, (p. 37 *Deflection of Beams*) viz.,

$$V = \frac{n'' f_o c_2}{E y_o}$$

where $n'' = 1$, $f_o = 1.5$ tons, $c = 72$ inches, $y_o = 2{\cdot}7$ inches,

whence V = .4 inches.

The deflection would be somewhat more than this actually, and it is too large for good construction. The proper deflection should not exceed $\frac{1}{140}$ inch = ·3 inch, (p. 40), and the depth of the beam should be increased, the dimensions being re-calculated.

It must be borne in mind that if the top flange is to be drilled for bolt holes, their diameter must be deducted from the width of the flange in calculation.

CASTING OF IRON GIRDERS, &c.

As to the quality of cast iron suitable for Engineering works, the points to be attended to in designing and testing castings, &c., see *Rankine's Civil Engineering*, p. 498 to 503, and p. 524, 525.

Where castings are large, and consequently too expensive to test by actual breaking, it is usual to specify that bars shall be cast at the same time as the castings, and tested. The usual size of such bars is either 1″ square or 1″ wide × 2″ deep, the latter being less affected by small flaws than the former, and giving more trustworthy results. They are placed on a clear bearing of from 3′ to 4′ 6″, and broken by a central weight.

The test weight varies according to the opinion of the Engineer; Mr. Dempsey specified 26 cwt., which seems to be too high, and Mr. Barlow 1800 lbs. for a bearing of 4′ 6″, as the weight which bars 1″ × 2″, should bear without breaking. (See *The Engineer*, 11th October, 1867.) According to Handyside and Co.'s *Works in Iron*, p. 3, a test bar, 3′ between bearings, 1″ × 2″, should take, as a breaking weight, from 25 cwt. to 30 cwt., the tougher irons standing the greater test.

It is well to insert in the specification the provision that the bearing of the central load shall not exceed 1 inch in length of the bar.

ROLLED WROUGHT IRON BEAMS.

Rolled wrought iron beams of considerable size and strength are much used in various constructions, such as joists for fireproof floors, or short bridges, bressummers, &c.

They are, of course, of uniform section throughout. The sections in ordinary use for joists or bressummers, where the beam is loaded transversely only, have the flanges symmetrical; and it is proposed in this paper to deal with such beams only. Rolled beams which are under thrust, as well as transversely loaded, such as the principal rafters in some roofs, are dealt with in the paper on Roofs.

The sheets of sections published by all iron merchants and manufacturers give the dimensions of these beams, and the simplest plan is to select from these sheets a section apparently suitable for the intended purpose, and enquire whether it actually is suitable.

In selecting a section, it would be advisable, on purely theoretical considerations, to choose, 1st., a section with broad flanges, a broad flange being capable of exerting a greater resistance to bending sideways under compression than a narrow one; 2nd, a section with a thin web, a beam of such section (provided the web is strong enough to resist the shearing stress) being both stronger and stiffer, weight for weight, than a beam with an unnecessarily thick web.

But owing to the mode of manufacture of these beams it is advisable not to have the flanges broad, if they are at the same time much thinner at the edges than the rest of the beam, and it may be said that the form of section likely to give the best results is that in which there is little variation in the thickness of the metal.

The principles on which these beams are dealt with are identical with those already given for Timber and Cast Iron Beams.

EXAMPLE I.

It is proposed to use rolled wrought iron beams, of the section shewn in *Fig.* 1. to carry a Dennett's floor. The beams are to be built into the walls at each end, and may be considered as "fixed." The clear bearing is 16 feet. The working resistances of the iron may be taken as follows:

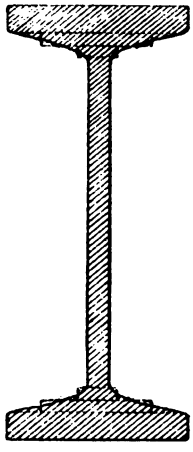
Tension	6 tons	} per square inch.
Compression	5 „	
Shearing	5 „	

Determine.

1st. The working load, uniformly distributed.

2nd. If the web is strong enough to resist the shearing stress.
3rd. The deflection.

Fig. 1.



Remarks.

This is the case of beams carrying an arched floor, similar to that in the billiard rooms at Brompton Barracks. In that floor the beams have equal flanges, and are "fixed" at the ends.

The working resistances are taken higher than in bridge work, because the load to be borne is considered as dead, it being, as usual, in floors, calculated for a number of persons so packed that they could not come on suddenly, or move, when on.

Preliminaries.

Let w tons = weight per inch run of length of beam.
„ d inches = depth of beam = 11·8 inches.
„ c inches = half the length of beam = 96 inches.
„ I = Moment of Inertia of the section about a horizontal axis passing through the centre of gravity of the section.
„ f_a tons = working resistance per square inch of wrought iron in compression = 5 tons.
„ y_a inches = distance from the neutral axis to the extreme fibre on the compression side of the section, at the section of greatest stress = in this case, $\frac{d}{2}$.
„ E lbs. = the Modulus of Elasticity (*U.R.T.*, p. 195) = 29,000,000 lbs.

Then (*Fixed Beams*, p. 56) the sections of greatest stress are at the points of fixing and the Moment of Flexure at each of those sections $= \frac{w\,l^2}{12} = \frac{w\,c^2}{3}$ (1)

The Moment of Resistance at any section of a beam (*Stress in Beams*, p. 17) $= \frac{f_o I}{y_o}$

where f_o is the working resistance per unit of section of the metal on that side of the section, which reaches its elastic limit first, and y_o is the distance from the neutral axis to the extreme fibre on that side.

In this case, as the neutral axis passes through the centre of the section, the extreme fibres are always exerting equal resistances; and those on the compression side of the section reach their elastic limit first, and determine the strength of the section.

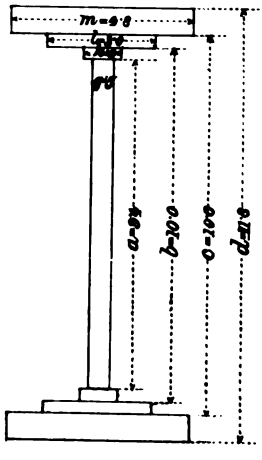
The Moment of Resistance therefore becomes $\frac{f_a I}{y_a}$ (2)

Then when equilibrium obtains (1) and (2) are equal and

$$\frac{w c^2}{3} = \frac{f_a I}{y_a} \quad (3)$$

To find the value of I, draw the section of the beam on a large scale, and describe rectangles about the flanges, so that the areas of the rectangles are equal to the area of the flanges. The more numerous these rectangles are, the more accurate will be the result, but in this case three are sufficient.

Fig. 2.



Then $I = \frac{1}{12}\left\{ga^3 + k(b^3 - a^3) + l(c^3 - b^3) + m(d^3 - c^3)\right\} = 276.$

Substituting the numerical values in (3)

$$\frac{w \times 96^2}{3} = \frac{5 \times 276}{5{\cdot}9} \text{ tons.}$$

whence $w = {\cdot}076$ tons per inch run of length of beam, and the total load uniformly distributed $= 16 \times 12 \times {\cdot}076 = 14{\cdot}59$ tons.

NOTE.—This method of calculation takes into consideration the resistance of the web to the direct stresses.

(2) The shearing stress is a maximum at the points of support (*Stress in Beams*, p. 20) where it is one half of the total load.

If then the web is strong enough at these points, it is more than strong enough at any other.

Shearing stress at the points of support $= \frac{14{\cdot}59}{2}$ tons $= 7{\cdot}29$ tons.

The sectional area of the web proper, *i.e.*, *ga* (*Fig.* 2) $= {\cdot}5'' \times 9{\cdot}4''$ $= 4{\cdot}7$ square inches.

The working resistance to shearing per square inch is given at 5 tons. Therefore the total resistance of the web to shearing is $4{\cdot}7 \times 5 = 23{\cdot}5$ tons which is about three times as strong as is necessary.

(3) The deflection of a beam of uniform section, uniformly loaded, and "fixed" at both ends is given by formula (*Fixed Beams*, p. 56), viz.

$$\text{Deflection} = V = \frac{W c^3}{E I} \frac{1}{48}$$

and substituting the values already known

$$V = \frac{14{\cdot}59 \times 2240 \times 96^3}{29{,}000{,}000 \times 276 \times 48} = {\cdot}075 \text{ inches.}$$

To find the value of I, where the section is not symmetrical above and below the neutral axis, as in a foot-rail (*Fig.* 3), draw the section on a large scale, and describe rectangles as already directed. Find the centre of gravity of the section thus transformed, and draw a horizontal line, OO, through it (*Fig.* 4).

Then $I = \frac{1}{3}\left\{ ha^3 + k(b^3 - a^3) + l(c^3 - b^3) + m(d^3 - c^3) + ne^3 + p(f^3 - e^3) + q(g^3 - f^3) \right\}$

Fig. 3.

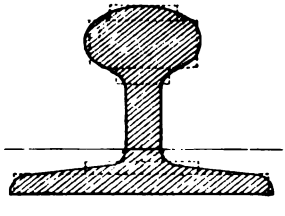
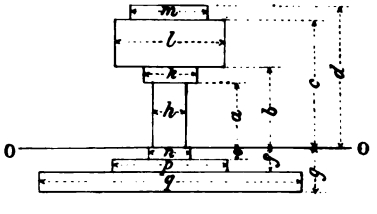


Fig. 4.



TENSION AND COMPRESSION BARS.

The beams of timber, cast iron, and rolled wrought iron considered in previous papers have been each one single piece, and of small dimensions.

Beams of large dimensions, and trusses, are often made up of timber, wrought and cast iron, each being used in the position most suitable for it; or, less frequently, of cast and wrought iron only, the use of these two materials together being objected to by some on account of their unequal expansion and contraction, (*H.I.B.*, p. 85,) and by others (*R.C.E.*, p. 560; *U.I.B.R.*, p. 70) on account of the greater pliability and brittleness of cast iron, and the rapid diminution of its resistance to crushing as the proportion of length to diameter increases; or, most generally, of a number of pieces of wrought iron, the connections in each case consisting of bolts, rivets, straps, or other contrivances.

The pieces composing the beam or truss are some in tension, some in compression, some, with a moving load, alternately in tension and compression, and it is necessary for scientific construction that all these pieces, as well as the connecting pieces, shall, as far as the practical conditions of the work allows, be equally strong to resist the maxima stresses that come upon them at any time.

In designing such a beam or truss the stresses of each kind on each piece are first ascertained, an approximate weight (*M.E.F.*; *U.I.B.R.*) determined by previous examples is assumed for the whole structure, the dimensions are then calculated, and the weight of the structure more accurately ascertained, a second calculation being made, if the difference between the assumed and actual weight of the structure is found to be considerable.

The method of determining the stresses in braced structures will be given in future papers, and it is proposed in this paper to assume that the stresses are known, and to show how to proportion the material to meet them.

TENSION BARS.

A tension bar being simply exposed to a stress of tension in the direction of its length has no tendency to break except by direct tearing.

Its strength is, therefore, simply its effective section multiplied by the working resistance per unit of section.

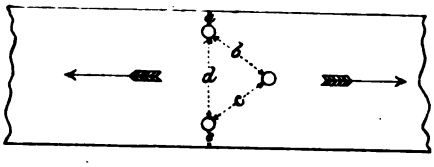
This effective section is the thickness of the bar multiplied by the length of the line along which fracture tends first to take place.

This line is called by Latham (*L.W.I.B.*, p. 15) the line of fracture, and he defines it as a "line drawn through the plate or bar (on its surface) crossing all the lines of strain in such a way that the section of the plate along it is a minimum.

"It is found by experiments on ordinary fibrous plate that the plate is more likely to break in such a line, even though it be irregular and zigzag, than in any other."

In *Fig.* 1 if $a + d + e$ is less than $a + b + c + e$, $a + d + e$ will be the line of fracture which measures the strength of the plate or bar, but if $a + b + c + e$ be the shorter of the two, it will be the line of fracture.

Fig. 1.



In computing the working strength of a tension bar of any material, it is therefore only necessary to determine the line of fracture, multiply it by the thickness of the bar and by the working resistance per unit.

The connection of tension bars or plates with each other, or with other parts of a structure, belongs to the subject of joints, and is explained further on. (*Joints and Fastenings.*)

EXAMPLE I.

The bracing of the cross girders of the Charing Cross Bridge as designed (*H.I.B.*, *Pl.* 63), consists of channel iron, $3\frac{1}{4}'' \times 1\frac{1}{4}'' \times \frac{3}{8}''$, (*Fig.* 2) connected with the flanges by rivets at each end, arranged as shown in *Fig.* 3, and they are also rivetted together at their intersections as shewn.

Fig. 2.

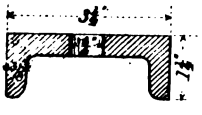
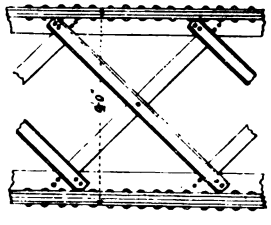


Fig. 3.



Determine the working strength of the braces in tension, taking the working resistance of wrought iron in tension at 5 tons per square inch.

The line of fracture in this case is straight through any one rivet hole, *i.e.*, $3\frac{1}{4}'' - \frac{3}{4}'' = 2\frac{1}{2}$ inches.

The effective section of the brace is therefore $(2\frac{1}{2} + \frac{1}{8}) \frac{3}{8} = \frac{61}{64}$ square inches, and the working strength is $\frac{61}{64}'' \times 5$ tons = 8 tons nearly.

COMPRESSION BARS.

The subject of compression bars is fully gone into in *Rankine's Applied Mechanics*, p. 360 to 366; *Rankine's Civil Engineering*, p. 232 to 239, and p. 520 to 524; and some further information is given in p. 49, *Unwin's Iron Bridges and Roofs*.

It is not therefore proposed to do more in this paper than to state generally the principles on which the results are arrived at, and to illustrate the same by examples.

A compression bar tends to fail in one of two ways, according as the ratio which its length bears to its smallest transverse dimension exceeds or does not exceed a certain limit.

Approximately, so long as the length of a compression bar, pillar, or strut of cast iron does not exceed five times its least dimension, it tends to fail by direct crushing; while if its length exceeds this proportion, it tends to fail by bending sideways and cross breaking.

In wrought iron, the proportion of length to least dimension, up to which failure tends to take place by direct crushing, is approximately 10, while in dry timber it is approximately 20.

It is possible that steel of average hardness would occupy an intermediate place between cast and wrought iron, but there is at present no definite information on the subject.

NOTE.—Much simpler formulæ for long compression bars are given in Reuleaux's Der Constructeur. (Ed.: 1869, p. 48). Reuleaux also makes the ratio of length to least transverse dimension at which failure by bending begins to tend to occur, vary with the manner in which the ends are secured, which is manifestly correct. As, however, the formulæ given further on have been found to give safe results, and as Reuleaux's formulæ with the same factor of safety gives smaller dimensions to meet a given thrust, it has been considered advisable to adhere to the formulæ in general use in this country.

SHORT COMPRESSION BARS.

To find the working resistance of a short compression bar, the length of which does not bear to its least dimension a greater ratio than above mentioned, it is only necessary to multiply the effective sectional area by the working resistance per unit.

The effective sectional area in the case of a post with a tenon let into a stone, iron, or hard wood sill, is the sectional area of the post minus the sectional area of the tenon.

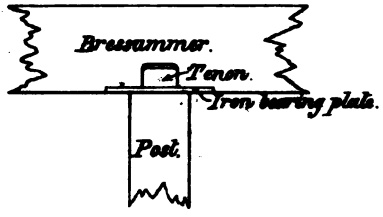
In compression bars drilled or punched for bolts or rivets the effective sectional area is the whole sectional area of the bar, no deduction being made for rivet or bolt holes, which, in good work are completely filled up by the bolts or rivets.

EXAMPLE II.

A dry fir (red pine) post, square in section, and 8 feet long, has to carry a weight of 10 tons.

Its head is tenoned into a bressummer, and is prevented cutting into the timber of the bressummer by a wrought iron bearing plate, as shown in *Fig.* 4.

Fig. 4.



Its foot is fitted with an iron spud, the projecting part or tenon of which is mortised into a stone sill.

Determine its dimensions, the tenon being taken at $4'' \times 1\frac{1}{2}''$, and the working resistance of the timber in compression being obtained from the Tables (*U.R.T.*, p. 198).

Remarks.

This is the ordinary case of a timber story post. The object of the tenons at the head and foot of the post is to prevent lateral motion and to afford the means of connecting the work together.

The tenons are cut shorter than the mortises, so as to ensure the bearing coming on the shoulders of the post, and do not therefore sustain any thrust.

The object of the bearing plate is to spread the pressure over a larger surface, and to prevent the post sinking into the bressummer, which would cause a loose joint.

Without this precaution, the safe pressure on the shoulders of the post could not exceed 1000 lbs. on the square inch. (*H.T.C.*, p. 103.)

If the bressummer were of oak, the safe pressure might be increased to 1400 lbs. on the square inch. (*H.T.C.*, p. 104.)

Preliminaries.

Let x inches = side of post.

l ,, = length = 96 inches.

f lbs. = ultimate resistance to crushing per square inch of dry fir (red pine), (*U.R.T.*, p. 198,) = 6000 lbs.

5 = factor of safety. (*U.R.T.*, p. 205.)

W lbs. = total weight to be borne = 10×2240 lbs.

On the supposition that the length will not exceed the side more than 20 times, the post will not bend.

Then the area of the section under compression $= x^2 - 4 \times 1\frac{1}{2} = x^2 - 6$, and the total working resistance $= (x^2 - 6)\frac{6000}{5}$

When equilibrium obtains

$$(x^2 - 6)1200 = 10 \times 2240$$

$$x^2 - 6 = \frac{10 \times 2240}{1200} = \frac{224}{12} = 18{\cdot}66$$

$$x^2 = 24{\cdot}66$$

$$x = 5 \text{ inches.}$$

This proportion is just within the ratio given, as safe against bending.

If the post were 10 feet long the dimensions would be calculated in the way shown in *Example* V., as a post 10 feet long and 5 inches square would be liable to bend under the weight.

If it were not considered desirable to put in a bearing plate, the safe compression on the shoulders with a fir bressummer would be 1000 lbs. per inch, and x would then be 5·3 inches.

LONG COMPRESSION BARS, PILLARS OR STRUTS.

When the length of a compression bar exceeds, approximately, the proportion already stated, it tends to fail by bending sideways and breaking across, and not by direct crushing.

The theory on this subject is by no means complete, and experimental data are much wanted, but the formulæ which have been arrived at, although approximate only, give results which experience shows the Engineer can depend upon, without apparently incurring any very serious waste of material.

The general principles on which the formulæ, originated by Professor Gordon, and given by Professor Rankine, are based, may be stated as follows:

When a pressure is applied to a long bar in the direction of its length the centre of pressure being at the centre of figure of the transverse section, the first effect is to compress the fibres uniformly all over its transverse section.

This gives a pressure per unit of sectional area (intensity of stress)

$$= \frac{\text{Total pressure}}{\text{Total sectional area}} = \frac{P}{S}$$

But, if the pressure be sufficient, a long bar bends in the direction of its smallest transverse dimension, by which bending the pressure becomes unequally distributed over the transverse section, a further compression of the fibres on the concave side taking place, while the fibres of the convex side are relieved to an equivalent extent.

In page 360, *Rankine's Applied Mechanics*, it is shewn that the additional stress due to bending is to the stress due to the direct pressure in a ratio, which increases as the square of the proportion, in which the length of the pillar exceeds the least diameter, and the whole intensity of the stress on the concave side, *i.e.*, where it is greatest, becomes

$$\frac{P}{S} + \frac{P}{S}\, a \frac{l^2}{h^2}$$

a being a constant coefficient, deduced from experiment, for different materials, and h being the least transverse dimension, or least dimension of a triangle or rectangle circumscribing the section of the bar, *i.e.*,

$$f = \frac{P}{S}\left(1 + a\,\frac{l^2}{h^2}\right)$$

where f is the ultimate resistance to crushing of a short column of the

material, and a is a constant deduced from Mr. Hodgkinson's experiments on the actual breaking of long pillars, fixed at the ends.

The values for f and a for pillars fixed at both ends are as follows (*R.C.E.*, p. 237.)

	f lbs. per inch.	a
Wrought iron (rectangular struts)	36,000	$\frac{1}{3000}$
Cast iron (cylindrical struts)	80,000	$\frac{1}{6400}$
Timber (rectangular pillars)	7,200	$\frac{1}{3000}$

and for a pillar rounded or jointed at both ends

$$f = \frac{P}{S}\left(1 + 4a\frac{l^2}{h^2}\right)$$

Where wrought iron of the form of angle iron, channel iron, T iron, double T or ✚ iron is used, (*Fig.* 5) the value of h may be taken as the least dimension of a triangle or rectangle circumscribed about the bar; thus

Fig. 5.

and where a strut is made up of irons stiffened by bracing as in the case of the Charing Cross bridge, *Example* IV, h may be taken as the diameter of the compound strut in that direction in which it is most flexible.

Where cast iron of the forms

is used for struts, fixed at both ends, Professor Rankine (*R.C.E.*, p. 520) gives the value for a as $\frac{3}{4000}$, $\frac{3}{8000}$, and $\frac{1}{4000}$ respectively, each being multiplied by 4 when the strut is hinged at both ends.

In page 522, *Civil Engineering*, he modifies the formula as above given for rectangular wrought iron bars, by taking into account the least radius of gyration r of the cross section measured from a neutral axis traversing its centre of gravity in that direction which makes r least. And in *Useful Rules and Tables*, p. 210, he makes this same formula applicable to struts of wrought iron, cast iron, and timber, adding a rule (XXIV) for a strut fixed at one end and jointed at the other, the strength of which Hodgkinson found to be an arithmetical mean between the strength of two pillars of the same length and diameter, one fixed at both ends, and the other rounded at both ends.

All these formulæ being approximate only, it has been considered best in this paper to work the Examples by the methods which have been hitherto in most general use; 1st, because they have been found to give safe results, and 2nd, because they are in many cases simpler and easier to work.

The factor of safety should be such as to reduce the value of the ultimate strength of the material to the working resistance per unit in compression of other parts of the same structure.

EXAMPLE III.

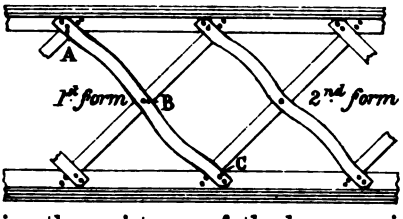
Determine the working strength of the braces in compression at the centre of the cross girders of the Charing Cross Bridge as designed (*H.I.B., Pl.* 63), *Example* I, the working resistance per square inch in compression being taken at 4 tons.

Remarks.

The rivetting together of the braces at their intersection stiffens the compression braces against bending in the direction of the length of the girder, and adds somewhat to the stiffness in the direction transverse to the girder. It is, however, impossible to say how much it adds to the stiffness, and as it cannot be much, it is thought better to leave it out of consideration.

If bending were to take place in the first direction, the compression braces would assume one of the forms shown to an exaggerated scale in *Fig.* 6.

Fig. 6.



and in calculating the resistance of the brace against bending in this direction, l may be taken as the central distance between the rivets A and B; and h as the width of the channel iron; viz.: $3\frac{1}{4}$ inches.

The part of the brace between A and B is fixed at one end and hinged at the other, but if there were two rivets at the centre it would be fixed at both ends.

In calculating the resistance of the brace against bending transversely to the girder, l must be taken as the central distance between the rivets A and C, and h as the depth of the channel iron; viz.: $1\frac{1}{4}$ inches.

Preliminaries.

Let P tons = total working resistance of brace.

„ S square inches = sectional area of brace = 1·875 square inches, rivet holes not being deducted in compression braces.

„ l inches = length = 24 inches in first case and 48 inches in second case.

„ h inches = dimension in direction in which bending will take place = $3\frac{1}{4}$ inches in first case, and $1\frac{1}{4}$ inches in second case.

Let $\frac{f \text{ tons}}{\text{factor of safety}}$ = working resistance of iron per square inch in compression = 4 tons.

„ $a = \frac{1}{3000}$, if the brace is fixed at both ends.

„ $a = \frac{1}{1500}$, if the brace is hinged at both ends.

Case 1.—Bending in direction of the length of the girder,

The ratio of l to h in this case is only a little more than 7 to 1, and there is no tendency to bend in this direction.

Had this ratio been greater than 10 to 1, it would have been necessary to calculate the brace both as a pillar fixed at both ends, and as a pillar hinged at both ends, the safe load being given by the mean of these two.

Case 2.—Bending transversely to the girder.

$$\frac{P}{S} = f \div \left(1 + a\frac{l^2}{h^2}\right) = 4 \div \left(1 + \tfrac{1}{3000}\frac{48^2}{(\frac{1}{2})^2}\right)$$

$$= \frac{4}{1{\cdot}49} = 2{\cdot}7 \text{ tons nearly}$$

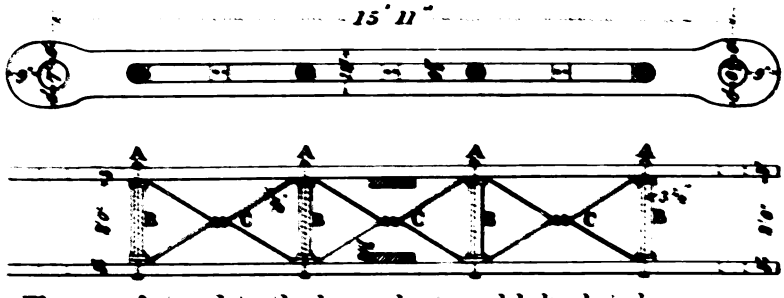
$$\text{and } P = 5{\cdot}06 \text{ tons.}$$

The strength of the tension brace is therefore greater than the corresponding compression brace, but this is often unavoidable, particularly in small girders, where it is not possible to reduce the bars to the theoretical dimensions, or where uniformity of construction is of more importance than the saving of a small quantity of metal.

EXAMPLE IV.

The end compression bars of the main girders of the Charing Cross bridge are of the form and dimensions shown in *Fig.* 7.

Fig. 7.



They are fastened to the booms by turned bolts, but they are not fastened to the tension bars at their intersection with them.

Determine the working resistance per square inch of which they are capable, and their total resistance, the working resistance of iron in compression per square inch being taken at 4 tons.

Remarks.

These bars consist of two iron bars, each 12" × 3", kept together by

screw bolts A, A, passing through distance pieces B, B, and stiffened by bracing C, C.

The duty of all the parts, except the two bars themselves, is merely to stiffen the two bars, and it is their section only which resists the compression.

The value of h in direction of the length of the girder is therefore 12 inches, while transversely to the girder it is 30 inches.

The length from centre to centre of bolts being 15′ 11″, the ratio of l to h in the second case is 6·4 to 1, and there is no danger of bending transversely.

It is possible also for compression bars of this kind to buckle between the points A, A, if these are too far apart, and it is necessary to inquire into this.

In the first case, the ratio of l to h is 16 to 1 nearly.

Preliminaries.

Let P tons = total safe compression on the bar.

„ S square inches = sectional area = 72 inches.

„ l inches = length of bar from centre to centre of bolt holes = 191 inches.

„ h inches = breadth of bar in direction in which bending tends to take place = 12 inches.

„ $\frac{f \text{ tons}}{\text{factor of safety}}$ = working resistance of iron in compression = 4 tons per square inch.

„ $a = \frac{4}{3000}$, the bars being hinged at the ends.

Then for bending in the plane of the girder,

$$\frac{P}{S} = f \div \left(1 + a\frac{l^2}{h^2}\right) = 4 \div \left(1 + \tfrac{4}{3000}\left(\tfrac{191}{12}\right)^2\right)$$

$= 3$ tons nearly = working resistance per square inch.

and P = 216 tons.

And for bending between A and A.

The distance between the points A, A, is 47″, and the ratio of l to h is 16 to 1 nearly, but these parts are fixed at the ends;

$$\therefore \frac{P}{S} = f \div \left(1 + a\frac{l^2}{h^2}\right) = 4 \div \left(1 + \frac{1}{3000}256\right)$$

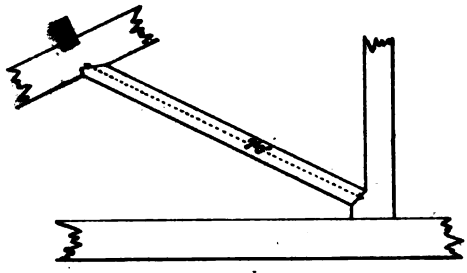
$$= \frac{4}{1{\cdot}08} = 3{\cdot}53 \text{ tons.}$$

i.e., the bar is less liable to buckle between A and A, than as a whole in the plane of the girder, and consequently the points A A are not too near together.

EXAMPLE V.

The strut of a queenpost roof is of dry fir, and is 3″ wide. Its length in the clear is 7 feet.

Fig. 8.



Determine its depth, so that it may resist without bending a compression of 5000 lbs.

Remarks.

As the length of the strut is 7 feet, and its breadth 3 inches, the ratio of l to h is 28 to 1, and the strut would tend to bend in a direction transverse to the plane of the truss, of which it forms a part.

The strength of the strut is measured by the greatest compression on any fibre, and the fibre on which the compression is greatest is on the concave side at the centre of the length of the strut. The strut would not therefore fail at the ends, and no deduction need be made for tenons.

The timber in all these cases is supposed to be seasoned enough for use in carpenters work, green timber being far weaker in compression than seasoned timber. *Note*, p. 198, *U.R.T.**

Preliminaries.

Let P lbs. = total compression = 5000 lbs.

„ d inches = depth of strut.

„ b „ = breadth of strut = 3 inches.

„ l „ = length = 84 inches.

„ S sq. inches = area of strut = $3d$ square inches.

„ $\frac{f \text{ lbs.}}{\text{factor of safety}}$ = working resistance of dry timber in compression per square inch = $\frac{7200}{5} = 1440$ lbs. (p. 86.)

„ $a = \frac{1}{3000}$, the strut being considered as fixed at the ends.

Then $\frac{P}{S} = f \div \left(1 + a\frac{l^2}{h^2}\right)$

$$\frac{P}{S} = 1440 \div (1 + \tfrac{1}{3000}(28)^2) = \frac{1440}{4{\cdot}13} = 349 \text{ lbs.}$$

* The term "dry" timber is here evidently applied to timber which is sufficiently "seasoned" for common use, *i.e.*, when it has lost $\frac{1}{4}$ of the weight it had at the time of felling (*H.T.C.*, p. 343), which requires not less than two years air seasoning. The term "dry" is applied to timber which has lost $\frac{1}{5}$ of its weight, and is fit for the joiners purposes. (*H.T.C.*, p. 344.) This takes four years air seasoning from the time of felling.

Whence $S = \frac{P}{349}$

$$d = \frac{5000}{3 \times 349} = 5 \text{ inches nearly.}$$

Tredgold gives 5 × 3 for the strut of a queenpost roof of 45 feet span; which would correspond nearly with the strut in this example.

Supposing the tenons at the ends of the strut to be $4'' \times 1\frac{1}{2}''$, the bearing area of the shoulders is $15'' - 6'' = 9''$, and the pressure on them per square inch is 555 lbs., which is quite safe.

EXAMPLE VI.

Determine the external diameter of a cast iron hollow column, 10 feet high, to carry a dead weight of 10 tons, the ends to be flat and firmly fixed (*Fig.* 9), and the weight borne over the centre of the column, the thickness of metal to be $\frac{1}{12}$ of the external diameter.

Remarks.

This is the case of an ordinary column under a bressummer.

The column may bend in any direction, so that h equals the diameter.

Preliminaries.

Let d inches = external diameter = h.

„ l inches = length = 120 inches.

„ P lbs. = total compression = 10 × 2240 lbs.

„ S sq. inches = $\frac{\pi}{4}\left(d^2 - \frac{11}{12}d^2\right) = \frac{\pi}{4} \times \frac{1}{12}d^2$

„ f lbs. = ultimate resistance of cast iron to crushing per sq. inch. = 80000 lbs. (*U.R.T.*, p. 211.)

„ 4 = factor of safety, which gives 20000 lbs. as the working resistance per sq. inch.

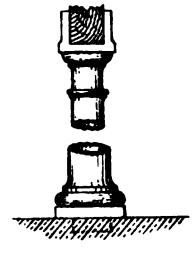
„ $a = \frac{1}{400}$ for cast iron columns fixed at both ends.

Then $\frac{P}{S} = 20000 \div \left(1 + \frac{1}{400}\frac{120^2}{d^2}\right)$

$$\frac{22400}{\frac{\pi}{4} \times \frac{1}{12}d^2} = \frac{20000 \times 400d^2}{400d^2 + 120^2}$$

whence $d = 4$ inches nearly.

Fig. 9.



JOINTS AND FASTENINGS.

The principles which should guide the design and execution of all kinds of joints and fastenings in carpentry are stated by Professor Rankine, (*R.C.E.*, p. 454,) as follows:

I. To cut the joints and arrange the fastenings so as to weaken the pieces of timber that they connect as little as possible.

II. To place each abutting surface in a joint as nearly as possible perpendicular to the pressure which it has to transmit.

III. To proportion the area of each surface to the pressure which it has to bear, so that the timber may be safe against injury under the heaviest load which occurs in practice; and to form and fit every pair of such surfaces accurately, in order to distribute the stress uniformly.

IV. To proportion the fastenings, so that they may be of equal strength with the pieces which they connect.

V. To place the fastenings in each piece of timber, so that there shall be sufficient resistance to the giving way of the joint by the fastenings shearing or crushing their way through the timber.

The same principles apply in every respect to joints in iron work, and to both may be added,

VI. To arrange the joints and fastenings in such a way that the stress in each piece shall be distributed as nearly as possible over its cross section (p. 102, *et seq.*)

It will probably best suit the purpose of these papers to work a few typical examples, referring to the following authors for the information on which they are worked:

Hurst's Tredgold's Carpentry, p. 297, *et seq.*
Rankine's Civil Engineering, p. 454, *et. seq.*
Latham's Wrought Iron Bridges, p. 15, *et. seq.*
Humber's Iron Bridges, p. 148, *et. seq.*
Unwin's Iron Bridges and Roofs.

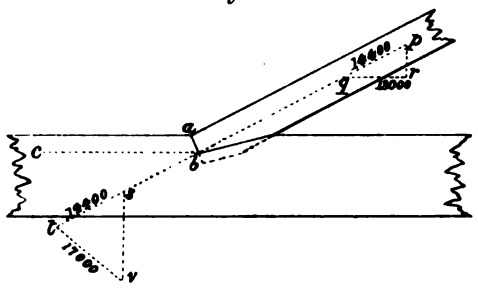
EXAMPLE I.

The principal rafter and tie of a king-post roof are of red pine (Scotch fir), and are connected together, as shewn in *Fig.* 1.

The principal is 4″ × 4″, the tie beam is 10″ × 4″, the abutment *a b* of the rafter is 2″ deep, the tenon occupies $\frac{1}{5}$ of the width of the joint and does not bear on the mortise.

The principal is 6 feet long between the points of support, and being only 16 times as long as its smallest dimension, will not tend to fail by bending.

Fig. 1.



Determine.

1st. What thrust the principal is competent to bear with safety, following the rule given in *Hurst's Tredgold's Carpentry*, p. 103, that the transverse pressure on yellow fir should not be greater than 1000 lbs. per square inch. The pressure in this case is not directly transvere to the beam, and the limit might be taken somewhat higher.

2nd. The length of the part *bc*, so that, supposing no iron straps to be used, there may be no danger of the part of the tie above *b c* being shorn off by the horizontal component of the thrust.

3rd. The minimum dimensions of an iron strap of the form and in the position shewn in *Figs.* 2 and 3, supposing that *bc*, (*Fig.* 1) cannot be made long enough to resist the horizontal component of the thrust.

Fig. 2.

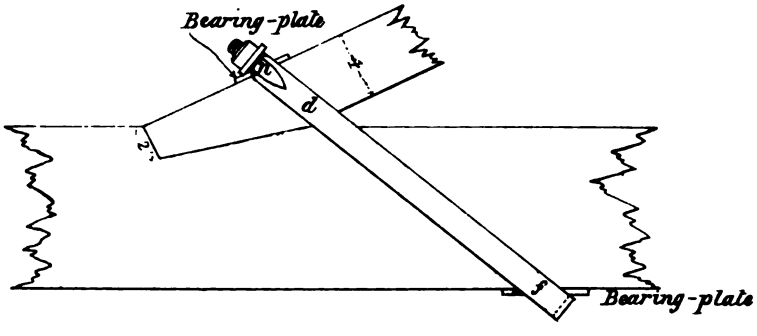
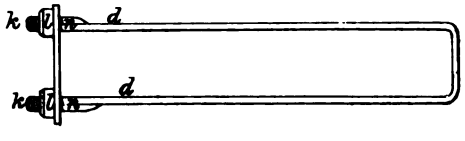


Fig. 3.



Remarks.

This is a common form for the joint connecting a principal rafter and tie beam.

There are numerous other forms, given in books on carpentry, having different advantages, which may be treated in a manner similar to this example.

The necessary strength of the joint is determined by the safe thrust down the principal, which is the bearing surface of the end of the rafter on the tie, measured perpendicularly to the thrust, multiplied by the safe compression per square inch.

Failure in a joint of this form may tend to occur.

1st. By the crushing of the fibres of the tie under the thrust, by which a loose joint, and derangement of the parts of the truss are caused.

2nd. By the shearing of the fibres of the tie along *bc*, (*Fig.* 1).

3rd. By the strap tearing across at *d d*, (*Fig.* 3).

4th. By the bottom of the strap cutting into the tie at *f*, or the top cutting into the principal at *a*, causing a loose joint, (*Fig.* 2).

5th. By the screw of the strap tearing across *s s*, (*Fig.* 3).

6th. By the screw of the strap shearing along *kl*, *kl*, (*Fig.* 3).

Preliminaries.

Let T lbs. = total safe thrust down the principal.

„ A square inches = bearing surface of principal rafter on tie beam.

„ x inches = length of part *b c*.

„ 1000 lbs. = safe thrust per square inch of bearing surface. (*H.T.C.*, p. 103.)

„ 600 lbs. = ultimate resistance to shearing per square inch of average fir. (*U.R.T.*, p. 197.)

„ 5 = factor of safety. (*U.R.T.*, p. 205.)

(1.) The bearing surface A at right angles to the direction of the thrust = $4'' \times 4'' - 2'' (\frac{1}{2} \times 4'') = 16 - 1{\cdot}6 = 14{\cdot}4$ square inches.

Then the total safe thrust, $T = \overset{\text{sq inches.}}{14{\cdot}4} \times \overset{\text{lbs.}}{1000} = \overset{\text{lbs.}}{14{\cdot}400}$.

(2.) Resolve this horizontally and vertically, see *Fig.* 1, where $pq = 14{,}400$, qr = horizontal component of the thrust, which is the force tending to produce shearing along $b\,c = 13{,}000$ lbs. (1)

The area of the surface resisting shearing $= 4 \times x$, and the total resistance to shearing $= 4 \times x \times \frac{600}{5}$ (2)

When equilibrium obtains (1) and (2) are equal, and $13{,}000 = 4 \times x \times 120$.

$$x = 27{\cdot}1 \text{ inches.}$$

It is seldom possible to give the end of a fir tie-beam sufficient length to resist shearing by itself, and an iron fastening of some kind is resorted to.

The most common form is shown in the figure, and the more nearly it approaches the horizontal direction, the more directly it resists the shearing force, and the smaller it may be made.

(3.) The tension on the strap is ascertained, as follows: Set down *st* (*Fig.* 1.) on any scale to represent the thrust T = 14,400 lbs., and resolve it vertically and parallel to the direction of the strap; then *tv* represents on the same scale the tension on the strap = 17,000 lbs. = 7·6 tons. (3)

Let the strap be of $\frac{3}{8}$-inch iron, and let its breadth be b; also let the working resistance of bar iron in tension be taken at 6 tons per square inch.

Then as the strap has two sections resisting tension, viz., d, d (*Fig.* 3), the total area resisting tension $= 2 \times b \times \frac{3}{8}$, and the total resistance $= 2 \times b \times \frac{3}{8} \times 6$ tons. (4)

As (3) and (4) are equal, when equilibrium obtains,

$$7{\cdot}6 = 2 \times b \times \tfrac{3}{8} \times 6$$
$$b = 1{\cdot}7, \text{ say } 1\tfrac{3}{4} \text{ inches.}$$

(4.) The pressure of the strap on the under side of the tie is equal to the tension on the strap = 17,000 lbs., and the surface over which it is distributed is $4'' \times 1\frac{3}{4}'' = 7$ square inches.

The transverse pressure of the iron on the fir should not exceed 1000 lbs. per square inch.

It therefore becomes necessary to employ some means of distributing the pressure of the strap on the tie over a large surface, which may be done by inserting a plate of iron having a surface of 17 square inches between the strap and the tie (*Fig.* 2), and a similar plate between the strap and the principal (*Fig.* 2).

(5.) The tension on each of the screws is equal to half the total tension on the strap, and the sectional area of each screw, clear of the threads, must therefore be equal to the sectional area of the strap, *i.e.* if d = diameter of screw, clear of threads,

$$\frac{\pi d^2}{4} = 1\tfrac{3}{4} \times \tfrac{3}{8}$$
$$d = {\cdot}9 \text{ inch.}$$

If the screws be Whitworth's, cut on $1\frac{1}{4}$ inch round iron (*M.E.F.*, p. 79, 80,) they will have sufficient area, and their clear diameter will be $1{\cdot}25 - 2\,(\frac{1}{7}) = {\cdot}96$ inch.

(6.) The shearing force, tending to strip each screw, is equal to the tension on each screw, *i.e.*, half the whole tension on the strap.

A screw may be subjected to a stress of 2 tons* (*H.I.B.*, p. 151) per square inch of its shearing surface, *i.e.*, its circumference at the bottom of the thread multiplied by the length of nut.

If l be this length, the shearing surface $= {\cdot}96 \times 3{\cdot}1416 \times l$ and the resistance to shearing $= {\cdot}96 \times 3{\cdot}1416\, l \times 2$ tons. (5)

$$\text{Hence } {\cdot}96 \times 3{\cdot}1416 \times l \times 2 = \frac{7{\cdot}6}{2}$$
$$\text{or } l = {\cdot}58.$$

The table (*M.E.F.*, p. 79,) gives $1\frac{1}{4}''$ as the proper depth for the nut of a $1\frac{1}{4}''$ bolt.

The dimensions of the straps in this Example are larger than are ordinarily used in practice, but where smaller straps are used, the

* This, from experiments made for the War Office, appears to be too great an intensity of stress. The ultimate shearing resistance in bolts of 3″ diameter, with 5 semi-circular threads to the inch, was found to be 6 tons per sq. inch; while, in bolts of 2″ diameter with 10 semi-circular threads to the inch, it was found to be only 4 tons per square inch. For structures for moving loads a working resistance of about 1 ton per square inch might perhaps be taken and for structures for dead loads, a working resistance of about 1,5 tons per square inch.

intensity of the stress must, at times, be greater than 6 tons for tension and 2 tons for shearing of the screws.

The maximum stress is produced very occasionally, *i.e.*, during the heaviest gales of wind blowing broadside on: and it may, therefore, generally be safe to take higher values of the working resistances.

EXAMPLE II.

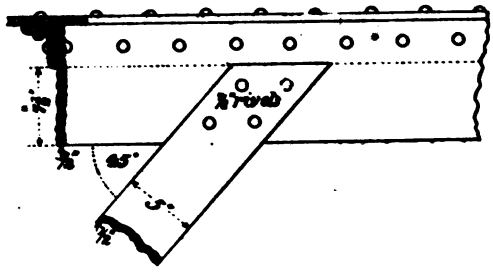
A tension bar of a lattice girder is rivetted to the flanges.

The bar is $5'' \times \frac{1}{2}''$ in section, and is subject to a stress of 5 tons per square inch of the effective section.

The rivets are $\frac{7}{8}''$ diameter in rows of two. (*Fig.* 4.)

Determine the number of rivets necessary for shearing resistance at 4 tons, and bearing resistance at 5 tons per square inch, no rivet hole being punched nearer to the edge of the bar than its own diameter, or nearer to the next rivet than $1\frac{1}{2}$ times its own diameter, clear interval.

Fig 4.



Remarks.

This is the case of the joint of a tension brace of a lattice girder. The rivets should be so arranged, that as many as possible of them may also hold the compression brace A, which is rivetted on to the other side of the vertical plate of the flange.

The necessary strength of the joint is determined by the strength of the brace which is the effective section of the brace multiplied by the working resistance of the iron in tension, viz., 5 tons per square inch.

Failure in this joint may tend to occur: 1st, by shearing of the rivets; 2nd, by insufficiency of bearing area.

The diameter of the rivets is that in ordinary use (*U.R.T.*, p. 209, *Rule* XXI), and this diameter should be kept as small as possible, consistently with good construction, in order to avoid weakening the brace.

As it is impossible that, even in a structure of moderate dimensions, all the rivets should be equally well fitted, and under equal stress, some of them must have upon them a greater stress than the mean stress, and it is necessary in order to prevent any weakness arising from this source, to allow for it by making all the rivets somewhat larger than is necessary to meet the mean stress (*U.R.T.*, p. 209, *Rule* XX.), or the same result is attained, as in this case, by taking a low shearing resistance.

The rules as to punching holes are taken from *Latham's Wrought Iron Bridges*, p. 11.

The arrangement of the rivets should be such that, while they firmly fix the brace to the flange, the line of fracture (*Tension and Compression Bars*, p. 82) should be as long as possible.

Preliminaries.

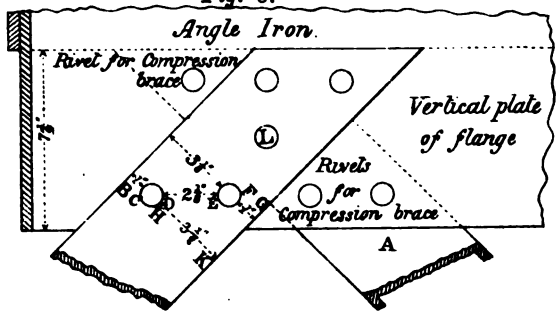
Let n = number of rivets, and let them be arranged in two horizontal rows, as shown in *Figs.* 4 and 5.

„ 5 tons = working resistance per square inch in tension.

„ 5* tons = working resistance per square inch of bearing area.

„ 4 „ = „ „ „ to shearing.

Fig. 5.



The line of fracture will be either BC, DE, FG, or BC, HK, these being equal, and the shortest lines of solid iron, through the first row of rivets.

The effective section of the brace is therefore $4\frac{1}{8}'' \times \frac{1}{2}'' = \frac{33}{16}$ square inches, and the total stress to be conveyed through the rivets is $\frac{33}{16} \times 5$ tons $= 10\frac{5}{16}$ tons. (1)

(1) To find the number of rivets to resist shearing.

The resistance to shearing of all the rivets together is

$$n \times \frac{\pi}{4} \times \left(\frac{7}{8}\right)^2 \times 4 \text{ tons.} \qquad (2)$$

As (1) and (2) are equal

$$10\tfrac{5}{16} = n \times \tfrac{\pi}{4} \times (\tfrac{7}{8})^2 \times 4,$$

whence $n = 4{\cdot}3$ nearly.

Two rows of rivets would not suffice, while 3 rows would give one too many, which might be met by inserting a rivet L near the centre.

(2) To find the number of rivets necessary to give sufficient bearing area.

The bearing area of a rivet on a plate is the diameter of the rivet multiplied by the thickness of the plate. (*L.W.I.B.*, p. 19.)

$$\text{Bearing area} = n \times \tfrac{7}{8} \times \tfrac{1}{2}.$$

$$\text{Total bearing resistance} = n \times \tfrac{7}{16} \times 5 \text{ tons.} \qquad (3)$$

* Mr. Unwin, p. 105, *Iron Bridges and Roofs*, thinks that in chain rivetted joints 7½ tons per square inch of bearing area might be taken as the working resistance.

As (1) and (3) are equal.

$$10\tfrac{9}{16} = n \times \tfrac{11}{4}$$

whence $n = 4{\cdot}7$

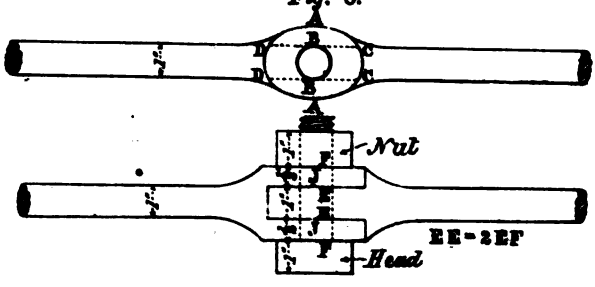
so that the number of rivets necessary for strength is also sufficient for bearing area.

Note.—For a better arrangement of the rivets in this joint, see p. 105, et seq.

EXAMPLE III.

A wrought iron tension rod, 1-inch diameter, and bearing a tensile stress of 5 tons to the inch, is jointed in the manner shewn in *Fig.* 6.

Fig. 6.



Determine:

1st. The diameter of the bolt necessary to resist shearing; and 2nd, to have sufficient bearing area.

3rd. The depth of the parts AB, so that they shall have on them the same intensity of stress as the rod itself.

4th. The depth of the parts BC, BD, so that they may have on them only a proper shearing stress.

Remarks.

This is a common method of forming the joints of tie rods in iron roofs.

The necessary strength of the joint is determined by the tension on the rod, *i.e.*, the section of the rod, multiplied by the working resistance per square inch of the iron in tension, which is given at 5 tons.

Failure in this joint may tend to occur:

1st. By the shearing of the bolt on the surfaces at E, E.

2nd. By the bolt cutting into the eye of the rod, or the eye of the rod into the bolt, producing a loose joint, consequent unequal distribution of the stress, and bending of the head and sides of the eye. (*U.I.B.R.*, p. 105, 106.)

3rd. By the tearing, either of the jaws, or of the tongue of the eye of the tie rod, at AB, AB.

4th. By the shearing, either of the eye of the jaws of the tie rod along BC, BC, or of the eye of the tongue along BD, BD.

In making a joint of this kind, the rod should be gradually rounded off into the tongue and jaws as shown in the figure.

Preliminaries.

Let d = diameter of bolt in inches.
„ b = breadth of eye at AB, in inches.
„ c = depth of eye at BD or BC „
„ t = thickness EE of tongue T „
Then $\frac{1}{2}t$ = thickness EF of jaws J, J „
Let 5 tons = working resistance per square inch in tension.
„ 4 tons = „ „ „ „ „ „ to shearing.
„ 5 tons = „ „ „ „ „ „ of bearing area.

(1.) The tension on the tie rod $= \frac{\pi}{4} \times 1 \times 5$ tons (1)

The bolt has two shearing surfaces, E, E, each of which has an area $= \frac{\pi}{4} \times d^2$, and the total working resistance, supposing the bolt to fit tight, $= 2 \times \frac{\pi}{4} \times d^2 \times 4$ tons.

If, however, full allowance be made for the joint not being tight, the working resistance should only be taken at $\frac{3}{4}$th of this (*U.R.T.*, p. 209, *Rule* XX.); and the working resistance becomes $2 \times \frac{3}{4} \times \frac{\pi}{4} \times d^2 \times 4$ (2)

As (1) and (2) are equal.

$$\frac{\pi}{4} \times 1 \times 5 = 2 \times \tfrac{3}{4} \times \frac{\pi}{4} \times d^2 \times 4$$

whence $d^2 = \frac{5}{6}$

$$d = \cdot 91$$

(2.) The bearing area of the bolt and eye on each other is the diameter of the bolt, multiplied by the width of the surfaces in contact, *i.e.*, the thickness of metal (*L.W.I.B.*, p. 19) $= d \times t$, and the working resistance of the bolt and eye each $= dt \times 5$ tons. (3)

As (1) and (3) are equal.

$$\frac{\pi}{4} \times 1 \times 5 = dt \times 5 = d \times 1 \times 5$$

whence $d = \cdot 7854$,

so that a smaller diameter being required for sufficient bearing area, than for resistance to shearing, the larger diameter would be adopted.

(3.) The sectional areas at AB, AB, are equal in the tongue and in the jaws, and are $2\,bt$, or $4\,b\frac{t}{2}$.

The working resistance to tearing is, therefore, $2\,bt \times 5$ tons. (4)

As (1) and (4) are equal.

$$\frac{\pi}{4} \times 1 \times 5 = 2bt \times 5 = 2b \times 1 \times 5$$

whence $b = \cdot 39$

As allowance has been made for a loose joint in the bolt, an allowance should also be made in this dimension. Sir Charles Fox (*U.I.B.R.*, p. 106) recommends in the case of suspension links, that the metal round the bolt should be $\frac{1}{10}$th wider than the width of the body of the link, the diameter of the bolt being not less than $\frac{2}{3}$rds the width of the body.

This is equivalent to an increase of $\frac{1}{10}$th of the sectional area, and following this recommendation, the breadth may be increased to ·43 inch.

Professor Rankine (*R.C.E.*, p. 518, 519) recommends, on account of the tension being unequally distributed over the sides of the eye, that their area should be made one-half greater than would be necessary, if the tension were uniformly distributed. This would make $p = \cdot 6$ nearly.

(4) The shearing surfaces are, in the case of the jaw, 4 in number, and in the case of the tongue 2 in number, the former being each $\frac{1}{2} t \times c$, the latter being each $t \times c$.

The total resistance to shearing in each case is $2t \times c \times 4$ tons (5)

The force producing shearing is given by equation (1)

As (1) and (5) are equal,

$$\frac{\pi}{4} \times 1 \times 5 = 2t \times c \times 4 = 2c \times 4$$

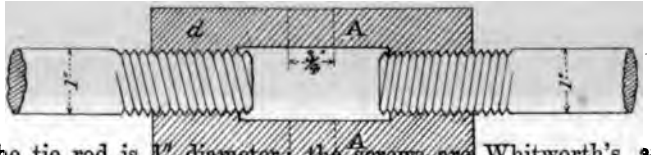
whence $c = \cdot 5$ inches.

It would probably be made somewhat larger, and having regard to unequal distribution of stress, it would be a wise precaution to make it half as large again, *i.e.*, ·75 inches.

EXAMPLE IV.

The tie rod of an iron roof is jointed in the centre by a coupling screw (*Fig.* 7).

Fig. 7.



The tie rod is 1″ diameter; the screws are Whitworth's, angular threads, 8 threads to the inch. Depth of thread = pitch of screws = $\frac{1}{8}$ inch.

The tension on the rod is 5 tons per square inch.

Determine.

1st. The minimum length of the female screw necessary to give a proper shearing resistance, it being estimated that half the resistance to shearing is lost by weakening the metal by the thread of the screw.

2nd. The thickness of the metal at d of the female screw, supposing that there is a hole on each side $\frac{1}{4}$″ diameter to allow of the insertion of a bar for turning the screw.

Remarks.

This is a common arrangement to enable the tie rods of an iron roof frame to be adjusted. It is also used in the suspension rods of suspension bridges.

The information respecting Whitworth's screws is taken from *Molesworth's Engineering Formulæ*, p. 79, 80.

The strength of the joint is determined by the strength of the tie rod, which is the effective section of the rod multiplied by the working resistance of the iron in tension, viz., 5 tons per square inch.

Failure in this joint may tend to occur:

1st. By the stripping or shearing of the male or female screw.

2nd. By the tearing of the coupling screw between the threads, *i.e.*, at A, A.

The resistance of the threads of a screw to shearing may be taken (*H.I.B.*, p. 151) at one half of that of solid iron, *i.e.*, at 2 tons per square inch (*See also Note*, p. 95).

The principles involved in this Example apply equally to screw bolts, which are to be subjected to tension.

Preliminaries.

Let l = length of screw in inches.

„ t = thickness of metal of female screw, exclusive of depth of thread in inches.

„ 5 tons = working resistance per square inch in tension.

„ 2 tons = „ „ „ „ to shearing of threads of screw.

The effective section of the tie rod $= (1'' - 2 \times \frac{1}{8})^2 \times \frac{\pi}{4}$ and the total tension on the tie rod $= (\frac{3}{4})^2 \times \frac{\pi}{4} \times 5$ tons (1)

(1.) The surface of the screws, which resists shearing with the least total force is $l \times \frac{3}{4}'' \times \pi$, and the total working resistance is therefore $l \times \frac{3}{4}\pi \times 2$ tons (2)

As (1) and (2) are equal.

$$(\tfrac{3}{4})^2 \times \pi \times 5 = l \times \tfrac{3}{4}\pi \times 2$$

whence $l = 1\frac{5}{8}$ inch.

The length given in the table of hexagon nuts and bolt heads (*M.E.F.*, p. 79) is 1″, which is therefore amply safe.

(2.) The area of the annulus of the female screw is $\left((1+2t)^2 - 1\right)\frac{\pi}{4}$,

$= (4t + 4t^2)\frac{\pi}{4}$, from which the diameter of the two lever holes multiplied by the thickness of the metal has to be deducted, whence the effective section of the annulus $= (4t + 4t^2)\frac{\pi}{4} - 1{\cdot}5t$.

The total working resistance of the annulus is therefore

$$\left\{(4t + 4t^2)\tfrac{\pi}{4} - 1{\cdot}5\ t\right\} 5 \text{ tons} = (3{\cdot}1416\ t^2 + 1{\cdot}6416\ t)\ 5 \text{ tons} \quad (3)$$

As (1) and (3) are equal,

$$(\tfrac{3}{4})^2 \times \pi \times 5 = (3{\cdot}1416\, t^2 + 1{\cdot}6416\ t) \times 5$$

whence $t = {\cdot}19$ inch exclusive of depth of thread.

which it is to be remembered is a minimum. It would be more convenient to make it larger.

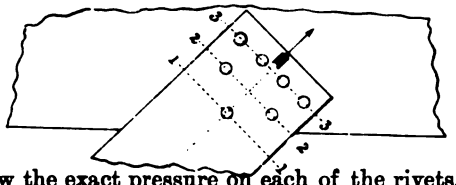
The following extract from Lecture I. of the Course of Lectures on *Wrought Iron Bridges* delivered at the School of Military Engineering, in February, 1871, by Mr. W. Cawthorne Unwin, B. Sc., sets forth principles, which should be attended to in the arrangement of rivetted joints. The complete explanation and illustration of these principles is given in two papers by Mr. Calcott Reilly, C.E., in Vols. XXIV. and XXIX, *Minutes of Proceedings of the Institution of Civil Engineers*,

the papers being *Uniform Stress in Girder Work*, and *Studies of Iron Bridges*:

"In the arrangement of rivetted joints there is one point of capital importance which hitherto has been a great deal too often neglected in the design of iron girders, and that is the arrangement of the rivetting so as to avoid bending stresses in the bars which are rivetted. This will be best explained by an example:

"In attaching a tie-bar to the flanges of a girder, we may, as is very well known, reduce the loss of section due to the rivet holes to that due to a single rivet. To do this, we must arrange the rivetting in rows at right angles to the longitudinal axis of the bar. In the row nearest the centre of the bar we must have not more than one rivet, and the section 1 — 1 *Fig.* 8, is accordingly reduced in strength by one

Fig. 8.

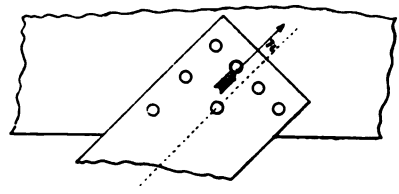


rivet hole. Now the exact pressure on each of the rivets, in a joint of this kind, depending on the relative resistance of the different sections of the parts connected, is unknown. But the following analysis is sufficiently accurate for practical purposes: In the section 1 — 1 the area corresponding to one rivet hole is punched out, and the bar is reduced in strength by that amount. In the section 2 — 2 there are two rivet holes, but if the bar tear at that section it must simultaneously shear the rivet in the section 1 — 1; hence if the resistance of one rivet to shear be not less than the resistance to tearing of a strip of plate one diameter wide (that is, if $d =$ or $> 1{\cdot}27\, t\frac{s_1}{s_2}$, where $d =$ diam. of rivet, $t =$ thickness of plate, s_1, s_2, = limiting safe stress in tension and shear) the section 2—2 will be equally strong with section 1—1. Similarly the third section might have 4, the fourth section 8, and the nth section $2^{(n-1)}$ rivet holes without the reduction of the strength of the bar below that of the section 1—1, or virtually by one rivet hole.*

"This condition as to the number of rivets in each row being fulfilled, it is too often supposed that the arrangement of the rivets in other respects is a matter of indifference, and an arrangement such as that in *Fig.* 9, has been frequently adopted. Now the effect of this arrangement is, to move the resultant of the forces applied by the rivets to the bar away from the mean fibre of the section. By the mean fibre, I understand the line through the centres of gravity of all the cross

* "This mode of calculation is somewhat different from that adopted by many engineers who regard each rivet in a joint with n rivets as carrying 1-nth of the load. Without more knowledge than we possess at present of the distribution of stress in rivetted joints, it is impossible to say which method is most accurate."

Fig. 9.



sections of the bar perpendicular to the applied force. When the resultant does not act in the direction of the mean fibre, the stress on the cross sections of the bar is no longer uniformly distributed, but is uniformly varying, and greater at one edge than the other. And at any point the magnitude of the stress will be found by compounding with the mean stress $\frac{P}{a}$, which would be the stress if the resultant force acted along the mean fibre, the stress due to a bending moment, Pn, where n is the distance between the resultant of the forces applied by the rivets and the mean fibre. And it is sufficiently accurate for practical purposes to assume that the resultant passes through the centre of gravity of the group of rivets. Let P be the load which the bar will carry safely; s the limiting stress, and a the area of cross section at any point; n the distance of resultant of forces acting at rivets from centre of gravity of section, measured parallel to section; x the distance of most strained edge from centre of gravity of section; I the moment of inertia of section; then

$$P = \frac{sa}{1 + \frac{nax}{I}} \quad \ldots\ldots(1)$$

If the section is rectangular

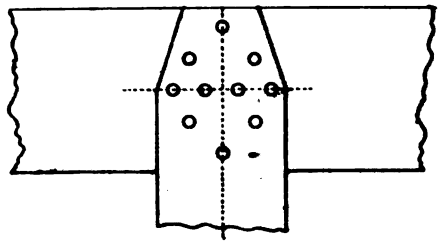
$$P = \frac{sbh}{1 + 6\frac{n}{h}} \quad \ldots\ldots(2)$$

Obviously the bar in this case will carry safely a less load than if the rivets were so arranged that the centre of gravity of the group coincided with the mean fibre of the bar. (For example, if $n = \frac{h}{2}$, a square bar will carry only $\frac{1}{4}$ as much as if $n=o$). Now it is open to us as circumstances may dictate to arrange the rivets symmetrically with the mean fibre or not. But in the latter case we require a heavier tie bar, in consequence of the existence of a bending couple as well as a direct tension. And the same is true of bars in compression.

"Thus far we have considered only a single bar. But the same considerations apply to the member to which the bar is attached. The centre of gravity of the group of rivets should fall not only on the mean fibre of the tie bar but also on that of the boom. Further, if instead of considering the bar as a whole we examined the state of stress in

the sections through the rivets, we should find that, to secure perfect uniformity of stress each row of rivets should be independently symmetrical about both the mean fibre of the boom and the tie bar. This state of symmetry can only be perfectly realised, in joints of two bars, when the bars are at right angles (*Fig.* 10); and in other cases by re-

Fig. 10.



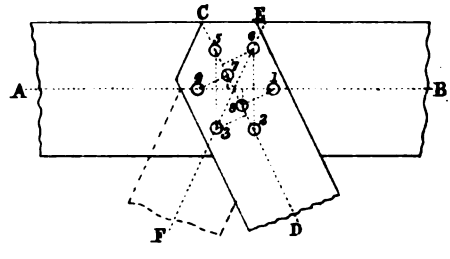
placing the group of rivets by a single pin. But for practical purposes it is sufficient to fulfil two conditions, the statement of which we owe to Mr. Calcott Reilly.

"(1). The centre of gravity of the group of rivets must fall on the intersection of the mean fibres of the bars.

"(2). The first row of rivets in each bar, that is the row on the side towards which the stress is transmitted, must be symmetrical with the mean fibre of that bar, or as Mr. Reilly puts it, and this in general amounts to the same thing, the leading rivet in each bar must be on the mean fibre. In designing a joint we fulfil these conditions very simply. We arrange the rivets as symmetrically as possible with each mean fibre, and for those rivets which cannot be placed in pairs symmetrically, we make the sum of the distances of the unsymmetrical rivets on one side of the mean fibre equal to the sum of the distances of the unsymmetrical rivets on the other side."

Thus in *Fig.* 11 two bars AB, CD are shewn, the main fibres being respectively AB, and CD. Then as regards AB, rivets 1 and 4 are on the mean fibre, rivets 2, 3, 5, 6, are symmetrical with regard to it, while 7 and 8, though unsymmetrical, are at equal distances on opposite sides of it.

Fig. 11.



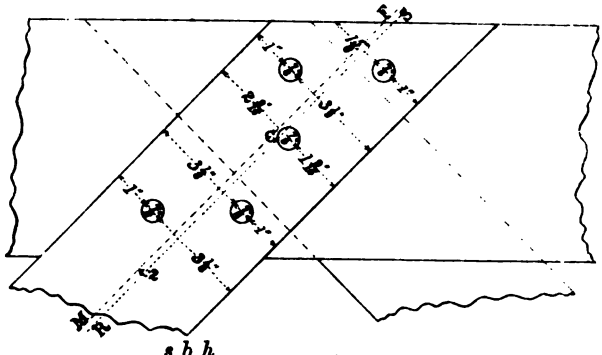
As regards CD, rivets 2, 5, 7 and 8 are on the mean fibre, while 3, 4, 6 and 1 are unsymmetrical, but at equal distances on opposite sides of it.

If another bar EF were attached, the same rivets would fulfil the conditions required in regard to it. But in this case only 3 and 6 are on the mean fibre, and the others are unsymmetrical, but at equal distances on opposite sides of it.

Let it be required on these principles to examine into the strength of the tension bar in *Fig.* 5, Example II, p. 97, supposing it to be rivetted as therein arranged.

The mean fibre traverses the centre of the bar MF (*Fig.* 12). The centre of gravity of the group of rivets is at C, and the resultant stress RS passes through it. Then the distance between the two lines, n, = ·2 inch.

Fig. 12.



Whence, as $P = \frac{s\,b\,h}{1+6\frac{n}{h}}$

$$s = \frac{P\left(1+6\frac{n}{h}\right)}{bh}$$

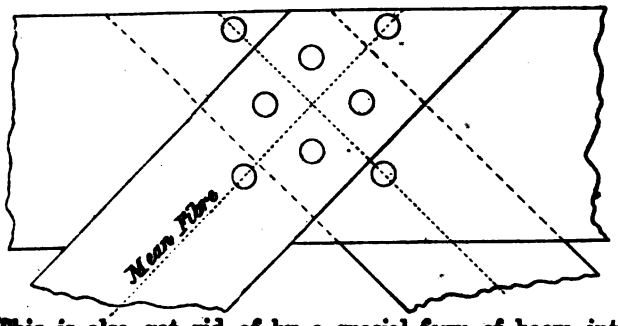
where P = $10\frac{8}{16}$ tons, $b = \frac{1}{2}$ inch, $h = 5$ inches, $n = ·2$ inch,
whence $s = 5·42$ tons per square inch.

i.e., the maximum intensity of the stress under a total tension of $10\frac{8}{16}$ tons exceeds the mean intensity by ·42 tons per square inch, or about 8 per cent.

Mr. Reilly gives (p. 482, Vol. XXIX., *M.P.I.C.E.*) a case of ordinary occurrence in which the increase of stress on one side, due to the faulty arrangement of the rivetting amounts to as much as 26 per cent.

The increase may be avoided by arranging the rivets as shewn in *Fig.* 13. It is to be borne in mind that in this case owing to the form of the boom, which, as shewn in *Fig.* 5, is of trough form, it is not possible to make the mean fibre of the bars, and of the boom intersect in a point; and that there is still a bending stress in the boom due to the rivetting, although it has been got rid of in both the tension and compression bars.

Fig. 13.



This is also got rid of by a special form of boom, introduced by Mr. Calcott Reilly, C.E., which is referred to again in the paper on *Built-up Wrought Iron Beams.*

The effect of neglecting the proper arrangement of the rivets is illustrated in a remarkable manner in the following extract from the lecture already quoted:

"It is extremely common, in employing angle, T and channel iron in the form of struts, to attach them in such a manner that the line of action of the external forces does not pass through the mean fibre. For instance it is extremely common to find them rivetted by one only of the flanges. It is of course open to us to construct struts in this way, and it may often be both the readiest and most economical way of proceeding, but we ought to remember that the bar so rivetted is weaker than if the attachment were made in a more perfect manner, and that the ordinary formulæ for long columns do not apply with strictness to bars so attached. We have experimental evidence of this, for some years ago Mr. Davies of the Crumlin Works tested to fracture a series of struts of angle, T, channel, and cross iron; some of them squared at the ends and compressed between parallel plates; others rivetted by one flange.

Angle irons,	squared ends	$3 \times 3 \times \frac{5}{16}$	crushed with	$18\frac{3}{4}$ tons.
„	rivetted by one flange	„	„	$12\frac{1}{2}$ „
Tee Iron,	squared ends	$3 \times 3 \times \frac{5}{8}$	„	$21\frac{1}{8}$ „
„	rivetted by one flange	„	„	$18\frac{3}{4}$ „
Channel Iron,	squared ends	$3 \times 1\frac{1}{4} \times \frac{5}{8}$	„	17·1 „
„	rivetted by one flange	„	„	14·1 „
Cross Iron,	squared ends	$3 \times 2\frac{1}{2} \times \frac{5}{8}$	„	17·1 „
„	rivetted by one flange	„	„	15·6 „

"The loss of strength was greatest in those cases in which the deviation of the line of compression from the line of mean fibre was greatest.

It is often very convenient to use angle irons in that way and we are not precluded from so using them; only if we do use them we must remember they are not so strong as the strength which would be calculated from the ordinary formulæ for long columns. In ordinary cases,

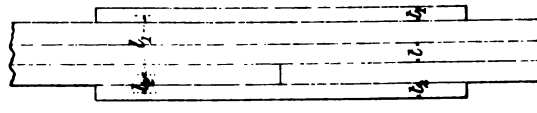
where the formulæ 1 and 2, given above, show a lower result than the ordinary formulæ for long columns, then we ought to calculate the strength by means of them."

And the effect on the cover plates of wrought iron girders is explained in the following extract from the same lectures, and is illustrated in Example II. *Built-up Wrought Iron Beams:*

"There is one other point on which light is thrown by considering the effect of the deviations of the line of action of the straining force from the mean fibre, and that is the proper proportions of cover plates. In many of the earliest bridges, the joints in the booms were supposed to be rendered of equal strength with the booms, by putting a cover plate of equal strength with the plate in which the joint occurred, on one side of the joint. The effect of this is to introduce a bending couple whose magnitude is the straining force multiplied by the thickness of the plates, and the joint is in consequence considerably weaker than the rest of the structure. When this was recognised, and partly, I believe, through the action of the Board of Trade, it became the custom to place invariably a cover plate on each side of the joint, and these covers varied in thickness according to the whim of the designer. In this case it is clear that we eliminate the bending action and preserve uniform strength by using two covers each of half the thickness of the plates to be joined.

When the boom consists of a number of plates (*Fig.* 14), the case is more complex.

Fig. 14.



"In this case, we ought to have $t_1 + t_2 = t$ (1)

and $t_1 l_1 = t_2 l_2$ (2)

It is easy to assume approximate values for l_1 and l_2 and then determine t_1 and t_2.

There is yet another case, when the same covers are used for the joints in several tiers of plates. In this case we may determine t_1 and t_2 for each of the joints and make each of the covers of the greatest thickness found for any of the joints. Of course it is impossible practically to proportion cover plates exactly by these rules. But they may usefully guide the selection of a convenient thickness."

The greatest inequality in the distribution of the stress on the two covers occurs, whatever the number of plates, when the joint is in the outermost plate on one side or the other (*Fig.* 14), and in this case, if u is the number of plates,

$$l_1 = (u - \tfrac{1}{2})\, t + \tfrac{1}{2}\, t_1$$
$$l_2 = \tfrac{1}{2}\,(t + t_2)$$

and substituting these values in equations (1) and (2)

$$t_2 = \frac{u}{u+1}\, t.$$

When the joint falls in the plate next to the outside plate.

$$t_1 = (u - \tfrac{1}{2})\, t + \tfrac{1}{2}\, t_1$$
$$t_2 = \tfrac{1}{2}\, t + \tfrac{1}{2}\, t_2$$

and by substituting these values in equations (1) and (2) the value of t_2 for this case is obtained.

Similarly for other positions of the joint.

The number of rivets (and consequently the length of the covers) is greater than would be required for a joint in a single plate with two covers, and it must be determined by the stress on the cover nearest to the joint, *i.e.*, on t_2.

Let n = number of rows of rivets on one side of the joint required to resist shear.

m = number of rows of rivets on one side of the joint required to give sufficient bearing surface.

t = thickness of one plate.

t_2 = „ of cover plate (as just determined).

b = breadth of plate (exclusive of rivet holes).

c = number of rivets in the breadth of the plate, *i.e.*, in each row.

s_1 = working resistance of iron in tension.

s_2 = „ „ „ in shear.

s_3 = „ „ „ in bearing

d = diameter of rivet.

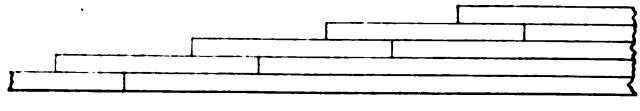
Then for shear $$b\, t_2\, s_1 = \frac{\pi}{4} d^2\, s_2 \times n\, c \qquad (3)$$

„ bearing on plate $b\, t\, s_1 = t \times d \times s_3 \times m\, c$ (4)

„ bearing on cover $b\, t_2\, s_1 = t_2 \times d \times s_3 \times m\, c.$ (5)

There are three methods in ordinary use of arranging the plates in the tension boom of a girder, where the plates are chain rivetted, viz.: (1) The plates are so arranged that each joint has its own pair of covers, which method has just been explained. (*Fig.* 14.) (2) The plates are so arranged that the joints occur at regular intervals, and an extra plate, over and above the number calculated as necessary to resist the tension on the boom, is put on the whole length of the girder to act as a cover. (*Fig.* 15). (3) The plates are so arranged that each joint in succession is distant from that in the plate next to it, twice the longitudinal pitch of the rivetting, the contiguous joints in all the plates being brought under one pair of covers. (*Fig.* 16.)

Fig. 15.



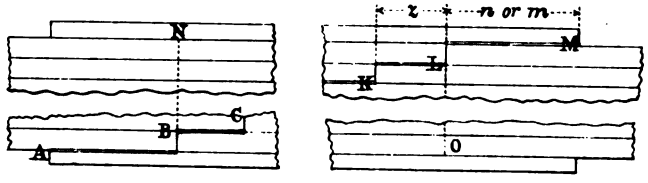
Either the first or third method must be better construction than the second, which is practically equivalent to using only one cover.

The third method is stated (*U.I.B.R.* p. 102) to save both weight and workmanship. It does not seem necessary to adhere to *twice* the

longitudinal pitch, and it is sometimes better to increase the distance between the contiguous joints, reducing to a corresponding extent the number of rows of rivets at each end of the cover.

For instance, let *Fig.* 16 represent a chain rivetted tension boom, with the plates arranged according to the third method.

Fig. 16.



Then as these plates are held together throughout their whole length by the rivets connecting them with the angle irons, as well as by others at their edges, they may be considered as one plate of which the joint is the double line ABC . . . KLM.

This joint may tend to fail in one of four ways, viz.: (1) The rivets may shear along the double line A . . . M. (2) The rivets between B and L may shear at the same moment that the covers tear across through the rows of rivets next to B and L. (3) The rivets on either side of BL may cut into the plates, or the plates into the rivets. (4) The rivets may cut into the covers between A and B, and between L and M, at the same time that the rivets between B L and B N, or between B L and L O cut into the plates.

Using the same notation as before, and calling u the number of plates, and z the number of rows of rivets between two contiguous joints (*Fig.* 16), the following equations are obtained;

For shear, $$u\ b\ t\ s_1 = (2\ c\ n + (u - 1)\ z\ c)\ \frac{\pi\ d^2}{4}\ s_2 \qquad (6)$$

„ shear and tear, $$u\ b\ t\ s_1 = (u - 1)\ z\ c\ \frac{\pi\ d^2\ s_3}{4} + 2\ b\ t_2\ s_1 \qquad (7)$$

„ bearing on plates, $$u\ b\ t\ s_1 = c\ (u\ m + \frac{u}{2}(u - 1)\ z)\ t\ d\ s_3 \qquad (8)$$

„ bearing on covers and plates, $$u\ b\ t\ s_1 = c\ (2\ m\ t_2 + \frac{u}{2}\ (u-1)z\ t)\ d\ s_3 \qquad (9)$$

In equation (6) $c\ (2\ n + (u - 1)\ z)$ is the total number of rivets in the cover, and for any case where b, t, s_1, c, d, and s_2 are constant, *i.e.*, for any given boom, the total number necessary for shearing is constant, and it does not matter how they are arranged, although it is clearly better to keep the end joints well within the covers.

In equation (8), $(u\ m + \frac{u}{2}\ (u - 1)\ z)\ c$ is half the total number of surfaces of rivets bearing on the plates, and as this equals $\frac{u\ t\ b\ s_1}{t\ d\ s_3}$, which

is constant, the arrangement of the rivets does not alter the number required, *i.e.*, whatever value is given to s, the total number remains the same.

In equation (7) the larger s is made, the smaller t_2 becomes, and it is therefore advantageous to make s large, because the cover is not lengthened thereby, while t_2 is lessened.

In order to keep the end joints well within the covers, it is better not to make s greater than n or m.

In equation (9), if s be made large, m t_2 is reduced, and making s as large as it can be made conveniently, as determined by equation (6) and giving t_2 the value determined by equation (7), the value of m is obtained.

If m be greater than n, s may be increased so long as it is not made to exceed m.

Applying these equations to a particular case, say $a=4$, $b=18$, $t=\frac{1}{2}$, $s_1=5$, $c=4$, $d=\frac{3}{4}$, $s_2=4$, $s_3=5$.

In equation (6), $\quad 180 = (8n+12\,s)\,\frac{11}{16}$

and whatever value s has, the total number of rivets is nearly the same, thus, when $s=2$, $n=9{\cdot}7$, and there are 104 rivets

when $s=3$, $n=8{\cdot}2$, „ „ 108 „
when $s=4$, $n=6{\cdot}7$, „ „ 104 „
when $s=5$, $n=5{\cdot}2$, „ „ 108 „
when $s=6$, $n=3{\cdot}7$, „ „ 104 „

In equation (7), $\quad 180=3\times s\times 4\times\frac{11}{16}+2\times 18\times t_2\times 5$

and as the largest advantageous value for s is 4, $t_2=\frac{11}{180}=\frac{9}{15}$"

In equation (8), $\quad 180=4\,(6m+2\times 3s)\,\frac{1}{2}\times\frac{3}{4}\times 5.$

$$m=4-s$$
$$m=0$$

or the joint is stronger in bearing than in shearing.

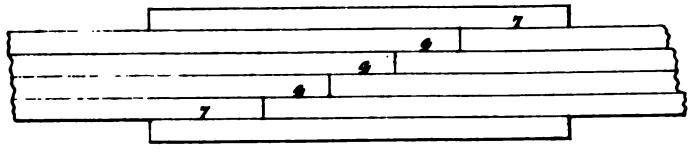
In equation (9), $\quad 180=4\,(2m\,t_2+6s\,\frac{1}{2})\,\frac{3}{4}\times 5$

$$12=2m\,t_2+3s$$
$$m=0$$

or here again the bearing strength of the joint is in excess.

This joint would be as shown in *Fig.* 17, and the weight of the covers would be $2\times\frac{26\times 3}{12}\times\frac{11}{12}\times 22{\cdot}5$ lbs. $=512$ lbs.

Fig. 17.



To compare this method with the first method;

For a single joint where there are 4 half-inch plates, and a joint in one outside plate,

$$t_2 = \frac{u}{u+1}\, t = \tfrac{4}{5} \times \tfrac{1}{2} = \tfrac{4}{5} = \tfrac{7}{16} \text{ inch}$$

In equation (3). $18 \times \frac{7}{16} \times 5 = \frac{9}{16} \times n \times 4$

$$n = 6$$

In equation (4). $18 \times \frac{8}{16} \times 5 = \frac{1}{2} \times \frac{3}{4} \times 5 \times m \times 4$

$$m = 6$$

In equation (5). $18 \times \frac{7}{16} \times 5 = \frac{7}{16} \times \frac{3}{4} \times 5 \times m \times 4$

$$m = 6$$

For a single joint where there are 4 half-inch plates and a joint in one inside plate,

$$l_1 = (4 - \tfrac{3}{2})\, \tfrac{1}{2} + \tfrac{1}{2}\, t_1 = \tfrac{5}{4} + \tfrac{1}{2}\, t_1$$

$$l_2 = \tfrac{3}{4} + \tfrac{1}{2}\, t_2$$

$$t_1 = \tfrac{1}{2} - t_2$$

$$t_1\, l_1 = t_2\, l_2$$

whence $t_2 = \frac{5}{20} = \frac{5}{16}$ inch

In equation (3). $18 \times \frac{5}{16} \times 5 = \frac{9}{16} \times n \times 4$

$$n = 5$$

In equation (4), the value is the same as in last case, viz., $m = 6$.

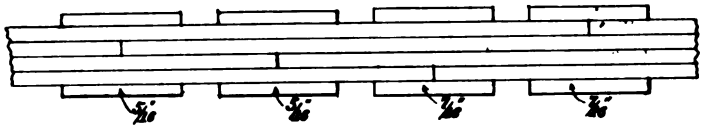
In equation (5). $18 \times \frac{5}{16} \times 5 = \frac{5}{16} \times \frac{3}{4} \times 5 \times m \times 4$

$$m = 6$$

so that the difference between these two inside joints and the two in the outside plates is only in the thickness of the covers.

Arranging the four joints, as shewn in *Fig.* 18, the weight of the 8 covers $= 8 \times \dfrac{12 \times 3}{12} \times \dfrac{21}{12} \times 15$ lbs. $= 630$ lbs.

Fig. 18.



This shews that in this particular case there is a considerable saving by bringing the joints under one pair of covers, and there is also a saving of workmanship.

BUILT-UP WROUGHT IRON BEAMS.

These beams are built-up of wrought iron of various forms; plate, bar, angle, tee and channel, and occasionally quadrant, connected together by rivets of iron, or by bolts of iron or steel.

Fig. 1.

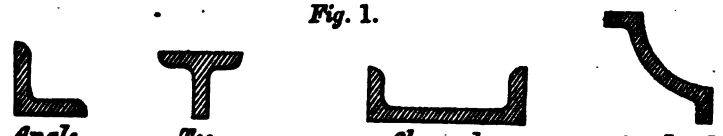


Plate iron is from $\frac{1}{4}$ to 1 inch thick, increasing in thickness by $\frac{1}{16}$ inch; ordinarily, plates do not exceed $\frac{3}{4}$ inch in thickness.

An extra price, per ton, is charged if the plate is over a certain weight, generally about 4 cwt.; or if over a certain size, about 24 superficial feet; or if over a certain length, about 15 feet; or if over a certain width, about 4 feet; or if less than a certain width, about 12 inches.

Plates of less than $\frac{1}{4}$ inch thick are called sheet iron, are made of iron of better quality, and are more expensive.

Bar iron includes round, square, and flat iron.

Round and square iron increases by $\frac{1}{16}$ inch, from $\frac{5}{8}$ inch to about $8\frac{1}{2}$ inches in side or diameter without increase of charge.

Flat iron increases $\frac{1}{8}$ inch in breadth, and $\frac{1}{16}$ inch in thickness, from $1'' \times \frac{1}{4}''$ to $6 \times 1''$ without increase of charge.

An extra price is charged for irons, either larger or smaller than the dimensions just stated.

Angle and T irons are of an almost infinite variety of section, full sized drawings of which are published by all iron merchants.

An extra price is charged for angle or T iron exceeding eight "united inches," the term united inches meaning the sum of its extreme breadth and depth; or, if over a certain length, about 25 feet; or for small sized angle or T irons, say for $1\frac{1}{2}'' \times 1\frac{1}{2}'' \times \frac{3}{16}''$ and under, or for obtuse or acute irons.

T iron is generally a little more costly than angle iron, and an extra price is charged usually for lengths over 21 feet.

The charge for channel iron appears to vary very much with different makers; but for ordinary sections the price and extras are generally

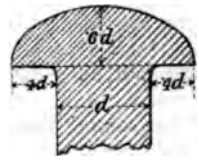
about the same as for T iron, the "united inches" in this case being the width added to twice the depth.

Quadrant iron consists of a flat bar bent to a quadrant of a circle, with two flat flanges. Four bars rivetted together form a tube, with four wings or feathers. It is in that form especially adapted for struts.

Iron for rivets is manufactured specially for the purpose, greater care being taken in its manufacture with a view to rendering it tough. It costs considerably more than the ordinary bar iron.

The quantity of iron in the head of an ordinary rivet may be taken to be the quantity in a cylinder having the diameter of the rivet hole, and a length of $1\frac{1}{16}$ times that diameter. This gives a rivet head of the form shown in *Fig.* 2.

Fig. 2.



Points to be attended to in designing built-up wrought iron work.

In designing a piece of built-up wrought iron work of any kind, the Engineer should, as far as possible, use the sections and lengths of iron usually in the market, of course recollecting that it may frequently be more economical to use special irons, if the use of such irons saves labor, as by reducing the number of joints; or weight, as by reducing the number of cover plates or wrappers.

Rivets, for which the holes are to be punched, must be so arranged that the iron will not be unnecessarily injured by punching the holes.

A rivet hole cannot be punched nearer to the edge of a plate than its own diameter, or nearer to another rivet hole than one and a half times its own diameter, without risk of breaking through the side of the plate or punching the two holes into one. (*L.W.I.B.*, p. 11.)

If the rivet holes are to be drilled, which is always desirable where many plates are to be rivetted together, the holes can be put closer together, and the arrangement of rivetted joints is thus frequently facilitated; but in no case must they be so close together that the heads of two adjoining rivets are in each other's way.

Whether the holes are punched or drilled, the rivets must be so arranged that both sides of each can be got at during construction, as otherwise the rivetting cannot be accomplished.

The dimensions of rivets in ordinary use are given in *Useful Rules and Tables* (Rule XXI., p. 209), and another is given in *Unwin's Iron Bridges and Roofs*, p. 109, 110. For calculations of strength of rivets, see *Joints and Fastenings*.

Experiments made for the War Department by Mr. Kirkaldy shew that the "ultimate" resistance to shearing of rivets in rivetted joints is largely increased by rounding the angles of either drilled or punched holes.

It is perhaps not worth while to go to this expense in ordinary constructions, although in the event of accident the safety of the structure would be thereby increased.

It is not considered desirable in girder work to use angle iron of thickness less than $\frac{1}{7}$ the length of one side. (*H.I.B.*, p. 84).

To lessen the evils of corrosion, it is not usual in girder work to use any plates of less than $\frac{1}{4}$ inch thickness. Thin large plates are apt to lose their flatness during the riveting of their edges.

All built-up iron work should, as far as possible, be so designed that all parts of it can be got at periodically for scraping and painting.

Different types of beams.

Built-up wrought iron beams may be divided into three great types:

1st. Beams with both flanges horizontal, and having continuous plate webs, of which the saddle-backed girder, frequently used on railways, is a modification.

2nd. Beams with parallel horizontal flanges having open webs, of which lattice and warren girders are instances.

3rd. Beams with one flange horizontal, the other curved from the centre of the beam to the ends where it is joined to the horizontal flange, the web being open or continuous, generally the former. This is the bowstring girder in use on railways and aqueducts.

Each of these types require a somewhat different method of calculation, which again varies as regards the web, according as the load to be borne is a moving or a dead load.

Beams with both flanges horizontal, and having continuous plate webs.

In these beams, the mass of the metal is collected in the flanges or booms, which are connected and kept apart by either one or two very thin vertical webs, which webs are prevented buckling under the stress by stiffeners placed at intervals, the stiffeners serving also to distribute the stress over the web.

It has been shewn (*Stress in Beams*, p. 25) that the intensity of the shearing stress is nothing at the outer edges of the section of a beam, and greatest at the neutral axis; and on the other hand, it has been shewn (*Stress in Beams*, p. 15) that the intensity of the direct stresses is nothing at the neutral axis, and a maximum at the outer edges of the beam.

If, therefore, the flanges be made wide and shallow, and the web very thin (comparatively), the flanges will be called upon to resist very little of the shearing stress, while the web will be called upon to resist very little of the direct stresses, and the formula for the Moment of Resistance, obtained by neglecting the resistance of the web to the direct stresses, and considering the resistance of the two flanges as

equal (*Cast Iron Beams*, p. 62), and constituting a couple, having an arm equal in length to the distance between the centres of tension and compression, becomes nearly correct; and as it gives results erring a little on the side of safety, it may be and is adopted in practice for the calculation of the dimensions of the flanges.

On the other hand as the web is called upon to resist a very small proportion of the direct stresses, while the flanges are called upon to resist very little of the shearing stress, a very slight error, and that on the side of safety, is introduced by considering the web as having to resist the whole of the latter and none of the former, and this method is also adopted in practice.

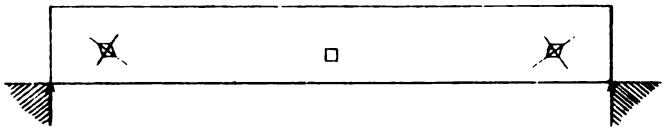
In calculating the dimensions of the web and its stiffeners, it is necessary to consider how the shearing stresses are received by the web, and how they pass through the web to the piers.

If the load be on the top of the beam, the stiffener acts as a pillar, receiving the stress, and distributing it over the web; if it be suspended from or resting on the bottom flange of the beam, the stiffener is a tension bar but still plays the part of distributing the stress over the web. The stiffeners over the points of supports are pillars in both cases.

The first effect of the load on any stiffener is to tend to shear the plate of the web vertically through the rivet holes by which it is connected with the stiffener.

The web being made strong enough to resist this shearing is distorted in a vertical plane, each particle originally square becoming a rhomb, the longer diagonal of which inclines upwards and towards the point of support, the shorter diagonal upwards and away from the point of support, thus,

Fig. 3.

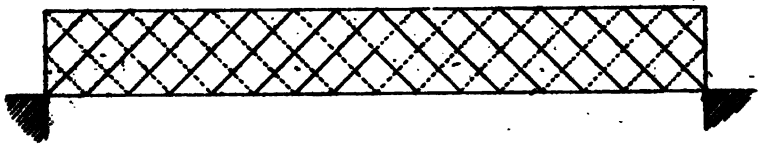


This distortion is greatest at the neutral axis, and lessens towards the outer edges of the beam, but between the neutral axis and the point of junction of the web and flange, the difference of distortion in beams of moderate depth is very little, and it is usual in consequence to consider it as uniform; in other words to take the shearing stress at any vertical section as uniformly distributed over the web at that section. Where there is no shearing stress, *i.e.*, in *Fig.* 3 at the centre of the beam, there is of course no distortion.

Professor Rankine has shewn (*R.C.E.*, p. 529, *U.I.B.R.*, p. 29), that the shearing forces producing distortion of any square particle may be replaced by two forces, one a tension and the other a thrust, having equal intensities with those which they replace, and on the assumption that the shearing stress is uniformly distributed, the whole of the shearing forces producing distortion may be replaced by a net work of

forces at right angles to each other, and inclined at 45° to the axis of the beam, these forces being tensions and compressions, under the

Fig. 4.



latter of which the web tends to buckle in lines at right angles to the direction of the thrust. They may be represented as shewn in *Fig.* 4, the broken lines being the tensions, and the dark lines the compressions or thrusts, the particular case shewn being that of a beam supported at both ends and uniformly loaded.

The web, therefore, must either be made strong enough to resist this thrust as a pillar, or must be strengthened by stiffeners at such intervals, as may be necessary to prevent buckling, the latter being the course actually adopted with built-up wrought iron beams in practice.

The application of this theory is illustrated in the Examples.

Figs. 5 to 10 are some of the more ordinary sections of this type of beam.

Fig. 5. *Fig.* 6. *Fig.* 9. *Fig.* 10.

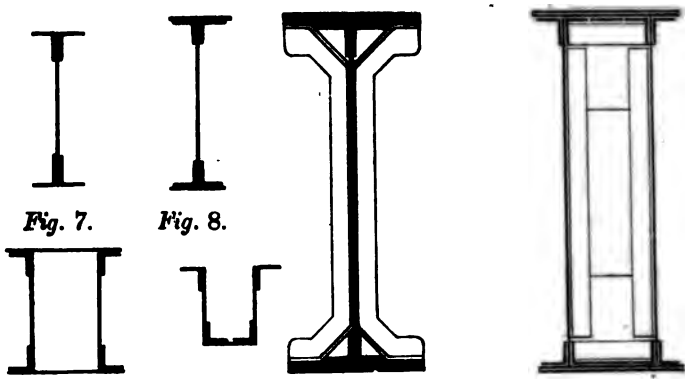


Fig. 7. *Fig.* 8.

Nos. 5 and 6 are useful for longitudinals and crossbeams for railway bridge platforms, or other small spans.

No. 7 for bressummers in dry situations. It is not a good form if it is to be exposed to the weather, unless it is large enough to allow of a boy getting inside to scrape and paint.

No. 8 for longitudinals of railway bridge platforms where the depth of the bridge platform is required to be reduced to a minimum.

No. 9 for main girders of railway bridges up to a span of about 70 feet.

No. 10 for main girders of railway bridges for larger spans, where the open web cannot be used.

In designing built-up beams, the following points should be kept in view:

(1) The length of the beam for the purpose of calculation should be the length from centre to centre of bearing, not the clear span.

(2) The effective depth of the beam is the vertical depth between the centres of tension and compression of the flanges, and the centres of tension and compression may without sensible error be considered to be at the centres of figure of the section of the flanges.

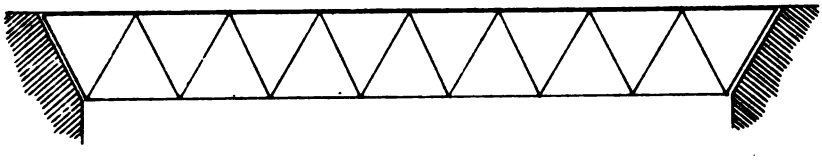
The depth varies in these girders from $\frac{1}{10}$ to $\frac{1}{15}$ of the length, $\frac{1}{12}$ being said to be the proportion which is most advantageous.

If the beam has a greater depth, the web is more liable to buckle, and extra iron has to be put in to stiffen it; if it has little depth, more metal has to be put into the flanges.

(3) In the tension flange, the metal removed in the rivet holes must be deducted in calculating the strength of the flange. The loss of strength due to rivet holes in the plates of a chain rivetted flange may sometimes be reduced as low as 15 per cent. of the whole. In the compression flange, if the work is properly done, the rivet holes being filled up again by the rivets, and the compression conveyed through them from one side of the hole to the other, no deduction need be made.

(4) The width of the tension flange may, if necessary, be reduced to what is required to give a proper bearing on the piers, or if the beam is suspended from the top flange (as in warren girders sometimes, *Fig.* 11) to a still smaller breadth.

Fig. 11.

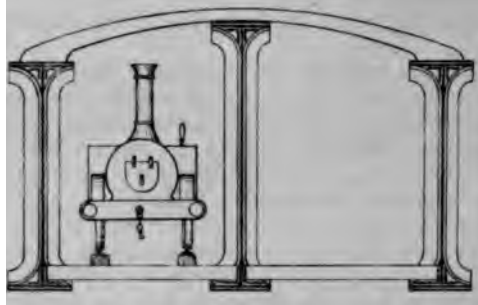


The width of the compression flange, when it is unsupported, as when the cross girders are on the bottom flange, must be enough to prevent buckling sideways (*i.e.*, in a horizontal plane) under the thrust. This is found from experience to be obtained by a width of $\frac{1}{30}$ to $\frac{1}{40}$ length of the beam. In long, and therefore deep, girders, a smaller width may be used, the top flange being stiffened against lateral bending by cross arches or girders (*Fig.* 12).

(5) The joints in the compression flange, not being a source of weakness, may be made as numerous as convenience requires. Those in the tension flange, being on the contrary a source of weakness, should be as few as possible.

The plates in the compression flange range in practice from 8 to about 12 feet in length, while those in the tension flange may range

Fig. 12.



from 12 feet upwards, the object of the greater length being to reduce the number of joints.

The plates in each flange should, as far as possible, be uniform in dimensions, and the rivets should be so arranged that the pattern for each plate is, as nearly as possible, the same. Any departure from this rule involves the risk of mistakes by the workmen. It is in fact worth while to waste a little metal to secure uniformity in the work generally.

Where the quantity of metal required in the flanges is great, it appears better to use a large number of thin plates than a small number of thick ones, the former being stronger in proportion to their thickness than the latter, and allowing of a more gradual reduction of the quantity of metal towards the points of support, thus rendering the deflection of the beam under its load more gradual.

(6) Horizontal joints in the webs of girders are to be avoided, as the upper sides of their covers afford a lodgment for the rain. They are necessary only in girders of unusual depth.

There is this advantage in reducing the number of all joints, that the number of places where rain can penetrate is thereby reduced.

(7) Built-up wrought iron beams are almost always constructed with a "camber," such that, when the beam is fully loaded it becomes horizontal or nearly so.

This "camber" is made equal to, or a little more than, the calculated deflection of the beam, the deflection in this case including a considerable amount of permanent set, due to the large number of parts and the unavoidable imperfections of their connections.

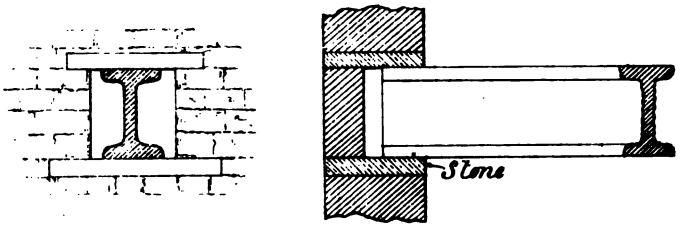
The effect of the two causes is to reduce the value of the modulus of elasticity, E, from about 29,000,000 lbs. to about 18,000,000 lbs. (E varies from 16,000,000 to 18,750,000, *R.C.E.*, p. 531); and, in calculating the camber, this value of E should be used.

With a beam of uniform strength, under stresses of 5 tons and 4 tons per square inch in tension and compression respectively, and having a depth of from $\frac{1}{10}$ to $\frac{1}{15}$ the span, the camber would be from about $\frac{1}{1050}$ to $\frac{1}{1400}$ of the span; and if, as may be inferred from the reduction in the value of E, about $\frac{1}{3}$ of this is absorbed by permanent set, the true deflection would be from $\frac{1}{1575}$ to $\frac{1}{2100}$ of the span.

It may be wise to increase this camber a little, say by $\frac{1}{3}$rd, the calculations for deflection being approximate only, as it is better to have too much than too little camber.

(8) Iron beams, if "fixed" with a view of getting the additional strength and stiffness due to fixing them, should still be free to expand and contract lengthways. This may in some cases, as in the case of floor girders, be effected by placing templates above the ends, as in *Fig.* 13, and weighting them.

Fig. 13.



Bridge girders should not be "fixed" at both ends; one end should be absolutely free to expand and contract, and usually they are merely placed on their supports without fastenings of any kind.

Girders for railways (double line, narrow guage) over about 80 feet in length generally have expansion rollers under one end; when less than that length, tarred felt, lead, or planedcast iron plates are often used under one or both ends to lessen friction and allow of easy expansion and contraction.

EXAMPLE I.

It is required to design a box girder of the form shown in *Fig.* 14, and of uniform section to replace a partition wall under the gutter of a roof of the form shown in *Fig.* 15.

Fig. 14.

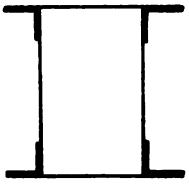
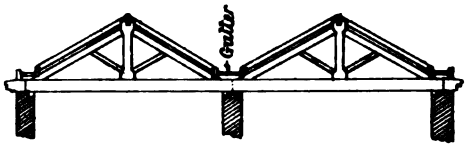


Fig. 15.



Its clear span is 30 feet, its width 14 inches, its effective depth is to be as small as possible, consistently with the girder fulfilling the two

following conditions: 1st, that the working resistances shall not exceed:

In Tension	5 tons	per square inch.
„ Compression	4 tons	
„ Shearing	5 tons	
On Bearing Surface	7 tons	

2nd, that the deflection shall not exceed $\frac{1}{4}$ inch under a uniformly distributed vertical load (including the weight of the girder) of 14 tons.

NOTE.—The Shearing and Bearing Surface resistances may be taken higher than usual, the load being dead, and the deflection not being influenced materially by them.

The girders will rest at each end on a brick wall 14″ thick, a stone template being placed under each end to distribute the weight over a sufficient surface of the wall.

The total length of the girder will be 32 ft. 4 ins., and the span from centre to centre of bearings may be taken at 31 feet.

It is not proposed to "fix" the ends of the girders.

(1) To find the depth necessary to fulfil the two given conditions.

This being a beam of uniform section, having a cross section of equal strength, the deflection is given by the formula.

$$V = \frac{n''(f_a + f_b)c^2}{Eh_0} \quad (\textit{Deflection of Beams}, \text{ p. 87}).$$

where the symbols have the meaning given in the page referred to.

In this case, $V = \frac{1}{10}$ inch, say $\frac{1}{4}$ inch, i.e. $\frac{1}{10}$ inch per ft. run (p. 40).

$n'' = \frac{5}{12}$ (the beam being supported at both ends and uniformly loaded).

$f_b = 5$ tons.

$f_a = 4$ tons.

$c = 186$ inches.

E = 18,500,000 lbs. the Modulus of Elasticity for plate beams, which may be taken high in this case, as the number of joints will not be large (*R.C.E.*, p. 531).

h_0 = effective depth in inches, required.

Then $\frac{1}{4} = \frac{5}{12} \frac{9 \times 2240 \times 186^2}{18{,}500{,}000 \times h_0}$

whence $h_0 = 20{\cdot}9$ inches, say 21 inches.

(2) To determine the section of the flanges.

The section of greatest stress being at the centre (*Stress in Beams*, p. 7), and the beam being of uniform section, it is sufficient to determine the dimensions of the flanges at the centre.

On the assumption referred to at page 114, that the whole of the direct stresses are taken by the flanges, and the whole shearing stress by the web,

The Moment of Flexure, $\frac{2}{w}c^2 = f_b A d$, or $f_a A_1 d$, Moment of

Resistance.

In this case w per inch run $= \frac{14 \text{ tons}}{31 \times 12} = \frac{7}{186}$ tons.

$c = 186$ inches.

$f_b = 5$ tons.

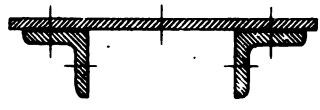
$f_a = 4$ tons.

whence $A = 6{\cdot}2$ square inches = effective section of lower flange.

$A_1 = 7{\cdot}75$ „ „ = gross section of upper flange.

Each flange may be taken to consist of the plate and two angle irons (*Fig.* 16), and the plate may be supposed to have three rows of rivets in it (the centre row being for the cover-plates), and the angle irons two rows each, one in each arm.

Fig. 16.



It is apparent, from the small sectional area required, that one plate will be enough, and it is as well (not necessary) to have the angle irons and plate the same thickness.

Assume the angle iron to be 3″ × 3″ which will be sufficient to allow of the wrappers being rivetted on, make the rivets $\frac{9}{16}$ inch diameter, and call the thickness of plate and angle irons, t.

Then for the tension flange.

The effective section (*Tension and Compression Bars*, p. 82) is $14 - 3 \times \frac{9}{16}) t$ (for the plate) + $2 (6 - t - 2 \times \frac{9}{16}) t$ (for the angle irons) and this effective section = A = 6·2 square inches,

whence $t = \frac{5}{16}$ inch.

For a plate $\frac{5}{16}$ inch thick, the proper diameter of rivet, according to ordinary practice (*U.R.T.*, p. 209; *U.I.B.R.*, p. 109) is about $\frac{9}{16}$ inch, which is the assumed diameter.

The loss of strength in the plate, due to the rivet holes, is about 13 per cent; the loss of strength on the whole flange is about 17 per cent.

For the compression flange.

The effective, which is the gross section, is $14\,t$ (for the plate) + $2\,(6—t)\,t$ (for the angle irons) and this gross section = $A_1 = 7{\cdot}75$ square inches,

whence $t = \frac{5}{16}$ inch.

(3) To determine the dimensions of the cover plates and of the wrappers for the angle irons.

The latter might be got rid of altogether, angle iron being easily obtainable of the length of this girder, but for the sake of showing how to calculate the wrapper, it will be assumed that there is a joint in the angle iron at the centre of the girder.

In order to keep the plates within the length for which extra charge is made, let there be three lengths of plate in each flange, the joints in each plate being thus about 5 feet on each side of the centre of the girder.

The stress on one side of the joints is first taken by the rivets, by them transferred to the cover plate, which in turn transfers it to the rivets on the other side of the joint, and so to the plate on that side.

The total stress at 5 feet from the centre $= \frac{\text{Moment of Flexure}}{\text{depth of girder.}}$

$= \frac{w(a^2-x^2)}{2ad}$ (*Stress in Beams*, p. 7) $= \frac{7 \times (186^2-60^2)}{186 \times 2 \times 21} = 27{\cdot}8$ tons nearly.

For the tension flange;

the effective section of the plate $= 9'' \times \frac{5}{16} = \frac{45}{16}$

and the effective section of the angle irons $= \frac{2 \times 355}{256} = \frac{44}{16}$

Of the total stress of 27·8 tons, the plate has to bear $\frac{45}{89} \times 27{\cdot}8$ tons $= 16$ tons nearly.

For the number, n, of rivets in the plate on each side of the joint, in order to be secure against shearing.

$$n \times \frac{\pi}{4} \times (\tfrac{10}{16})^2 \times 5 \text{ tons} = 16 \text{ tons}$$

$$\text{whence } n = 10{\cdot}4$$

For bearing surface,

$$n \times \tfrac{5}{16} \times \tfrac{10}{16} \times 7 \text{ tons} = 16 \text{ tons}$$

$$\text{whence } n = 11{\cdot}7$$

and the latter number must be adopted.

Let the pitch of the rivets in these plates be taken at 3 inches. Then as there will be four rows on each side of the joint, the cover plates will be 24″ long, and it will be seen afterwards that these rivets range with the other rivets in the work. As there is only one cover it should be $\frac{6}{16}$ thick. It would be far better construction to have two covers $\frac{4}{16}$ each, although it would take a little more metal.

For the compression flange;

the gross section of the plate $= 14'' \times \frac{5}{16} = \frac{70}{16} = \frac{440}{256}$

and the gross section of the angle irons $= 2 \times \frac{455}{256} = \frac{455}{128}$

The plate has on it $\frac{560}{1015} \times 27{\cdot}8$ tons $= 15{\cdot}3$ tons,

$$\text{For shearing, } n \times \frac{\pi}{4} \times (\tfrac{10}{16})^2 \times 5 \text{ tons} = 15{\cdot}3 \text{ tons},$$

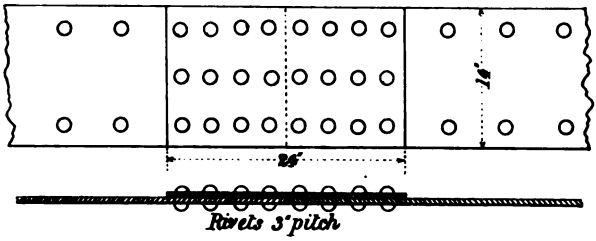
$$\text{whence } n = 9{\cdot}97.$$

$$\text{For bearing surface, } n \times \tfrac{5}{16} \times \tfrac{10}{16} \times 7 \text{ tons} = 15{\cdot}3 \text{ tons},$$

$$\text{whence } n = 11{\cdot}1 \text{ nearly.}$$

The cover plates in each flange may therefore be the same, and they will be as shown in *Fig.* 17, subject to alteration, if it be found necessary in order to make their rivets range with those in the rest of the work.

Fig. 17.



The angle iron wrapper.

The joint being at the centre of the girder in the tension flange, the relative effective section of the plate, and angle irons being the same as given just now, and the stress at the centre being 6·2 square inches × 5 tons = 31, the angle irons have on them $\frac{3\,5\,5}{8\,4\,0}$ × 31 = 13·1 tons.

For shearing, $n \times \frac{\pi}{4} \times (\frac{1\,0}{1\,6})^2 \times 5$ tons $= \frac{13{\cdot}1}{2}$ tons, there being two angle irons,

whence, n, the number of rivets in each angle iron = 4·3.

For bearing surface, $n \times \frac{1\,0}{1\,6} \times \frac{5}{1\,6} \times 7$ tons $= \frac{13{\cdot}1}{2}$ tons

whence n = 4·8.

or three rivets on each side of the joint, in one arm of the angle iron, and two in the other, will be required, or three in each may be used, if more convenient.

In the compression flange, the angle irons have on them $\frac{4\,5\,5}{1\,0\,1\,5}$ × 31 tons = 14 tons nearly.

For shearing, n = 4·5

„ bearing surface, n = 5·1

so that the wrappers may be the same for each flange.

A wrapper 19 inches long, and having a section equal to that of the angle iron may be used. It may be $2\frac{1}{4} \times 2\frac{1}{4} \times \frac{3}{8}$.

(4) To determine the thickness of the web, the number of rivets necessary in the web covers, the size of these covers, and whether stiffeners are necessary.

The web may be in 3 lengths, so that there will be a cover plate over the vertical joints at a distance on each side of the centre of, say 5 ft.

The web being of uniform thickness, it will be sufficient to determine the thickness of web necessary at the piers, where the shearing stress is greatest.

The shearing stress here is 7 tons, and as there are two rivet holes in the web, its depth to resist shearing is $21 - 2 \times \frac{1\,0}{1\,6}$ inches = 19·75 inches.

Let it be assumed that the shearing stress is equally distributed over the web (*Stress in Beams*, p. 27,) and call the thickness t.

Then for shearing, $19{\cdot}75\ t \times 5$ tons = 7 tons

$$t = \frac{7}{98{\cdot}75} \text{ inch, an impossible thickness.}$$

The webs must each have, then, the minimum thickness, *i.e.* $\frac{1}{4}$ inch.

It is however, necessary to determine how many rivets are necessary to convey the shearing stress from the web into the cover.

The shearing stress at 5 feet from the centre $= w \times 5 = \frac{14}{4} \text{ tons} \times 5 = \frac{70}{4} \text{ tons} = \frac{35}{2} \text{ tons}$ on each web (*Stress in Beams*, p. 20.)

Let n be the number of rivets necessary for shearing or bearing resistance.

Then for shearing, $n \times (\frac{10}{16})^2 \times \frac{\pi}{4} \times 5 \text{ tons} = \frac{35}{2} \text{ tons}$.

whence $n = 1$ barely.

For bearing surface, $n \times \frac{10}{16}'' \times \frac{1}{4}'' \times 7 \text{ tons} = \frac{35}{2} \text{ tons}$.

whence $n = 1{\cdot}03$.

In order to make the cover lie close to the web, two rivets are necessary on each side of the vertical joint, as shewn in *Fig.* 18. The cover may be the same thickness as the web, or, as there is but one cover on each web a little thicker, say $\frac{5}{16}$.

The width of the cover must be enough to allow of the rivet holes being punched both in the plates and in the cover (*Fig.* 18), $\frac{60}{16}''$ minimum, but it will be better to say 5 inches and make the rivets range with those in the angle irons.

Fig. 18.

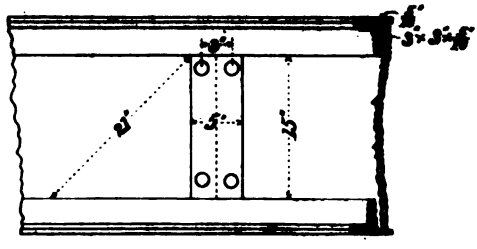
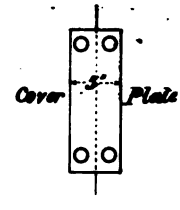


Fig. 19.



It is evidently unnecessary to enquire whether the two additional holes in the web render it unable to resist the shearing stress at this section, its thickness being so much in excess of that which is necessary.

To determine if stiffeners are necessary.

The shearing stress being greatest at the piers, and the web uniform in thickness, if stiffeners are necessary at all, they will be required at the piers.

The shearing stress at the piers is 7 tons, which is carried on two webs 19·75 inches in height after deducting the rivet holes.

Then the stress on 1 inch in height of each web is $\frac{7}{19{\cdot}75 \times 2}$ tons, and as the web is $\frac{1}{4}$ inch thick, the stress per square inch $= \frac{28}{39{\cdot}5}$ tons $= \frac{1}{1\cdot 0}$ ton nearly.

It has been shewn that the vertical and horizontal shearing stresses

having equal intensities may be resolved into two stresses of the same intensity, one a tension, and the other a thrust, acting at angles of 45° with the horizontal and at right angles to one another, and that the web tends to buckle under the thrust.

The intensity of the thrust $\frac{S}{P}$ (*Tension and Compression Bars*, p. 86) must not exceed what the material can safely bear, *i.e.*, $f_s \div \left(1+a\frac{l^2}{h^2}\right)$

In this case $\frac{P}{S} = \frac{7}{10}$ ton.

$f_s = 4$ tons.

$a = \frac{1}{3000}$

$h = \frac{1}{4}$ inch.

and l is unknown,

whence $l = 29$ inches, l being measured between the angle irons at an angle of 45°.

The distance between the angle irons vertically being 15 inches (*Fig.* 18) the distance measured at an angle of 45° is 21 inches, so that no stiffeners are necessary.

(5) The number of rivets necessary to connect the angle irons with the web and with the plates of the flanges.

It is usual to assume (*Stress in Beams*, p. 27), that the shearing stress is uniformly distributed over the web, and to calculate the number of rivets on this assumption.

Then, as the riveting is to be uniform throughout, it will be sufficient to determine the number of rivets necessary at the pier where the shearing stress is greatest.

At the pier, the shearing stress = 7 tons, and the web is 21 inches high; so that, on one foot in height of the two webs, the shearing stress $= \frac{12}{21} \times 7$ tons = 4 tons or 2 tons on each web.

The horizontal shearing stress, which tends to shear the rivets connecting the angle irons with the web, is equal to this; and hence, if n be the number per foot run of web,

$$\text{For shear, } n \times \frac{\pi}{4} \times \left(\tfrac{9}{16}\right)^2 \times 5 \text{ tons} = 2 \text{ tons.}$$
$$n = 1{\cdot}3.$$

$$\text{For bearing surface, } n \times \tfrac{9}{16} \times \tfrac{1}{4} \times 7 \text{ tons} = 2 \text{ tons.}$$
$$n = 2 \text{ nearly.}$$

Rivets at 6″ pitch will give the number required; and as the plate is thicker than the web, and there is the same shearing stress, it is evident that this number of rivets will be enough to connect the plate and angle iron. The actual depth of the girder will be 21·5 inches, the additional ·5 inch being the distance from the centres of figure of the flanges to the outside of the beam.

The girder may now be designed, as shown in *Plate* I. the rivets being so arranged that each can be rivetted up in the following order.

The lower flange, with angle irons and cover plates, is first rivetted

up; then the webs are put together and connected with the lower flange; then the angle irons for the top flange are rivetted to the webs; and finally, the plates of the top flange are put together, and connected with the webs, the last rows of rivets being those connecting the top plates with the angle irons.

Plate I. shews the rivets at 3″ pitch in the covers, and 6″ pitch clear of the covers. If the pitch were made 4″ throughout, the work would look better, but it would increase the number of rivets, and thereby the amount of work as well as lengthen the covers.

The centre row of rivets in the plates exists only at the covers, and in calculating the centre section (p. 121) the centre rivet hole might be left out of consideration, there being no cover there, but as the girder is to be a beam of *uniform section*, it is more correct to suppose it to be present.

EXAMPLE II.

A double line of narrow gauge railway is to be carried over a clear opening of 62 feet, the effective span, or span from centre to centre of bearings is to be 66 feet, and the total length 70 feet.

The bridge is to be of the form shewn in *Plate* II. The rails are to be carried on longitudinal timbers, 12″ × 6″ (which are not to be taken into account as affording any strength to the bridge); these timbers resting on longitudinal iron beams, which in turn are carried on, and rivetted to cross beams at central intervals of 5′6″.

The cross beams rest on the lower flanges, and are rivetted to the web and stiffeners of the main girders.

The platform between the longitudinals, cross beams, and main girders consists of ¼ inch plates, on which ballast, having an average thickness of 4 inches, is laid.

The rolling load on the bridge may be taken at 1¼ tons per foot run for each line of way.

The working resistances, taking the live load as if it were an equivalent dead load, are to be as follows:

In tension	5 tons	per square inch.
„ compression	4 „	
„ shearing	4 „	
On bearing surfaces	5 „	

The main girders are to be placed on lead on the piers, and are not to be fastened in any way.

Remarks.

The width of the bridge is determined by the Board of Trade Rules, which are as follows:

"*Rule* 15.—No standing work should be nearer to the side of the widest carriage in use on the line than 2 feet 4 inches at any point between the level of 2 feet 6 inches above the rails and the level of the upper parts of the highest carriage doors. This applies to all arches, abutments, piers, supports, girders, tunnels, bridges, roofs, walls, posts,

tanks, signals, fences, and other works, and to all projections at the side of a railway constructed to any gauge."

"*Rule* 16. The intervals between adjacent lines of rails, or between lines of rails and sidings, should not be less than 6 feet."

The condition that the longitudinal sleepers carrying the rails are not to be taken into account as affording any strength to the bridge is necessary from the consideration, that these timbers may and probably will be renewed by men knowing nothing of their functions as part of the bridge, and consequently if relied on for strength, their repair may involve the safety of the bridge. (*U.I.B.B.*, p. 64.)

The distance apart of the cross beams has been selected as being suitable to make a stiff platform. It would be cheaper to put them farther apart, but the longitudinals would then have to be stronger. This distance, or any distance up to about 7 or 8 feet has the advantage of ensuring that only one driving wheel of an engine can come on the longitudinals at one time. Cross beams are sometimes put as far apart as 12 or 14 feet.

The curved form adopted for the plates of the platform is for the purpose of drainage, and a row of small holes should be punched at the bottom of the curve and provision made for carrying off the water.

The rolling load on the bridge is that used ordinarily in calculations for bridges of this length. (*Foot Note*, p. 69). The working resistances are in accordance with ordinary practice, and do not exceed the limit laid down in the Board of Trade Rules.

"*Rule* 11. In a wrought-iron bridge the greatest load which can be brought upon it, added to the weight of the super-structure, should not produce a greater strain on any part of the material than five tons per square inch.

The heaviest engines in use on railways afford a measure of the greatest moving loads to which a bridge can be subjected. These rules apply equally to the main and the transverse girders. The latter should be so proportioned as to carry the heaviest weights on the driving wheels of locomotive engines."

THE LONGITUDINAL BEAMS.

These girders have a span of 5·5 feet.

They are rivetted at the ends to the cross beams, and are in the condition of "imperfectly" fixed beams. It is not generally possible to "fix" them absolutely, as there is not room for the requisite number of rivets.

The flanges of these beams are, however, really stronger than they are calculated to be, and this is a good provision, because the rolling load, which constitutes nearly the whole load on them, comes on very suddenly and produces actually a much greater intensity of stress, (it approaches twice the intensity, *R.C.E.*, p. 278), than that at which they are calculated, an effect which is also produced on the cross beams but not to nearly the same extent on the main girders. (*U.I.B.B.* p. 35).

The dead load on them consists of 5·5 feet run of rail, say at 80 lbs.* to the yard including fastenings, 5·5 feet run of timber 12" × 6", at 40 lbs. per foot cube, about 25 feet super of $\frac{1}{4}$" plate, covered by 4" of ballast at 93 lbs. to the cubic foot, and finally their own weight.

The live load consists of the weight on one driving wheel, say 7 tons, applied as a single concentrated load, which will produce the greatest direct stresses when it is at the centre of the longitudinal, and the greatest shearing stress when it is just clear of the cross beam, either coming on, or going off the longitudinal.

Although the ratio of the live to the dead load in this case is so great, that the latter might be neglected without appreciable error, the weight of the longitudinals is here approximately determined for the sake of illustration, by the formula given by Mr. Unwin. (*U.I.B.R.*, p. 41).

Then W = 147 lbs. + 110 lbs. + 250 lbs. + 775 lbs. = 1282 lbs.
= ·57 tons + 7 tons central = 14·57 tons uniformly distributed.

W_1 = weight of longitudinal in tons.
l = 5·5 feet.
d = 1 foot = effective depth.
r = 5·5. = ratio of length to depth.
C = 1500. (C varies for small wrought iron plate girders from 1400 to 1500).
s = 4 tons.

$$\text{Then } W_1 = \frac{W\,l\,r}{C\,s - l\,r} = \frac{14{\cdot}57 \times 5{\cdot}5 \times 5{\cdot}5}{1500 \times 4 - 5{\cdot}5 \times 5{\cdot}5} = {\cdot}073 \text{ tons, each,}$$

whence total distributed load = 14·643 tons = ·222 tons per inch run of length.

The Flanges.

For the tension flange at the centre. (*Stress in Beams*, p. 7.)

$$\frac{w\,c^2}{2} = f_b \times A \times d, \text{ all in tons and inches.}$$

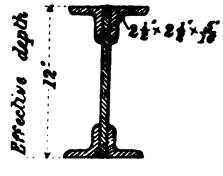
$$\frac{{\cdot}222 \times 33^2}{2} = 5 \times A \times 12$$

whence A = 2 square inches, effective section of tension flange.

For the compression flange, the gross section will be $\frac{5}{4}$ of the effective section of the tension flange, or $2\frac{1}{2}$ square inches.

Taking two angle irons and a web for the section,

Fig. 20.



* Rails are now used on main lines up to 84lbs. per yard run exclusive of chairs and fastenings.

making the angle irons $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{5}{16}''$, and assuming the rivets at $\frac{3}{4}''$,

$A = 2\ (5 - \frac{5}{16} - \frac{3}{4})\ \frac{5}{16} = 2{\cdot}5$ square inches which is enough; and for the compression flange,

$A_1 = 2\ (5 - \frac{5}{16})\ \frac{5}{16} = 2{\cdot}93$ square inches, which is more than enough.

The flanges will therefore be made similar.

The Web.

The greatest shearing stress is close to the ends and $= 7{\cdot}32$ tons when the driving wheel of the engine is either just coming on or rolling off.

The web must be thick enough to fulfil three conditions, viz.: it must have sufficient metal in it to resist the horizontal shearing along the joint of the web with the angle irons and the vertical shearing after deducting the metal lost by the rivet holes; and it must be thick enough not to buckle under the thrust in the manner described at p. 116.

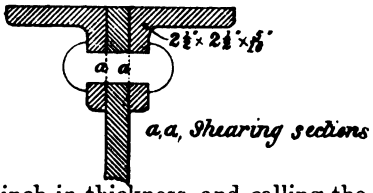
The shearing stress per foot in height of the web at the piers is 7·32 tons, the web being 1 foot high, and the horizontal shearing stress along the joints of the web with the flanges is the same per foot run of length.

Assuming the rivets at $\frac{3}{4}$ inch diameter, calling the number of rivets to resist shear, n, and recollecting that each rivet shears in two sections (*Fig.* 21).

$$n \times 2 \times \frac{\pi}{4} \times (\tfrac{3}{4})^2 \times 4 \text{ tons} = 7{\cdot}32 \text{ tons}$$

whence $n = 2$.

Fig. 21.



Assuming the web at $\frac{3}{8}$ inch in thickness, and calling the number of $\frac{3}{4}$ inch rivets necessary for bearing area, m,

$$m \times \tfrac{3}{4}'' \times \tfrac{3}{8}'' \times 5 \text{ tons} = 7{\cdot}32 \text{ tons.}$$

whence $m = 5{\cdot}2$

This is an inconvenient number, so that the web must either be thickened, or the rivets enlarged, or both.

Make the web $\frac{1}{2}$ inch.

Then

$$m \times \tfrac{3}{4}'' \times \tfrac{1}{2}'' = 7{\cdot}32 \text{ tons.}$$

and $m = 3{\cdot}9$, say 4.

This gives the pitch of the rivets as 3 inches, and there will remain, after deducting the rivet holes $(12 - 4 \times \frac{3}{4}), \frac{1}{2} = 4{\cdot}5$ inches solid metal to resist the shear, which is evidently more than enough.

This thickness, and this arrangement of rivets meet the case, and it remains to see whether there is any danger of buckling.

The clear distance between the angle irons is 7 inches, the shear on one foot in height of the web is 7·32 tons, which is equal, the web having 4·5 square inches of iron, to 1·63 tons on the square inch.

The thrust at an angle of 45° has therefore this same intensity, and in order that buckling may not occur, it must not be greater than a long column, of a length $7\sqrt{2}$ inches, and a least dimension of $\frac{1}{4}$ inch, can bear with safety.

The intensity of stress which such a column can safely bear is given by the right side of the equation (*Tension and Compression Bars*, p. 86), viz.:

$$f_c + (1 + a\frac{p}{k^2}), \text{ which in this case} = \frac{4 \text{ tons}}{1 + \frac{1}{3000}\frac{98}{\frac{1}{4}}} = 3{\cdot}53 \text{ tons.}$$

so that there is no danger of buckling.

The Deflection.

If the beam be regarded as supported at the ends, and loaded in the centre, and to be working under stresses of 7·5 and 6 tons respectively in consequence of the suddenness of the application of the load, the deflection will be given by formula 15 (*Deflection of Beams*, p. 37).

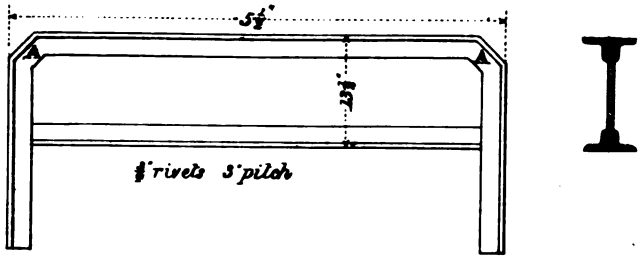
$$v = \frac{n'(f_c + f_t)\sigma^2}{E h_0} = \frac{1}{8} \cdot \frac{13{\cdot}5 \times 2240 \times 33^2}{18{,}000{,}000 \times 12} = {\cdot}05 \text{ inch}$$

which would be reduced (*Fixed Beams*, p. 58), to $\frac{{\cdot}50}{5} = {\cdot}01$ inch, if the "fixing" were complete.

The weight of the beam.

The section and elevation of the beam may now be drawn, and its weight calculated as below. It would make sounder work to put covers at the points A, A, instead of cranking the angle iron, as the weld in the latter method is very apt to injure the iron.

Fig. 22.



The extra $1\frac{1}{2}$ inch in depth is given by the distance from the centre of figure of the flanges to the outsides of the beam.

The weight is as follows:

2×13 feet, $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{5}{16}$, angle irons $= 125$ lbs.

$5{\cdot}5' \times \frac{13{\cdot}5''}{12} \times \frac{1}{2}''$, web $= 124$ „

104 rivet heads, each $\frac{1}{16} \times \frac{3}{4}''$ long, $\frac{3}{4}''$ round iron $= 12$ „

261 lbs. = ·116 ton.

or the actual weight is considerably greater than that given by the formula. The difference, however, is not enough to make recalculation necessary.

The connection of the longitudinal with the cross beam by rivets will be given after the thickness of the web of the latter is determined.

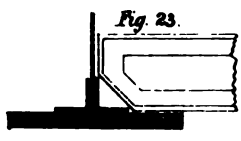
NOTE.—In very small beams, such as these, the area of the web bears so large a ratio to the whole section, that this method of calculation underestimates materially their transverse strength. In such beams, intended for a dead load, it is more accurate and quite proper to treat them in the same way as the rolled wrought iron beam, given at p. 78, care being taken to deduct the metal in the rivet holes for the parts in tension, in calculating the value of the Moment of Inertia, I. It is, perhaps, better to calculate small railway beams in the less accurate manner, because by so doing, a rough compensation is afforded for the sudden application of the load.

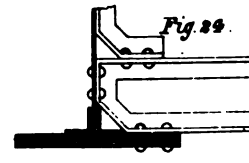
THE CROSS BEAMS.

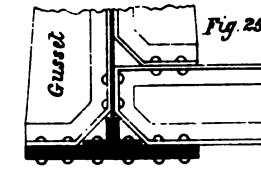
These beams may be either in the condition of beams "supported only at the ends," "imperfectly fixed," or "fixed," according to the manner in which they are connected with the main girders.

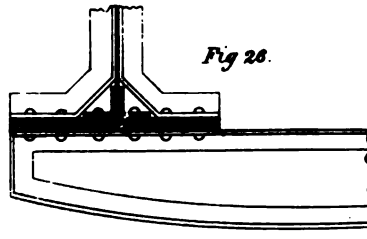
If they merely rest on the lower flange without having their ends rivetted to the web or stiffeners of the main girder (*Fig.* 23), they are nearly free to bend as a "supported" beam bends; but this is unusual, although it strains the stiffener of the main girder less, and is therefore better for the main girder. It is more common to see them rivetted firmly to the web (*Fig.* 24), and stiffeners of the main girder, and sometimes the connection is made more complete by gusset pieces (*Fig.* 25). In this case they are beams nearly perfectly fixed at the ends, the only defect from perfect fixing being due to the possible lateral bending of the stiffeners, which cannot be great.

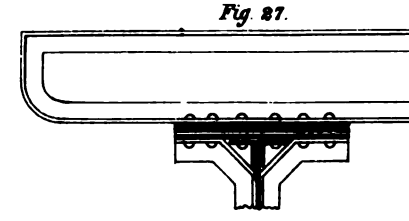
If they are suspended to the under side (*Fig.* 26), or rest on the top of the main girders (*Fig.* 27), being rivetted to the flanges, they are perfectly "fixed" beams.











It is customary to design them as if they were "supported only," the section of their flanges or their depth being reduced from the centre to the ends. But, if they are really "fixed" beams, it would surely be better to design them as such, in which case, as the section of greatest stress is at the ends, the quantity of metal in the flanges, or the depth, should be greatest there, and the reduction, if any, should be towards the centre.

Moreover, as the load is really applied "suddenly" and therefore produces at first a greater stress than the same amount of load gradually applied would produce, it would seem prudent to allow for this by multiplying it by some factor to reduce it to dead load, and this would perhaps be met by a factor of $1\frac{1}{2}$.

In this Example the cross beams are worked out by both methods.

To design the cross beams according to ordinary practice.

Their maximum load consists of four loads symmetrically placed, two on each side of the centre, each consisting of the weight on one driving wheel of an engine, added to the load on each longitudinal; and of the uniform load due to the weight of the beam itself.

It simplifies the calculations to reduce these loads to an uniformly distributed equivalent load, which may be done, with sufficient accuracy, by equating the moment of flexure at the centre of the beam due to the four loads, with the moment of flexure of an uniformly distributed load, the beam in both cases being considered as supported only, thus:

Moment of flexure of the 4 loads (*Stress in Beams*, p. 10) $= -2\,Wd - Wb$

Moment of flexure of uniform load (*Stress in Beams*, p. 7) $= -\frac{wc^2}{2}$

$$\text{Equating these, } w = \frac{4\,Wd + 2\,Wb}{c^2}$$

and substituting the values, W=7·7 tons, d=5′, b=5′, c=13′,

w=1·37 tons per ft. run.

The weight of the beam itself may be deduced as before.

In this case W=35·62 tons, l=26 feet, $d=\frac{1}{12}$ feet, r=12, C=1500. s=4 tons,

$$\text{and } W_1 = \frac{35{\cdot}62 \times 26 \times 12}{1500 \times 4 - 26 \times 12} = 1{\cdot}953 \text{ tons} = {\cdot}075 \text{ tons per ft. run} = w_1$$

The total value of w per ft. run $= 1{\cdot}37 + {\cdot}075 = 1.44$ tons.

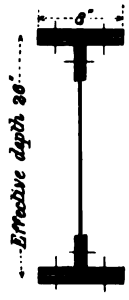
The Flanges.

It is proposed to build the beams of the section shewn in *Fig.* 28, one, two, or more plates being used at the centre of the beam, which are reduced in number towards the ends.

The simplest way to design the flanges is partly by calculation, partly by diagram, as follows:

Calculate the moments of flexure at convenient distances, say at 3′, 5·5′, 8′, 10·5′ on each side of the centre, when the whole load is on,

Fig. 28.



the direct stresses being then greatest (*Stress in Beams*, p. 25) and lay them down to a scale of parts, as in *Fig.* 29.

The values of $M = -\frac{w}{2}(c^2 - x^2)$ (*Stress in Beams*, p. 7) are as follows:

when $x =$ 0, *i.e.*, at the centre,	$M = -121{\cdot}68$	ft. tons.
$x =$ 3′	$M = -115{\cdot}2$	„ „
$x =$ 5·5′	$M = -\ 99{\cdot}9$	„ „
$x =$ 8′	$M = -\ 75.6$	„ „
$x = 10{\cdot}5'$	$M = -\ 42{\cdot}3$	„ „

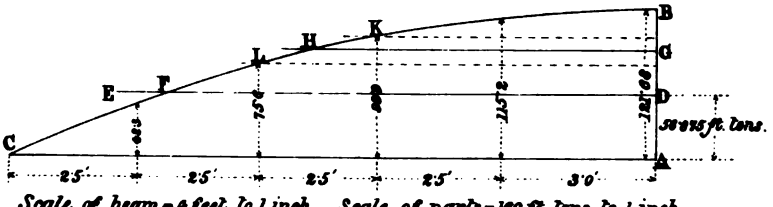
Assuming the angle irons to be $3\frac{3}{4}'' \times 3\frac{3}{4}'' \times \frac{1}{2}''$, and the rivets to be $\frac{7}{8}''$, two rows of rivets in each angle iron, and two rows in each plate, the effective section of the angle irons of the tension flange is $5\frac{1}{4}$ sq. ins.

The moment of resistance of these angle irons is

$\frac{26}{12}'$ (depth) $\times$ $\frac{5}{4}''$ (area) $\times$ 5 tons $= 56{\cdot}875$ ft. tons,

which is constant throughout the beam.

Fig. 29.



Scale of beam – 4 feet to 1 inch. Scale of parts – 100 ft. tons to 1 inch.

Set off on AB (*Fig.* 29), AD to represent 56·875 ft. tons, and draw D E parallel to A C; then D B represents the moment of resistance which the plates must furnish at the centre, and the ordinate between D E and F B anywhere gives the moment of resistance to be furnished by the plates at that section.

As this ordinate has no value at F, it is clear that no plate is required

between F and the end of the beam, although a plate is always used in order to make sound work.

Let there be two plates at the centre of the girder, then each supplies half of the moment of resistance still wanting. Bisect D B in G, and draw G H parallel to A C. Then it is evident that one of the plates becomes unnecessary at H, where the moment of resistance of the angle irons, and one plate are together equal to the moment of flexure.

Let the plates be made 8″ wide and let t be their thickness. Then in the tension flange the effective section of each is $(8-\frac{1}{8})\ t$ and the moment of resistance of each is

$$\tfrac{7}{8}t \times 5 \text{ tons} \times \tfrac{3}{12} \text{ feet} = 67{\cdot}7t \text{ ft. tons.} \qquad (1)$$

and the moment of flexure at the centre which each has to meet is

$$\frac{122-57}{2} = 33 \text{ ft. tons.} \qquad (2)$$

Equating (1) and (2) $t = {\cdot}49''$, the nearest thickness to which is $\frac{8}{16}$

The tension flange will therefore have two plates at the centre and for 6·5 feet on each side, and one plate from these points to the end. It might be better to have three plates at the centre, and drop the first at K, and the second at L; but $\frac{5}{16}''$ plates would give hardly enough section, while $\frac{6}{16}''$ would give too much.

In the compression flange, the effective section of the angle irons is the gross section, but the metal has a lower working resistance.

The moment of resistance of the angle irons is

$$\tfrac{3}{12}' \times 7'' \times 4 \text{ tons} = 60{\cdot}66 \text{ ft. tons.}$$

which is rather greater than in the tension flange.

Taking two plates as before, each has a moment of resistance of

$$8 \times t \times \tfrac{3}{12}' \times 4 \text{ tons} = 66t \text{ ft. tons.} \qquad (3)$$

and the moment of flexure which each has to meet is

$$\frac{122-61}{2} = 30{\cdot}5 \text{ ft. tons.} \qquad (4)$$

Equating (3) and (4) $t = {\cdot}46''$, to which the nearest thickness of plate is $\frac{8}{16}$.

The two flanges may therefore be made alike.

The Web.

The greatest shearing stress is at the ends of the beam, when the load is fully on, and then it equals $\frac{37{\cdot}44}{2} = 18{\cdot}72$ tons.

The shearing stress per ft. in height of the web

$$= \frac{18{\cdot}72 \times 12}{26} = 8{\cdot}64 \text{ tons.}$$

which equals the shearing stress per ft. run along the joints of the web with the angle irons.

Assuming the rivets at $\frac{7}{8}''$ diameter, calling the number of rivets necessary to resist shear, n, and recollecting that each rivet shears in two sections (*Fig.* 21).

$$n \times 2 \times \frac{\pi}{4} \times (\tfrac{7}{8})^2 \times 4 \text{ tons} = 8{\cdot}64 \text{ tons.}$$

$$\text{whence } n = 1{\cdot}8.$$

Assuming the web at $\frac{1}{2}''$ in thickness, and calling the number of rivets necessary for bearing area, m,

$$m \times \tfrac{1}{2}'' \times \tfrac{7}{8}'' \times 5 \text{ tons} = 8{\cdot}64 \text{ tons}$$

whence $m = 4$, which is a convenient number.

To see if there is enough metal in the web to resist shear.

Assume 4 rivets per ft. run in height, the metal remaining is $(12 - 4 \times \frac{7}{8}'') \times \frac{1}{2}'' = \frac{17}{32}''$ square inches which is capable of resisting a shear of 17 tons, or twice what it has to bear.

To see if the web at the piers will be liable to buckle.

At the piers the shearing stress per ft. in height = 8·64 tons, which is spread over $\frac{17}{4}$ square inches, so that the stress per square inch is 2 tons.

The thrust at an angle of 45° has therefore this same intensity, and in order that buckling may not occur it must not be greater than a long column of a length of $18{\cdot}5\sqrt{2}$ inches, and a least dimension of $\frac{1}{2}$ inch (*Tension and Compression Bars*, p. 86) can bear with safety, *i.e.*

$$f_a \div \left(1 + a\frac{l^2}{h^2}\right) > 2 \text{ tons.}$$

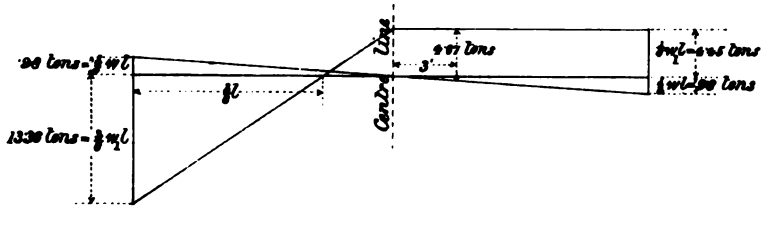
As the left side of the equation = 2·09 tons, the web will not buckle. and no stiffeners are necessary for the part of the web which is $\frac{1}{2}''$ thick,

If the web be made in 3 lengths, say two 10 ft. lengths, and a centre length of 6 ft., the shearing stress on the centre length being very small, it may be reduced probably to $\frac{1}{4}''$ thick.

The shearing stress on the centre part is greatest at one end of it, when the rails at the other end of it only are loaded, *i.e.* when only one train is on.

In this case the shearing stress may be represented graphically, as in *Fig.* 30.

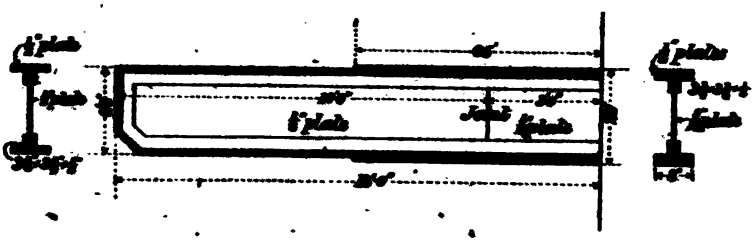
Fig. 30.



The maximum shear on the centre length of the web is only 4·67 tons, and calculating as for the other lengths it is found that $\frac{1}{4}''$ plate is barely sufficient for it. It should be made $\frac{5}{16}''$.

On these calculations the sections and elevation of the cross beam will be as shewn in *Fig* 31. and it remains to calculate the various connections.

Fig. 31.



The Connections.

It will be convenient to have the second plate of the flanges in one length, *i.e.* 13 feet and the other plate in three parts, which avoids a joint at the centre, where the direct stress is greatest.

By making each lower end plate $10\frac{11}{12}$ ft. long, it will be long enough to turn up at the end, meeting the upper flange at the angle.

The Covers.

For the cover plate at 5 ft. from the centre.

The greatest direct stress here is given by $\frac{w(c^2-x^2)}{2d} = \frac{1{\cdot}44(169-25)}{2\times\frac{11}{12}}$

=47·3 tons, which is taken up by the angle irons, and the two plates in proportion to their sections, *i.e.*, as $\frac{7}{8}^2$, $\frac{5}{8}^2$, $\frac{5}{8}^2$, so that each plate has on it 12·85 tons.

Then to carry this stress across the joint in the tension flange, if n be the number of rivets necessary to resist shearing,

$$n \times \frac{\pi}{4} \times (\tfrac{7}{8})^2 \times 4 \text{ tons} = 12{\cdot}85 \text{ tons},$$

whence $n=5{\cdot}4$ or 3 rows on each side of the joint; and for bearing area,

$$m \times \tfrac{7}{8}'' \times \tfrac{1}{2}'' \times 5 \text{ tons} = 12{\cdot}85 \text{ tons},$$

whence $m=6$ nearly or 3 rows.

Taking 3 rows and the pitch as 3″, the length of the cover plate will be about 18″ inches, and it may be $\frac{1}{2}''$ or $\frac{9}{16}''$ thick.

The same cover plate is found to answer for the compression flange (see note on covers of main girder).

The Wrappers.

No wrappers will be required, as it is easy to get angle irons 28 feet long, and there will be no joint except at the ends, where a wrapper is not necessary.

The number of rivets to connect the angle irons and plates.

The horizontal shear is greatest at the ends, and has already been shewn to be, at those points, 8·7 tons per foot run of length,

Then for shearing,

$$n \times \frac{\pi}{4} \times (\tfrac{7}{8})^2 \times 4 \text{ tons} = 8{\cdot}7,$$

whence $n = 3{\cdot}6$,

and as there are two rows, 2 per foot run will be enough.

For bearing area, $m \times \frac{7}{8}'' \times \frac{1}{2}'' \times 5$ tons $= 8{\cdot}7$ tons,

whence $m = 4$, or as there are two rows, 2 per foot run will suffice.

The Covers to the Joints in the Web.

The covers to the two joints in the web, may be the ends of the longitudinals, and the number of rivets to connect the longitudinals and cross beams may now be determined.

They have to resist a shear of 7·32 tons.

Taking them as $\frac{7}{8}''$ diameter;

To resist shear, $n \times \frac{\pi}{4} \times (\frac{7}{8})^2 \times 4$ tons $= 7{\cdot}32$ tons.

whence $n = 3$.

For bearing area, $m \times \frac{7}{8}'' \times \frac{1}{2}'' \times 5$ tons $= 7{\cdot}32$ tons,

whence $m = 3{\cdot}3$,

so that 4 rivets, or two in each angle iron will suffice.

The Deflection.

As the beam is nearly a beam of uniform strength and breadth, uniformly loaded, the deflection including camber is given by formula 15, (*Deflection of Beams*. p. 37).

$$V = \frac{n''(f_a + f_b)c^2}{E h_o}$$

$$= \frac{{\cdot}5708 \times 9 \times 2240 \times 156^2}{18{,}000{,}000 \times 26} = {\cdot}598 \text{ inches.}$$

The deflection would really be more than this, owing to the actual sudden application of the load, probably about $\frac{1}{4}$ more.

Weight of Beam.

The cross beam may now be drawn and its weight calculated.

Centre plates $2 \times 13' \times 8'' \times \frac{1}{2}''$ Other plates $2 \times 30{\cdot}4' \times 8'' \times \frac{1}{2}''$ Covers $2 \times 2 \times 19'' \times 8'' \times \frac{1}{2}''$	62 feet super. at 20 lbs. =	1240 lbs.
Angle irons $2 \times 2 \times 30{\cdot}4' \times 3\frac{3}{4}'' \times 3\frac{3}{4}'' \times \frac{1}{2}'' = 70\frac{9}{12}$ feet super. at 20 lbs.	=	1415 lbs
Web plates $2 \times 10'$ $\times 2'{\cdot}2'' \times \frac{1}{2}'' = 43\frac{4}{12}$ feet super. at 20 lbs.	=	867 lbs
„ „ $1 \times 6'$ $\times 2'{\cdot}2'' \times \frac{5}{16}'' = 13$ feet super. at 12·5 lbs.	=	163 lbs.
Packing $2 \times 8'{\cdot}2'' \times 3\frac{3}{4}''$ $\times \frac{3}{16}'' = 5\frac{1}{12}$ feet super. at 7·5 lbs.	=	38 lbs.
Rivet heads, $1332 \times \frac{7}{8}'' \times 1\frac{3}{16}$, of $\frac{7}{8}''$ round iron	=	227 lbs.
		3952 lbs.

THE CROSS BEAM CONSIDERED AS "FIXED."

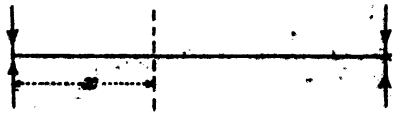
In this case the calculations will be as follows:

The Flanges.

The Moment of Flexure at any vertical section is obtained by steps similar to those given in full, in *Fixed Beams*, p. 52.

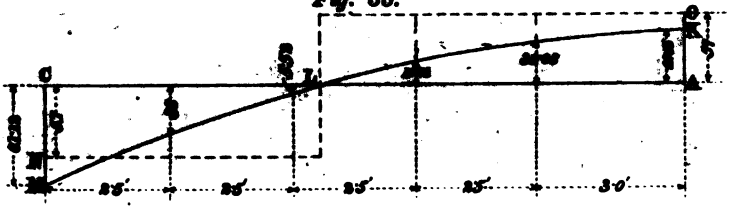
and $=\frac{w}{2}(l\sqrt{\frac{1}{3}}+x)^2-w\left(\frac{l}{2}+l\sqrt{\frac{1}{3}}\right)x$ which may be put for calculation in the form $\frac{wl^2}{2}\left(\frac{1}{3}-\frac{x}{l}+\frac{x^2}{l^2}\right)$

Fig. 32.



When $x=0$, *i.e.*, at the end of the beam, $M=+81{\cdot}12$ ft. tons.
$x=2{\cdot}5'$ $M=+38{\cdot}$ " "
$x=5'$ $M=+5{\cdot}52$ " "
$x=7{\cdot}5'$ $M=-18{\cdot}78$ " "
$x=10'$ $M=-34{\cdot}08$ " "
$x=15'$, *i.e.*, at the centre, $M=-40{\cdot}55$ " "

Fig. 33.



Laying this down to scale as before (p. 133), the curve of moments is given by KLM (*Fig.* 33), and setting off C N and A O, each equal to the moment of resistance of the angle irons, *i.e.*, 57 ft. tons, it is seen that when the maximum load is on, no plates are required except at a very small distance, about 2 feet, at the ends.

The stresses are somewhat different, when only one train is on the bridge, but they are then much smaller, and they are different only about the point of contra flexure (*Fixed Beams*, p. 52), *i.e.*, about L, which then approaches nearer to C, and it is evident that the angle irons alone are more than sufficient to meet any change in the stress here.

The thickness of the plate at the ends must be enough to take up $81{\cdot}12-57=24{\cdot}12$ ft. tons.

The effective breadth is $(8-\frac{1}{8}")$ inches, and the moment of resistance (as before, p. 134) $=67{\cdot}7t$ ft. tons.

$$\text{whence } t=\frac{24{\cdot}12}{67{\cdot}7}={\cdot}36"=\tfrac{6}{16}" \text{ nearly,}$$

and the beam might be made uniform in section, the flanges consisting of two angles and one $\frac{6}{16}"$ plate.

The Web.

The web would be the same as for the beam "supported only," the

shearing stress on the ends being the same, while the maximum shearing stress on the centre length is nearly the same in both cases.

The Deflection.

The deflection would be given by the formula (*Fixed Beams*, p. 56).

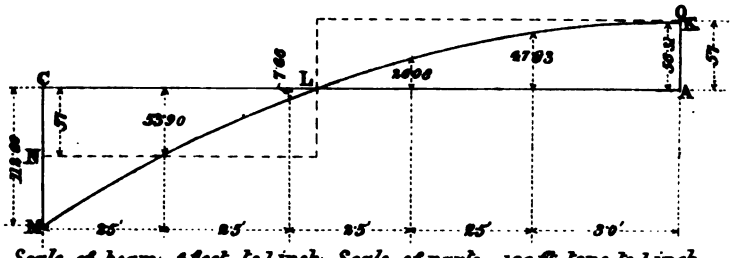
$$V = \frac{wl^4}{EI} \cdot \frac{1}{384}$$

Finding I by the method already explained (*Rolled Wrought Iron Beams*, p. 79, 80), recollecting to leave out the metal lost by the rivet holes in the parts in tension, and working out, V = ·13 inch.

But this makes no allowance for the actual effect of the quickly rolling load.

If allowance be made for this, as already suggested (p. 132), the load will be 2 tons per ft. run, and the curve of moments as derived from the formula, $M = \frac{w}{2}(l\sqrt{\frac{1}{8}}+x)^2 - w\left(\frac{l}{2}+l\sqrt{\frac{1}{8}}\right)x$ will be as shewn in *Fig.* 34.

Fig. 34.



Scale of beam 4 feet to 1 inch. Scale of parts = 160 ft. tons to 1 inch

where, when $x =$ 0, $M = +112{\cdot}66$ ft. tons.

$x = 2{\cdot}5'$, " $= + 53{\cdot}90$ " "

$x = 5'$, " $= + 7{\cdot}66$ " "

$x = 7{\cdot}5'$, " $= - 26{\cdot}08$ " "

$x = 10'$, " $= - 47{\cdot}33$ " "

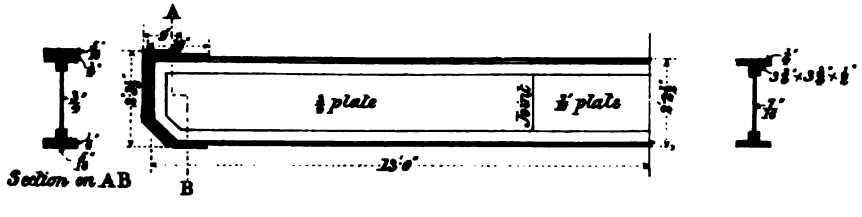
$x = 13'$, " $= - 56{\cdot}31$ " "

The thickness of the plate at the piers must be enough to take up $112{\cdot}66 - 57 = 55{\cdot}66$ ft. tons.

The moment of resistance of the plate as before shewn is $67{\cdot}7t$ ft. tons.

whence $t = \frac{55{\cdot}66}{67{\cdot}7} = {\cdot}82'' = 1\frac{1}{16}$

Fig. 35.



As this thickness is only necessary for a distance of 2 feet from the ends, it will be best to put one $\frac{1}{4}$" plate running through, and to make up the necessary thickness for about 18 inches at the ends with $\frac{5}{16}$" plates. (*Fig.* 35.)

The two flanges may be made similar.

The web should also be increased in thickness in proportion to increased load, *i.e.*, from $\frac{1}{2}$" to $\frac{3}{4}$" at the ends, and from $\frac{5}{16}$" to $\frac{7}{16}$" at the centre, and the rivets should be calculated.

The deflection will now be somewhat greater than before, the beam not being a beam of uniform section, but approaching a "fixed" beam of uniform strength.

THE MAIN GIRDERS.

The load on the main girders consists of the platform, the rolling load, and their own weight, which may all be taken as uniformly distributed.

There are 11 cross beams, carrying 40 longitudinals and 8 more longitudinals (4 longitudinals at each end) half of which are carried on the cross beams resting directly on the bearings; and there is also 66 ft. of platform on each side, 2·5′ wide, borne directly on the main girder.

The load is as follows:

44 longitudinals	**=44×** ·166 ton	**=**	**5·104 tons.**
Platform resting on ditto	**=44×** ·57 „	**=**	**25·08** „
11 cross beams	**=11×1·755** „	**=**	**19·305** „
2×66′ platform 2·5′ wide=330′ super. × 41 lbs.		**=**	**6·04** „
			55·529 „

The rolling load is $66 \times 2 \times 1\frac{1}{4}$ tons=165 tons.

Total load exclusive of main girders=220·5 tons=W. Then from the formula previously used (p. 128), the symbols having the same meaning,

$$W_1=\frac{Wlr}{Cs-lr}=\frac{221\times 66'\times 12}{1500\times 4-66'\times 12}=33\cdot 6 \text{ tons or } 16\cdot 8 \text{ tons each.}$$

Hence the total dead load on one girder is 44·6 tons.

„ rolling load „ „ 82·5 tons.

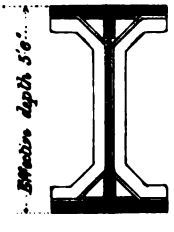
the weight per ft. run of dead load is ·68 tons=w

„ „ rolling load 1·25 tons=w_1

„ „ rolling and dead load together. } 1·93 tons.

It is proposed to make these girders of the form shewn in *Fig.* 36.

Fig. 36.

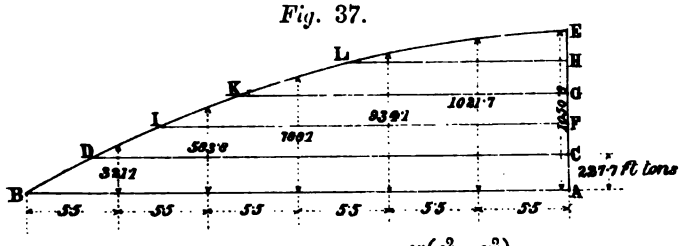


The Flanges or Booms.

As the greatest stress is produced in the flanges when the girder is fully loaded, it is only necessary to deal with this condition of the load.

Calculate the Moments of Flexure (*Stress in Beams*, p. 11) at intervals of, say, 5·5′ along the girder, and lay them down to scale, as in *Fig.* 37.

Fig. 37.



Then, when $x = 0$, *i.e.*, at the centre, $M = \frac{w(c^2-x^2)}{2} = 1050{\cdot}8$ ft. tons.

$x = 5{\cdot}5$	,,	$= 1021{\cdot}7$	,,
$x = 11$	,,	$= 934{\cdot}1$	,,
$x = 16{\cdot}5$	,,	$= 788{\cdot}1$	,,
$x = 22$	,,	$= 583{\cdot}8$	,,
$x = 27{\cdot}5$	,,	$= 321{\cdot}1$	,,
$x = 33$, *i.e.*, at the end	,,	$= 0{\cdot}0$	,,

Assuming the angle irons to be $4\frac{1}{2}'' \times 4\frac{1}{2}'' \times \frac{5}{8}''$, the rivets for safe punching should be about $\frac{5}{8}'' \times 1\frac{1}{2}$, say $\frac{7}{8}''$ diameter.

For the tension flange, the effective section, deducting rivet holes is $2\,(6\frac{1}{8} \times \frac{5}{8}) = 8{\cdot}28$ sq. inches.

The moment of resistance of the angle irons is

$8{\cdot}28'' \times 5{\cdot}5$ ft. (effective depth) $\times$ 5 tons $= 227{\cdot}7$ foot tons.

Set this off on *Fig* 37, and draw the horizontal line C D; then the ordinates between the line C D and the curve give, at every point, the moment of resistance to be supplied by the plates.

If the tension boom be made 19″ wide, *i.e.*, about $\frac{1}{40}$ of the span, with 4 rows of $\frac{7}{8}''$ rivets across it, the effective section will be $(19 - 4 \times \frac{7}{8})$ inches $\times\, t = 15{\cdot}5$ inches $\times\, t$, t being the total thickness of the plates at the centre, and the moment of resistance of the plates will be

$$15{\cdot}5t \times 5{\cdot}5 \text{ ft.} \times 5 \text{ tons} = 426{\cdot}25t \text{ ft. tons.} \qquad (1)$$

The moment of resistance required is $1050{\cdot}8 - 227{\cdot}7 = 823{\cdot}1$ ft. tons (2)

whence $t = 1{\cdot}93$ inches.

Let it be determined to use four half-inch plates, which is rather more than is necessary.

Divide C E into 4 equal parts in F,G,H, and draw FI, GK, HL. Then the points where these lines cut the curve B D E are the points in the girder where the plates may be dropped.

The Compression Flange.

The gross section of the angle irons is 10·47 sq. inches, and their

moment of resistance is $10{\cdot}47 \times 5{\cdot}5$ ft. $\times$ 4 tons$=230{\cdot}34$ ft. tons, or nearly the same as for the tension flange.

Assuming the thickness at 2 inches, and calling b_1 the total breadth, and proceeding as for the tension flange,

$$b_1 \times 2'' \times 5{\cdot}5 \text{ feet} \times 4 \text{ tons} = 1050{\cdot}8 - 230{\cdot}3 = 820{\cdot}5 \text{ ft. tons},$$

whence $b_1 = 18{\cdot}65$ inches.

The compression flange, for the sake of uniformity, may be made 19 inches wide. It would be better, in order to guard against buckling sideways, to make it broader, say 21 inches, and recalculate, increasing for the sake of uniformity, the width of the tension flange also.

As the point D in figure is practically the same for both flanges, the plates may be dropped in both at the same distances from the centre.

The Web.

The shearing stress being greatest at the piers, and diminishing to a comparatively small force at the centre, the web may be reduced towards the centre, but it must be thick enough; 1st, to give a sufficient bearing area upon the rivets connecting it with the angle irons. 2nd, to have enough metal in it after deducting the rivet holes connecting it with the stiffeners to resist shearing through the line of these holes. 3rd, to resist buckling without incurring the cost of putting in a great many stiffeners.

To ascertain the shearing stress, say at distances of 5·5 ft. along the girder.

The shearing stress due to the dead load is shewn (*Stress in Beams*, p. 20) to be $w\,x$, where w is the weight per unit of length and x any distance from the centre of the girder. In this case $w = {\cdot}68$ tons.

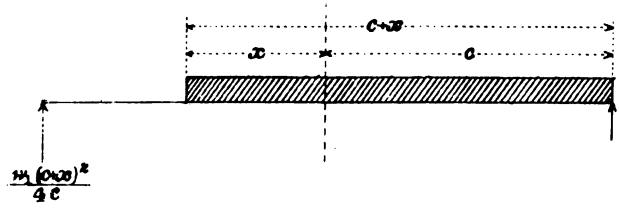
The maximum shearing stress due to the rolling load (taken as uniform per unit of length) occurs at any section, when the longer of the two parts into which the section divides the girder is fully loaded, and the other not loaded at all (*Stress in Beams*, p. 21 and 27).

Then by Rule 2 (*Stress in Beams*, p. 18), the maximum shearing stress equals the reaction at the nearest pier, which (*Fig.* 38) is

$$\frac{w_1(c+x)^2}{4c}$$

w_1 being the weight per unit of length of the rolling load.

Fig. 38.



The total shear, F, is therefore $w\,x+w_1\frac{(c+x)^2}{4c}$

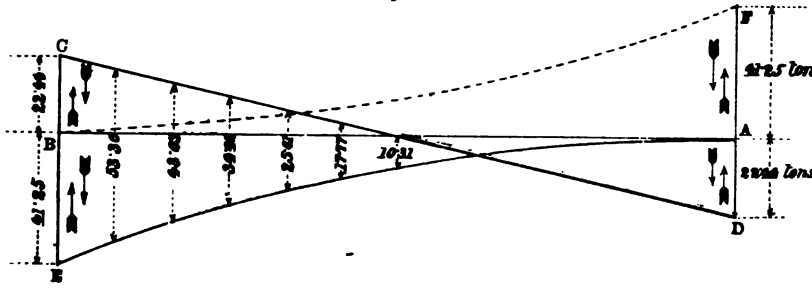
Substituting different values of x,

when $x=$ 0, *i.e.*, at the centre, F=10·31 tons.
$x=$ 5.5 „ =17·77 „
$x=11$ „ =25·81 „
$x=16{\cdot}5$ „ =34·42 „
$x=22$ „ =43·63 „
$x=27{\cdot}5$ „ =53·36 „
$x=33$, *i.e.*, at the end „ =63·69 „

This may also be done conveniently by diagram.

Make AB (*Fig.* 39) on any scale to represent the girder; and on any scale of parts make BC, AD each equal to half the dead load on the

Fig. 39.



girder, *i.e.*, 22·3 tons. Join CD. On the same scale make BE equal to half the maximum rolling load on the girder, *i.e.*, 41·25 tons. Describe a parabola having its apex at A, touching AB, and passing through E. Then the vertical between CD and the curve EA, represents at any section the maximum shearing stress occurring from the entry of the train on to the bridge at A, until it covers the whole bridge. As the train rolls off the bridge, the ordinate between the parabola BF and the line CD gives the maximum shearing stress.

If the length of BE be taken not greater than $\frac{1}{8}$ BA, the curve may be drawn as a circle, as in that case the parabola and circle agree nearly enough for the purpose of calculation.

To determine the thickness of web at distances 5·5′ along the girder necessary to give sufficient bearing area on the rivets connecting it with the angle irons.

Assume 4 rivets per foot run in length, all along the girder.

Then as the shearing stress is greatest at the piers, it is only necessary to enquire whether 4 rivets are enough to resist shearing at the piers.

The shearing stress here is 63·69 tons; which gives $\frac{63{\cdot}69}{5{\cdot}5}=11{\cdot}6$ tons per ft. of height of web; and the horizontal shearing stress is the same.

Therefore, calling the number of rivets to resist shear, n, and recollecting that each must shear on two surfaces,

$$n \times 2\,(\text{surfaces}) \times \tfrac{\pi}{4} \times (\tfrac{7}{8}'')^2 \times 4 \text{ tons} = 11{\cdot}6 \text{ tons.}$$

whence $n = 2{\cdot}4$, so that 4 rivets per foot run will do.

Call the thickness of web necessary to give sufficient bearing area, t.

Then $t \times 4$ (number) $\times \frac{7}{8}''$ (diam.) $\times$ 5 tons $= 11\,6_2$ tons

and $t = {\cdot}67'' = \frac{11}{16}''$ nearly.

Similarly at 5·5′ from the piers, $t = {\cdot}56'' = \frac{9}{16}''$

" 11·0′ " " $t = {\cdot}46'' = \frac{8}{16}''$

" 16·5′ " " $t = {\cdot}36'' = \frac{6}{16}''$

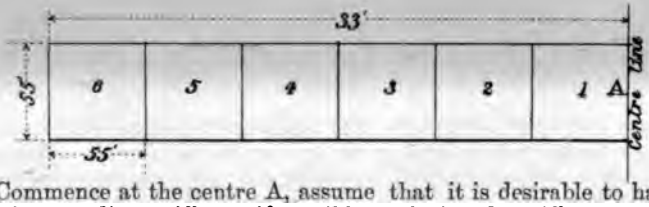
" 22·0′ " " $t = {\cdot}27'' = \frac{5}{16}''$

" 27·5′ " " $t = {\cdot}18'' = \frac{3}{16}''$

It is not necessary to go further, as $\frac{1}{4}''$ is the least thickness allowable. Next, to ascertain if these thicknesses will give sufficient metal, after deducting the rivet holes for connecting the web with the stiffeners, to resist the shear, and whether it will be necessary to have more than one intermediate stiffener between each pair of cross beams. There must be a stiffener at each cross beam to receive the load and distribute it over the web.

Let *Fig.* 40 represent the elevation of the web of half the girder.

Fig. 40.



Commence at the centre A, assume that it is desirable to have only one intermediate stiffener, if possible, and give the stiffeners a width of 6″ to allow room for the rivets; then the clear horizontal space between the stiffeners is 27″; also, take 4 rivets per foot in height of the stiffeners.

The shearing stress at A (*Fig.* 40) is 10·31 tons, which gives 1·87 tons per foot in height of web; the quantity of metal remaining, after deducting 4 rivet holes, is $(12'' - 4\frac{5}{8}'')\,\frac{1}{4} = \frac{17}{32}$ square inches and the shearing stress per square inch of metal $= 1{\cdot}87 \times \frac{8}{17} = {\cdot}87$ tons, which is safe as regards shear.

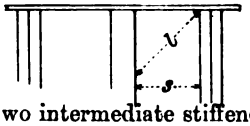
Applying this, as already explained (p. 135) in the formula,

$$\frac{P}{S} = f_a \div \left(1 + a\frac{l^2}{h^2}\right) \quad (\textit{Tension and Compression Bars}, \text{ p. } 86)$$

$${\cdot}88 = 4 \div \left(1 + \tfrac{1}{8000}\,\tfrac{l^2}{\frac{1}{16}}\right)$$

whence $l = 25{\cdot}7$,

and $s = 18''$



Hence it would be necessary to have two intermediate stiffeners with a web of $\frac{1}{4}''$ plate.

Increasing the plate to $\frac{6}{16}$", and recalculating, it is still found that one intermediate stiffener will be insufficient; and the next thickness of plate, viz.: $\frac{7}{8}$", is found somewhat more than is necessary.

$\frac{7}{8}$" must, therefore be adopted, unless two intermediate stiffeners are used.

Proceeding in a similar manner for the remaining spaces, it is found that with one intermediate stiffener,

In space 2 (*Fig.* 40) the web must be $\frac{7}{16}$" thick.
" 3 " " $\frac{8}{16}$" "
" 4 " " $\frac{9}{16}$" "
" 5 " " $\frac{10}{16}$" "
" 6 " " $\frac{11}{16}$" "

and in all, the quantity of metal left after punching four $\frac{7}{8}$" rivet holes per foot in height of the web is sufficient to resist shearing.

The Dimensions of the Stiffeners.

The main stiffeners, viz.: those at the cross beams, have to receive half the load on one cross beam and transmit it to the web, as well as to carry a part of the weight of the main girder 5·5′ in length; making a total load of about 17 tons.

As the load is on the bottom flange, they act in tension (if it were on the top, they would have to be treated as pillars), and taking them as 6" wide to allow of two $\frac{7}{8}$" rivet holes being punched in them and in the edges of the web plates, and calling t their thickness on each side of the web.

$$2\,t\,(6-1\tfrac{4}{8}) \times 5 \text{ tons} = 17 \text{ tons},$$
$$\text{whence } t = \tfrac{7}{16} \text{ inch.}$$

It is usual to make stiffeners either with angle or T irons in order to stiffen the whole girder, and these stiffeners might be made of T iron 6" × 3" × $\frac{8}{8}$".

The intermediate stiffeners may be 6" wide, each of the thickness of the web at the joint they cover.

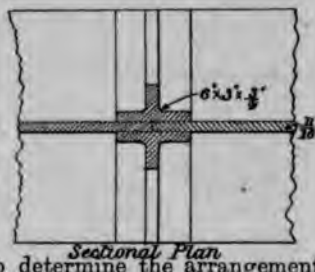
The stiffener over the bearing must be treated as a pillar. It has to resist a compression = to the shearing stress, *i.e.*, half the total load = 63·69 tons.

If the least dimension be made not less than $\frac{44}{10} = 6\frac{4}{10}$ inches, the pillar may be treated as a short column under compression (*Tension and Compression Bars, p.* 83), and in that case will require a sectional area $= \frac{63{\cdot}69}{4} = 15{\cdot}9$ square inches, which may be got by two T irons 6" × 3" × $\frac{8}{4}$". (*Fig.* 41.)

A very good form for this stiffener would be as shewn in *Fig.* 42, *i.e.*, 2 angles on each side with a gusset piece between them.

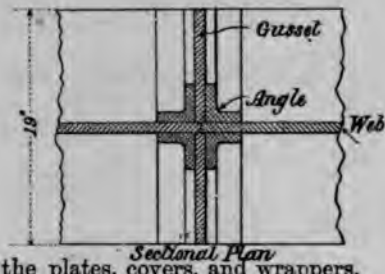
As the girder is built with a camber, and as, when it deflects under the load, the position of the centre of bearings alters, it is well to put in another stiffener at the inner end of the bearing. This is shown in *Plate* II.

Fig. 41.



Sectional Plan

Fig. 42.



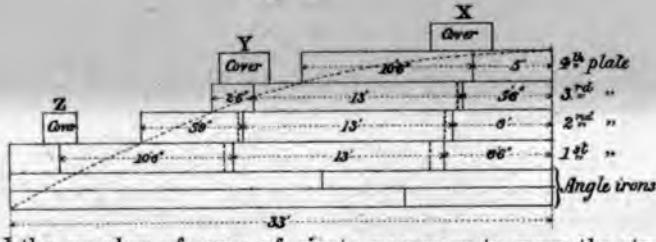
Sectional Plan

To determine the arrangement of the plates, covers, and wrappers, and their dimensions.

For the Tension Flange.

Let the plates be provisionally arranged as in *Fig.* 43. Then none of the plates will be in excess of 4 cwt. in weight, and only 3 covers will be necessary on each side of the centre, viz.: at X, Y, Z.

Fig. 43.



To find the number of rows of rivets necessary to carry the stress over a joint in one plate, with two covers.

There are 4 rivets in a row, and the stress on one plate is $(19-3\frac{1}{2}) \times \frac{1}{2}'' \times 5$ tons $= 38{\cdot}75$ tons.

For shearing,

$$2 \times 4 \times n \times \frac{\pi}{4} \times (\tfrac{7}{8})^2 \times 4 \text{ tons} = 38{\cdot}75 \text{ tons}$$

whence $n = 2$ rows.

For Bearing Surface.

$$4 \times m \times \tfrac{7}{8} \times \tfrac{1}{2} \times 5 \text{ tons} = 38{\cdot}75 \text{ tons},$$

whence $m = 4{\cdot}4$ rows.

As the stress is really a little below 5 tons to the inch, owing to the extra thickness put in the plate, and as 5 tons is not a high stress for bearing area, 4 rows will suffice, and when there is only one joint, as at Z, the covers will be $8 \times 3'' = 24''$ long, as in *Fig.* 44.

Where there are three joints, as at Y, the arrangement is given by equations (6) to (9), p. 109.

Fig. 44.

The values of the symbols are as follows:

$u=3, b=15{\cdot}5'', t=\frac{1}{2}'', s_1=5$ tons.

$c=4, d=\frac{7}{8}'', s_2=4$ tons, $s_3=5$ tons.

In equation (6), $\frac{232{\cdot}5}{2}=(8n+8z)\frac{38{\cdot}5}{16}$

when $z=2, n=4$, total number of rivets $=48$

$z=3, n=3$, „ „ „ $=48$

$z=4, n=2$, „ „ „ $=48$

In equation (7), making $z=3, n=3$;

$$\frac{232{\cdot}5}{2}=\frac{115{\cdot}5}{2}+155t_2,$$

$$t_2={\cdot}377=\tfrac{6}{16}\text{ inch.}$$

In equation (8), $\frac{232{\cdot}5}{2}(3m+9)\frac{5}{8}$

$$m=1{\cdot}4$$

or the joint is stronger in bearing than for shearing.

In equation (9), $\frac{232{\cdot}5}{2}=(\frac{3}{4}m+\frac{9}{2})\frac{5}{8}$

$$m=2{\cdot}8$$

or the joint is slightly stronger in bearing than for shearing.

This joint may be as shewn in *Fig.* 45, the cover being 36″ long.

Fig. 45.

Using the same equations for the joint at X, the following values are obtained:

$s=3$, $n=4$, total number of rivets $=68$,

$t_2=\frac{1}{16}''$, $m=$a minus quantity, shewing that the arrangement sufficient for shearing is more than sufficient for bearing.

This joint may be as shewn in *Fig.* 46, the cover being 51" long.

Fig. 46.

This way of arranging the rivets would necessitate a re-arrangement of the plates, as shewn in *Fig.* 48 by the broken lines.

For the Wrapper.

The tension on each wrapper=4·14 square inches × 5 tons=20·7 tons.

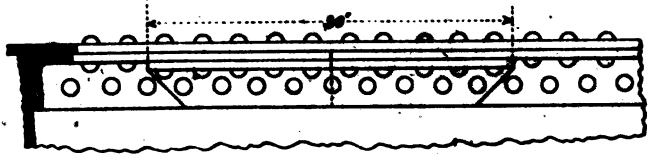
For shearing,

$$n \times \frac{\pi}{4} \times (\tfrac{7}{8})^2 \times 4 \text{ tons} = 20{\cdot}7 \text{ tons.}$$

$$n = 8{\cdot}6$$

or 4 in one arm and 5 in the other will suffice, *Fig.* 47.

Fig. 47.



For bearing

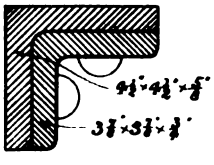
$$m \times \tfrac{7}{8} \times \tfrac{5}{8} \times 5 \text{ tons} = 20{\cdot}7 \text{ tons}$$

$$m = 8 \text{ nearly}$$

or the number necessary for shearing must be used.

The section of the wrapper must be equal to the section of the angle iron and may be shown as in *Fig.* 48.

Fig. 48.



For the compression flange, the plates may be arranged in a similar manner to those in the tension flange, and the covers and wrappers may also be similar.

NOTE.—If the workmanship were perfect, the cover plates and wrappers in the compression flange and their rivets would have no direct stress on them. But however well the work may be done, and however carefully the plates butting on each other may be fitted by planing, which in good work is always done, it is impossible to secure absolute contact in every joint, and therefore it is better to calculate the covers, wrappers, and rivets in the compression flange in a similar manner to those in the tension flange.

There may be some difficulty in ensuring the proper filling up of the rivet holes where there are a great many plates to connect in which case the following method may be adopted (*M.P.I.C.E.*, Vol. XXIX., p. 457.)

Long rivets, exceeding in length about three times the diameter, can be made to fill their holes by having them heated on their sides in a coke fire without a plate, slightly cooling the extreme point in water just previous to insertion, and clenching them with the snap tool and a 14 lb. sledge hammer, avoiding altogether the use of the ordinary riveting hammers; by this process the shank of the rivet is completely upset throughout its entire length, owing to the hardness of the cooled point, and if the latter is not cooled too much, it will recover sufficient heat by conduction from the shank, to permit the proper formation of the snap. This method was first applied at the Boyne Viaduct, vide Stoney, *Theory of Strains*, Vol. II., p. 315, London, 1869.

The position of the joints in the angle iron is unimportant. They should not be put where the wrapper will interfere with the putting together of the rest of the work, and they should break joint with one another. They are shewn on *Plate* II.

The Camber and Deflection.

The girder approaches to, but is somewhat stiffer than a beam of uniform strength and uniform breadth, having a cross section of equal strength.

Then (*Deflection of Beams*, p. 37).

$$V = \frac{n''(f_a + f_b)c^2}{E\, h_o}$$

Taking E=18,000,000 lbs., n''=·5708, f_a=4 tons, f_b=5 tons, c=33′ × 12, h_o=66″,

$$V = 1{\cdot}519 \text{ inches.}$$

Allowing $\frac{1}{3}$ of this for permanent set, (p. 118,) the true deflection is 1″, which being divided in direct proportion to the dead and rolling loads, gives ·36 inch for the former and ·64 inch for the latter, or $\frac{1}{12\frac{1}{2}\frac{1}{5}\frac{1}{7}}$ of the length for the rolling load.

It may be well to make an addition to the camber, to cover the effect of the sudden application of the live load. Probably about $\frac{1}{7}$ or $\frac{1}{8}$ would be sufficient for a bridge of this length (*U.I.B.R.*, p. 35), but even $\frac{1}{4}$ would not be too much.

The weight of the girder may now be taken out as follows:

Flange plates	$2 \times 31'$	$\times \frac{1}{1}\frac{0}{2}' \times \frac{1}{2}\frac{1}{4}'$	62′
	$2 \times 41{\cdot}5'$	$\times \frac{1}{1}\frac{0}{2}' \times \frac{1}{2}\frac{1}{4}'$	83′
	$2 \times 49{\cdot}5'$	$\times \frac{1}{1}\frac{0}{2}' \times \frac{1}{2}\frac{1}{4}'$	99′
	$2 \times 70'$	$\times \frac{1}{1}\frac{0}{2}' \times \frac{1}{2}\frac{1}{4}'$	140′
	$2 \times 5{\cdot}5'$	$\times \frac{1}{1}\frac{0}{2}' \times \frac{1}{2}\frac{1}{4}'$	11′
		$395' \times \frac{1}{1}\frac{0}{2}'$ at 20 lbs.	= 12508 lbs.
Covers,	$4 \times 2 \times \frac{8}{1}\frac{1}{2}' \times \frac{1}{1}\frac{0}{2}' \times \frac{7}{1}\frac{}{6}''$	= 54 feet super. at 17·5 lbs.	= 845 lbs.
,,	$4 \times 2 \times \frac{3}{1}\frac{6}{2}' \times \frac{1}{1}\frac{0}{2}' \times \frac{6}{1}\frac{}{6}''$	= 38 feet super. at 15 lbs.	= 570 ,,
,,	$4 \times 2 \times \frac{2}{1}\frac{4}{2}' \times \frac{1}{1}\frac{0}{2}' \times \frac{8}{1}\frac{}{6}''$	= 26 feet super. at 20 lbs	= 560 ,,
Angles,	$2 \times 2 \times 75{\cdot}5$ at 17·45 lbs. per foot run		= 5270 ,,
Wrappers,	$2 \times 6 \times \frac{3}{1}\frac{0}{2}'$ at 17·45 lbs. per foot run		= 524 ,,
		Carried over.....	20277 ,,

	Brought forward.....	20277 lbs.
Webs, $2 \times 5{\cdot}5' \times 5{\cdot}5' \times \frac{6}{16}''$		
$2 \quad " \quad " \quad \frac{7}{16}''$		
$2 \quad " \quad " \quad \frac{8}{16}''$	=363 ft. super. at 21·25 lbs. =	7714 „
$2 \quad " \quad " \quad \frac{9}{16}''$		
$2 \quad " \quad " \quad \frac{10}{16}''$		
$2 \quad " \quad " \quad \frac{11}{16}''$		
$2 \times 2' \times 5{\cdot}5' \times \frac{12}{16}''$	=22 feet super. at 30 lbs. =	660 „
Main Stiffeners $2 \times 11 \times 7' \times$ T $6'' \times 3'' \times \frac{3}{8}''$	=154′ at 11 lbs. per foot run. =	1694 „
Intermediate do. $2 \times 12 \times 5'{\cdot}5' \times 6'' \times \frac{17}{32}''$ (mean thickness)	=66 feet super. at 21·25 lbs. =	1403 „
End pillars $2 \times 2 \times 7' \times$ T $6'' \times 3'' \times \frac{3}{4}''$	=28 feet run at 21 lbs. =	588 „

Rivet heads,

In Plates, - $16 \times 150 \times 2 = 4800$
„ Web and angles, $2 \times 150 \times 2 = 1200$
„ Stiffeners $2 \times 2 \times 25 \times 20 = 2000$

Total, 8000

$8000 \times \frac{12}{16} \times \frac{1}{8}''$ of $\frac{1}{8}''$ round iron=692 feet run at 2 lbs.	=	**1384 „**
Total.....		**33720 „**
		=15·5 tons.

This gives a weight rather less than was provided for, but the difference is not enough to render recalculation necessary. There would be a slight addition to this weight for "packing."

The ends of the girders may be finished by rounding off the top flange and continuing the angles and plate to meet those of the lower flange.

Fig. 49.

There should be an extra plate on the lower flange at the bearings, into which the heads of the rivets are countersunk. In large girders this is sometimes of cast iron, planed, in which case it is bolted on to the flange.

Some of the ordinary methods of providing for the expansion and contraction of the girders are shewn in *Plate* III.

Bridges require to be braced both horizontally and vertically.

The following extract from Mr. Unwin's lectures on the Construction of Wrought Iron Bridges, Lecture III., February 15th, 1871, will be useful.

Functions of the Horizontal and Vertical Bracing.

"The rolling load in passing over a bridge does not produce a simple

vertical pressure on the bridge. In the case of a railway train, the inequalities of the way set up both vertical and horizontal oscillations of each carriage, which modify the vertical load and introduce horizontal pressure on the rails. The primary object of the horizontal bracing is to resist these horizontal forces, by forming with the booms of the bridge, and the platform, a horizontally placed girder of a depth equal to the width of the bridge. And the primary object of the vertical bracing is to bind together the two main girders so as to present a combined resistance to the torsion and tendency to overturning produced by the horizontal forces.

Whether the platform is on top or at bottom of the main girders, it is obvious that the horizontal bracing will be most effective when the rails, the booms, and the bracing, are as nearly as possible in one plane. This is well secured in the arrangement shewn in *Figs.* 1 and 2, *Plate* IV.

When there are no cross beams, the bracing bars are alternately struts and ties, according to the unknown direction of the horizontal forces. In such cases the horizontal bracing should consist of ⊤ iron bars placed back to back and rivetted to the flanges of the main girders. ⊤ irons $4 \times 3 \times \frac{3}{8}$ to $4 \times 4 \times \frac{1}{2}$ intersecting the axis of the bridge at angles of 30° to 40°, are sufficiently strong in ordinary cases. When there is a platform with cross beams, the cross beams may be utilized as struts in the horizontal bracing, and the bracing bars may be flat or round bars rivetted to the main girders and to the cross beams where they intersect them. These bars may be 6″ to 8″ wide and $\frac{3}{8}$″ to $\frac{1}{2}$″ thick. The magnitude of the horizontal force developed by the lateral oscillation of a train is, I believe, unknown. The wind pressure alone acting on the side of a train of carriages might produce a pressure of perhaps 3 cwts. per foot run, or say a horizontal force of $\frac{1}{4}$ the rolling load. So that we shall, I believe, make no excessive allowance if we suppose, in single line bridges, that the horizontal force acting on the rails may reach $\frac{1}{5}$ of the rolling load. When the platform is on top of the main girders this force tends to overturn the girders, and must be resisted by building the girders into the masonry of the piers, or we may adopt the plan shown in *Figs.* 3 and 4, *Plate* IV, and introduce over each pier a frame of vertical bracing, binding together all the girders and spreading over a wide space the resistance to overturning. Very considerable differences will be found in existing bridges as to the amount of this vertical bracing, but it seems reasonable that the frame, at each end of a span of a single line bridge, should be at least strong enough to resist a horizontal force of $\frac{1}{10}$ the rolling load on the span applied in the plane of the rails. And the resultant of this force and the half dead and live load of the span should of course fall within the frame. If the bridge has several spans, so that the girders of two spans are united on one vertical bracing frame over the pier, that frame requires double the strength of frame previously considered. When the bridge carries two lines of way, the effect of wind pressure bears a less proportion to the rolling load, but I am hardly prepared to recommend a less strength in the bracing. Of course the suggestions I have made here

are to be taken only as general guides. The exact nature and amount of the horizontal straining forces are unknown.

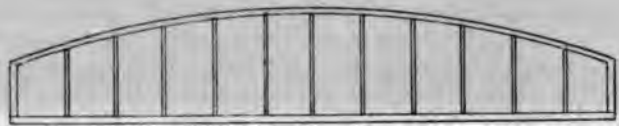
With girders of moderate span no other vertical bracing frames than those over the piers are necessary. But many engineers introduce vertical bracing frames at distances of about 20 to 30 feet along the span.

With the platform on top of the main girders the horizontal bracing has the subsidiary effect of stiffening the compression booms against lateral flexure. When the platform is below the main girders the top booms can have no aid from bracing, unless the depth of the main girders is about 15 feet, and then light top bracing is introduced to stiffen the booms. With the platform at bottom the vertical bracing is dispensed with."

The method of calculation given above applies to all the forms of continuous web girders having horizontal flanges.

It does not apply properly to the form of girder known as "saddle back" (*Fig.* 50) which is of doubtful advantage.

Fig. 50.



The rails on bridges should have a camber and it may be the same that the main girders have before the rolling load comes on. If the rails are laid level, the engine, soon after coming on the bridge begins to run up an incline, the springs are compressed, owing to the load on them not rising so quickly as the incline, and this compression produces a heavier pressure on the bridge than the mere load on the wheels.

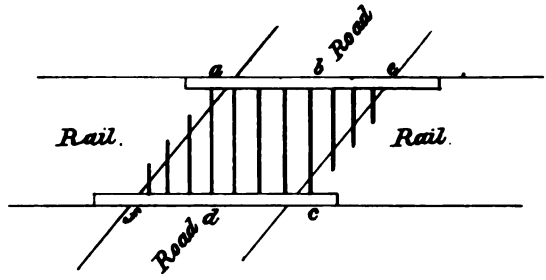
If the rails be laid with a camber, this takes place when the bridge is only very partially loaded, and the effect can then be borne without any risk, see *Latham's Wrought Iron Bridges*, p. 212 to 218.

Another rule is that the camber given to the rails on a bridge should be equal to $\frac{1}{3}$rds the measured deflection caused by the passage of a long train over the same bridge, when the rails are laid without camber (*M.P.I.C.E.*, vol. XXIX., p. 443).

When a railway crosses a road at a skew, it is worth while to remember that the load on the bridge is not uniform, and to design the girder accordingly.

Thus, the parts *a b*, *c d* of the main girders (*Fig.* 51) are each loaded with half the load on the rectangle *a b c d*, but the parts *b e*, *d f* are loaded with considerably less, and the girders may be reduced in strength accordingly.

Fig. 51.



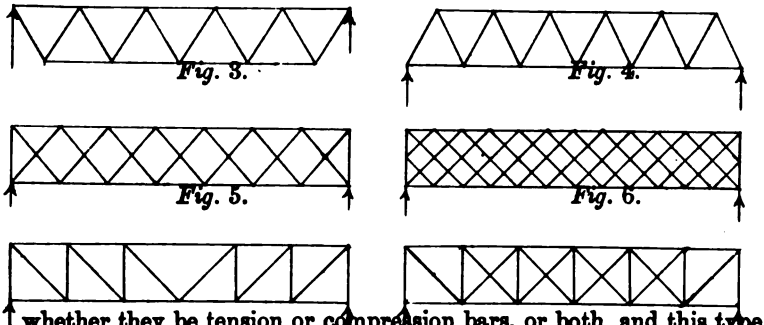
BEAMS WITH HORIZONTAL FLANGES AND OPEN WEBS.

In these beams, the web consists of a number of bars, all of which are inclined (*Figs.* 1 to 4), or some of which are inclined and some vertical (*Figs.* 5 and 6), connected with the flanges (or booms) by rivets or turned steel bolts.

When the load is invariable, some of these bars are in tension, some in compression, the stresses remaining the same always both in nature and amount, but where the load moves, as on a bridge, the stress in every bar varies in amount, as the load moves, and it varies in some bars both in amount and nature, so that these bars have to be constructed to bear both kinds of stress. This is called "counterbracing."

The term "brace" is applied to all the bars connecting the flanges,

Fig. 1. *Fig.* 2.



whether they be tension or compression bars, or both, and this type of beam is called a "braced" beam or girder.

A frame of any form braced in this manner is called a "truss."

In order to brace a frame of any kind completely, the lines representing the directions of the tensions or compressions in the different bars composing the frame must divide the space included within the external bars of the frame into a number of triangles, and each of the

bars must be competent to resist the particular kind of stress to which it will be subjected.

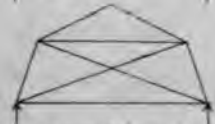
Thus a roof frame of the external form shown in *Fig.* 7 may be completely braced in any of the ways shown in *Figs.* 8, 9, 10, the bars in each case being of suitable dimensions.

Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.



As three bars of given length can only be arranged in a triangle of one form, it is evident that no change of form in any triangle can take place without an alteration in the length of one or more of the bars; and hence the division of the internal space in the triangles by means of bars of due strength, precludes any alteration in the form of the frame.

Where a frame is to be subjected to a moving load, the bracing is sometimes accomplished by bars crossing one another as in *Fig.* 10.

Fig. 11.

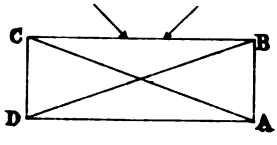
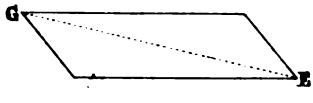


Fig. 12.



Suppose a rectangular frame hinged at all the joints, and acted upon by an inclined force from one side, say the right, *Fig.* 11.

Then without any bracing the frame would simply revolve on the hinges and collapse as shown in *Fig.* 12, but if a tension bar of sufficient strength be put in from A to C, *Fig.* 11, this collapse cannot take place, because, before it can assume the position shown in *Fig.* 12, the bar must elongate to the length E G.

But this bracing would be insufficient to resist a similar force coming from the left, because the tension bar would then come under compression and would buckle, unless constructed as a compression bar, as well as a tension bar.

If a second tension bar be put in from B to D, *Fig.* 11, no alteration of form can take place under either force, and the frame is as completely braced, as if the first bar A C had been constructed both as a tension and as a compression bar.

In calculating the stresses in these beams it is assumed that the flanges are not continuous, but consist of a number of bars connected with the braces at each point of junction, so that each point is a hinge round which motion may take place.

It is shewn (*L.W.I.B.*, p. 122) that the effect of constructing the flanges continuous from end to end is to relieve the braces, and to preserve evenness of stress, which might otherwise be endangered by the necessity, which arises in construction, of making some of the braces larger than is necessary for the stresses they have to bear.

It is further shewn that the rivetting together of the braces of lattice girders at their points of crossing has the effect of stiffening the girder, so that any error in the calculation, on the assumption that the booms are not continuous, and the braces not rivetted to one another at their points of crossing, is on the side of safety.

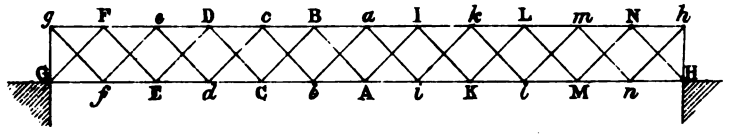
EXAMPLE I.

Determine the stresses in a lattice girder of the form shewn in *Fig.* 13, the rolling load (that of a railway train), being on the lower flange.

The distribution of the dead load on the girder is as follows:

On each upper apex, such as a, the weight of half of the girder divided by the number of spaces such as A b; call this w_1. On each

Fig. 13.



lower apex, such as A, the weight of half of the girder added to half of the platform, and of the rails, divided by the number of spaces, such as A b; call this w.

The rolling load may be taken as distributed equally at the lower apices, so that each apex will have on it the total rolling load carried by one main girder divided by the number of spaces; and it facilitates calculation without importing sensible error to suppose that as the train comes on, each apex is loaded fully in succession, while the apex immediately in front of it is not loaded at all. Call the rolling load on each apex W.

For the sake of shortening the tables, and of showing as simply as possible how Latham's method of dealing with these girders is arrived at, the load on the upper apices, which is small in comparison with that on the lower apices, is left out of consideration for the present.

Then considering, first, the dead load on the lower flange, and taking each load by itself, it is obvious that the load at the apices of one set of triangles produces no effect on the braces of the other set of triangles, thus: of the load at C, which is 8 spaces from H and 4 spaces

from G, $\frac{5}{12}$ is carried to the pier G through the braces C D, D E, E F, and F G, while $\frac{4}{12}$ is carried to H through C B, B A, A I, I K, K L, L M, M N, and N H, and similarly, of the load at d, $\frac{3}{12}$ is carried to G, through $d e$, $e f$, $f g$, g G, and $\frac{3}{12}$ through $d c$, $c b$, ... $n h$, h H to H.

Beginning with the weight w at E, a reaction of $\frac{5}{6} w$ is produced at G, and a reaction of $\frac{1}{6} w$ at H.

The reaction $\frac{5}{6} w$ at G is balanced by two forces, one a thrust in F G, and the other a tension in E G (*Fig.* 14) and these three forces are represented in magnitude and direction by the triangle f F G. (*Fig.* 15).

Fig. 14.

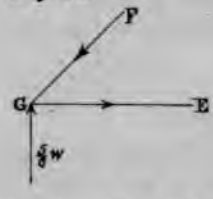
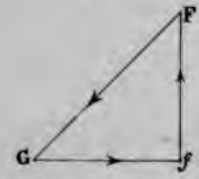


Fig. 15.



Now the thrust in F G : Reaction :: F G : F f : sin 90° : sin i, whence thrust in F G = Reaction × cosec. i = + $\frac{5}{6} w$ cosec. i.

Neglecting for the present the stresses in the flanges, and following the stresses in the braces, the thrust in F G is balanced at F by a thrust in D F, and a tension in F E (*Fig.* 16), and these three forces are represented in magnitude and direction by the triangle F G E. (*Fig.* 17.)

Fig. 16.

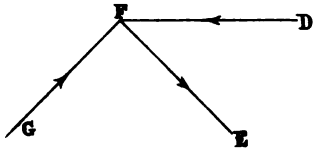
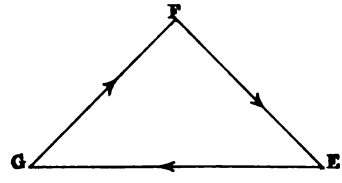


Fig. 17.



As this is an isosceles triangle, it is obvious that the tension in F E = the thrust in F G = − $\frac{5}{6} w$ cosec. i.

Commencing now from H, the reaction at H = $\frac{1}{6} w$ is balanced by two forces, one a thrust in N H, the other a tension in M H (*Fig.* 18), and these three forces are represented in magnitude and direction by the triangle N H h. (*Fig.* 19.)

Fig. 18.

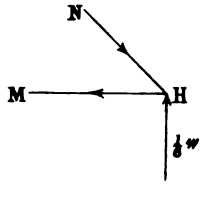
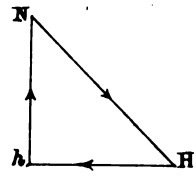


Fig. 19.



Reasoning as before, the thrust in N H = Reaction × cosec. $i = \frac{1}{6}\ w$ cosec. i, and this thrust is balanced at N, by a thrust in L N, and a tension in N M = $\frac{1}{6}\ w$ cosec. i.

Similarly this tension in N M is balanced at M by a tension in M K, and a thrust in L M = $\frac{1}{6}\ w$ cosec. i, which again is balanced at L by a thrust in I L and a tension in K L = $\frac{1}{6}\ w$ cosec. i, and so on, alternately compressions and tensions up to the load at E.

Calling cosec. i, n, the first line of *Table* I. is obtained.

In a similar manner, the second line is obtained by considering the load at C, which produces a reaction of $\frac{4}{6}\ w$ at G and $\frac{2}{6}\ w$ at H; and the rest of the table is obtained similarly from the loads at A, K, and M in succession.

Now as these stresses are always in existence and have a constant value, it is necessary to provide for the sum of them on each bar, and as tensions and compressions neutralize each other, the tensions may be taken as — and the compressions as + to obtain the sum.

This addition completes *Table* I.

Considering now the loads at f, d, b,.....n, the load at f produces a reaction of $\frac{1}{1\frac{1}{2}}\ w$ at G, and $\frac{1}{1\frac{1}{2}}\ w$ at H.

The reaction at G is balanced directly by a thrust equal to it in G g. The thrust in Gg is balanced at g by a thrust in Fg, and a tension in $g f$ (*Fig.* 20), and these three forces are represented in magnitude and direction by the triangle gF G (*Fig.* 21).

Fig. 20.

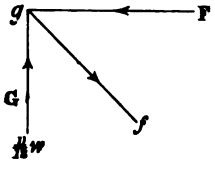
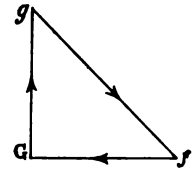


Fig. 21.



Now tension in $g f$: Reaction :: $g f$: $_g$G :: sin 90° : sin i, whence tension in $g f$ = Reaction × cosec. $i = \frac{1}{1\frac{1}{2}}\ w$ cosec. i.

Commencing from H, the reaction = $\frac{1}{1\frac{1}{2}}\ w$ at H is balanced by a thrust equal to it in H h.

The thrust in H h is balanced at h by a thrust in N h, and a tension in $n h = \frac{1}{1\frac{1}{2}}\ w$ cosec. i, and on the same principle as before, there are alternate compressions and tensions, each equal to $\frac{1}{1\frac{1}{2}}\ w$ cosec. i, in the braces from $n m$ to $e f$.

In this manner, the first line of *Table* II. is formed, and the other lines similarly, the total being obtained by adding the different columns together, regarding the compressions as +, and the tension as —, as in *Table* I.

Similar tables might be formed for any number of sets of triangles, or for the case where a portion of the load is on the top.

The rolling load is now to be considered; and its effect on the braces may be obtained from *Tables* I. and II.

Let the train be supposed to roll on from G, until the apex E only is

loaded. Then substituting W for w, the effect produced by the rolling load at E is given by the first line of *Table* I.; the effect produced when E and C are loaded is given by the two first lines taken together; that produced when E, C and A are loaded is given by the first three lines, and so on, until the bridge is fully loaded.

As the train rolls off, and the load at E is removed, the lowest four added together give the effect on the braces; when E and C are unloaded, the lowest three lines added together give the effect on the braces, and so until the bridge is unloaded.

At some time, then, during the passage of the load, the maximum of each kind of stress is produced on the bars, and instead of providing for the sum of the stresses, it is necessary to provide for the maximum stress of each kind.

Thus, considering the bar B C; when the train has advanced so far as to load E and C, B C has on it a tension $= \frac{3}{6} W n$, but when the apex A is loaded in addition, this tension, is altogether neutralized, and when the whole girder is loaded, B C has on it a compression $= \frac{3}{6} W n$. When the train has rolled off, so that E and C are unloaded, B C has on it nothing but compression, which is then a maximum, and equals $\frac{6}{6} W n$.

Similarly other bars are sometimes in compression and sometimes in tension from the rolling load.

Tables III. and IV. are formed by adding together the stresses of each kind on each bar in *Tables* I. and II., substituting W for w.

Considering, now, the dead and rolling load together; the stresses produced on each bar by the dead load either neutralize the stresses produced by the rolling load, if they are of different kinds, or are in addition to them if they are of the same kind, and it is necessary to add them both together, still considering compressions as + and tensions as —.

This gives *Tables* V. and VI. and completes the Tables necessary for the braces.

Examining these Tables it is seen that the stresses on the bars increase from the centre of the girder to the piers; that the end bars G g, G F, H h, H n, are always in compression, and those next to them $f g$, F E, $n h$, N M, are always in tension, while the others may be sometimes in tension, and sometimes in compression, this depending upon the ratio of the dead and rolling loads to one another.

Thus suppose the dead and rolling loads to be to one another as 3 to 6, then w: W :: 3 : 6 : or $w = \frac{W}{2}$.

Substituting these for any bar, it is at once seen whether the bar is a tension or compression bar or both.

Take the bar C B for example, from *Table* V.

$$\text{Its greatest compression} = + \tfrac{3}{6}\frac{W}{2} n + \tfrac{6}{6} W n = + \frac{15}{12} W n$$

$$\text{,, ,, tension} = + \tfrac{3}{6}\frac{W}{2} n - \tfrac{3}{6} W n = - \frac{3}{12} W n$$

or the bar is a compression bar sometimes, and is in slight tension at others.

Take the bar B A, from *Table* V.

$$\text{Its greatest compression} = -\tfrac{5}{6}\frac{W}{2}n + \tfrac{5}{6}Wn = +\frac{3W}{12}n$$

$$\text{,, ,, tension} = -\tfrac{6}{6}\frac{W}{2}n - \tfrac{6}{6}Wn = -\frac{15}{12}Wn$$

or the bar is sometimes in tension and sometimes in slight compression.

Both these braces must be designed to resist both kinds of stress. This is called "counterbracing."

In all braced bridges, it is necessary to "counterbrace" the central bars, the number of bars requiring counterbracing depending upon the ratio of the rolling load to the weight of the structure.

The flanges or booms.

As the greatest stress in the flanges is produced when the bridge is fully loaded, each lower apex may be taken, as loaded with a load $= w + W$.

Considering the effect of the load at E, on the lower flange, and reasoning as before, the tension on G E which balances the reaction $\frac{5}{6}(w+W)$ at G is obtained from the triangle of forces, G F f (*Fig.* 15), in which f G represents the tension on G E, F f representing the reaction at G.

As G f : f F :: cos i : sin i

$$\text{Tension on G E} = \text{Reaction} \times \cot i = \tfrac{5}{6}(w + W)\cot i.$$

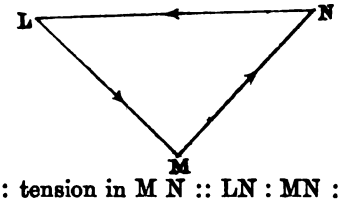
Commencing from the other end, and reasoning similarly,

$$\text{Tension on H E} = \tfrac{1}{6}(w + W)\cot i.$$

this tension acting all through from H to the point of application of the load.

Now it has already been seen, that at M there are two forces acting, one a tension $= \frac{1}{6}(w + W)$ cosec i in M N and the other a thrust $= \frac{1}{6}(w + W)$ cosec i in L M, which are balanced by a tension extending from M to E, and these three forces are represented in magnitude and direction by the triangle L M N (*Fig.* 22.).

Fig. 22.



Now tension in M E : tension in M N :: LN : MN :: sin. 2 i : sin. i.

Whence tension in ME $= \frac{1}{6}(w+W)$ cosec $i\frac{2 \text{ sin. } i \text{ cos. } i}{\text{sin. } i} = \frac{1}{3}(w+W)\cot i$.

Similarly tension in K E required to balance the forces in L K, K I $= \frac{1}{3}(w+W)$ cot. i.

Tension in A E to balance forces in A I and A B $= \frac{1}{3}(w+W)$ cot. i.
Tension in C E „ „ B C and C D $= \frac{1}{3}(w+W)$ cot. i.
So that on H M, tension $= \frac{1}{6}(w+W)$ cot. i.
„ on M K $= \frac{1}{6}(w+W)$ cot. $i. + \frac{1}{3}(w+W)$ cot $i. = \frac{3}{6}(w+W)$ cot. i.
„ on K A $= \frac{1}{6}(w+W)$ cot. $i. + \frac{1}{3}(w+W)$ cot. $i + \frac{1}{3}(w+W)$ cot. i.
$= \frac{5}{6}(w+W)$ cot. i.
„ on C E $= (w+W)$ cot. $i. + \ldots\ldots\ldots\ldots = \frac{9}{6}(w+W)$ cot. i.

Calling cot. i, m, this gives the first line of *Table* VII., and considering the loads at the other apices in succession, the whole Table is formed, the total stress being the sum of the stresses due to each load, which are all of the same kind.

In a similar manner, *Tables* VIII., IX., and X, are obtained.

Examining these Tables, it appears that with the exception of the end bars of both flanges, *i.e.*, G f, g F, H n, h N, each bar forms a part of two triangles, and has to bear the stress on both. For example, C b is a part of two triangles d c b and B C A, and has to bear the tension both on d b and C A, which by *Tables* VII. and IX. is found to be $\frac{4}{6}(w + W)$ cot. $i. + \frac{3}{6}(w+W)$ cot. $i.$; and similarly I k is a part of a k and I L, and has to bear the compression on both, which by *Tables* VIII. and X. is found to be $\frac{10.5}{12}(w + W)$ cot $i + \frac{3.4}{3}(w + W)$ cot. i.

Table XI. is formed in this manner.

It is seen from these Tables that the stresses in the flanges are greatest at the centre of the girder, and decrease towards the ends.

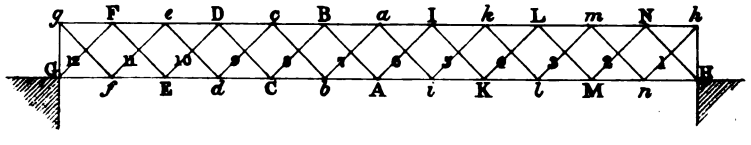
These Tables give the stresses in all the bars of a lattice girder with two sets of triangles, the load being on the lower flange, and the small portion of the load, which is actually borne at the upper apices, being left out of consideration.

It is obvious that a girder having any number of triangles, and loaded both on the top and bottom apices, might be dealt with in a similar manner, but it would be troublesome to form a very large number of Tables, and it is desirable to simplify the method if possible.

Latham's method, given in his *Wrought Iron Bridges* (a book now somewhat difficult to get) affords this simple method.

Commencing from the right side, H, of the girder, number all the parallel to h n, in succession 1, 2, ... 12 (*Fig.* 23).

Fig. 23.



Then an inspection of *Table* 1. shows that for the bar A I the stresses caused by the dead load are $\frac{1}{6}$ w n $(-1-2-3+2+1)$,
which may be put in the form $\frac{1}{12}$ w n $(-2-4-6+4+2)$.
Now it is obvious that the loads at E, C, and A, each produce tension

on A I, while those at K and M produce compression, and it is seen on inspection, that the apex A is distant 6 spaces, the apex C is distant 4 spaces, and the apex E is distant 2 spaces from G, while the apex K is distant 4 spaces, and the apex M is distant 2 spaces from H, so that the series given in the Table may be obtained by counting on the diagram of the girder the number of spaces that each load on the one side is distant from the pier on that side, and deducting from their sum the sum of the number of spaces that the loads on the other side are distant from the pier on that side, and writing before the series the terms $w\ n$ divided by the total number of spaces.

Take a bar in the other set of triangles, say $a\ b$.

The series given by the Table is

$$\tfrac{1}{12}\ w n\ (-1-3-5+5+3+1)$$

and it is seen that b, d, and f, the loads on which produce tension on $a\ b$, are respectively 5, 3 and 1 spaces from G, while i, l, and n, the loads on which produce compression on $a\ b$, are also respectively 5, 3, and 1 spaces from H.

Had a table been formed for the load w_1 on the upper apices, the result for the effect of these loads on the bar $a\ b$, would have been on the same principle $\tfrac{1}{12}\ w_1\ n\ (-4-2+6+4+2)$ because the loads at c, and e, which are respectively 4 and 2 spaces from G produce tension on $a\ b$, while the loads at a, k, and m, which are respectively 6, 4, and 2 spaces from H produce compression on $a\ b$.

When the effect of the rolling load on any bar is considered, the compression is greatest when none of the loads which produce tension on that bar are on, while the tension is greatest when none of the loads which produce compression are on.

Thus, for the bar $a\ b$, the compression is greatest when the loads at i, l, and n only are on, while the tension is greatest when the loads at f, d, and b only are on.

Referring now to *Table* II., the series obtained for maximum compression is

$$\tfrac{1}{12}\ w\ n\ (+5+3+1)$$

while that for the maximum tension is

$$\tfrac{1}{12}\ w\ n\ (-5-3-1)$$

For the bar A I, *Table* I., the series for maximum compression is

$$\tfrac{1}{12}\ w\ n\ (+4+2)$$

while that for the maximum tension is

$$\tfrac{1}{12}\ w\ n\ (-6-4-2)$$

so that for the rolling load, it is only necessary to repeat the series for the dead load, substituting W for w, and leaving out the compressions, or the tensions, according as the calculation is being made for tension or compression.

Applying this in the case under consideration, and supposing that there is a load w_1 at each of the upper apices, let it be supposed that the girder already considered has a span of 84 feet and a depth of 7 feet, and that the loads on the top and bottom of the girder are as follows:

Dead load at each top apex $=1$ ton $=w_1$
" " bottom apex $=4{\cdot}5$ " $=w$
Rolling load at each bottom apex $=9{\cdot}0$ " $=W$
Also let the bars be set at 45°.

Then $\frac{w_1 n}{12}=\cdot1178$ tons

" $\frac{w n}{12}=\cdot5303$ "

" $\frac{W n}{12}=1{\cdot}0606$ "

To find the maximum compression in the bars.

As 1 and 2 are always in tension, it is unnecessary to examine them in regard to compression.

Maximum compression in bar 3:

$$\begin{aligned}&\cdot1178\ (+2-8-6-4-2)\\ &+\ \cdot5303\ (+1-9-7-5-3-1)\\ &+1{\cdot}0606\ (+1)\\ =&\cdot1178\ (-18)+\cdot5303\ (-24)+1{\cdot}0606+(1)=-2{\cdot}1204-12{\cdot}7272+\\ &1{\cdot}0606=-13{\cdot}7870\text{ tons, so that this bar is always in tension.}\end{aligned}$$

Bar 4:

$$\begin{aligned}&\cdot1178\ (+3+1-7-5-3-1)\\ &+\cdot5303\ (+2-8-6-4-2)\\ &+1{\cdot}0606\ (+2)\\ =&\cdot1178\ (-12)+\cdot5303\ (-18)+1{\cdot}0606\ (+2)=-1.4136-9{\cdot}5454+\\ &2{\cdot}1212=-8{\cdot}8378\text{, so that this bar is always in tension.}\end{aligned}$$

Bar 5:

$$\begin{aligned}&\cdot1178\ (+4+2-6-4-2)\\ &+\cdot5303\ (+3+1-7-5-3-1)\\ &+1{\cdot}0606\ (+3+1)\\ =&\cdot1178\ (-6)+\cdot5303\ (-12)+1{\cdot}0606\ (+4)=-\cdot7068-6{\cdot}3636+\\ &4{\cdot}2424=-2{\cdot}8280\text{ tons, so that this bar is always in tension.}\end{aligned}$$

Bar 6:

$$\begin{aligned}&\cdot1178\ (+5+3+1-5-3-1)\\ &+\cdot5303\ (+4+2-6-4-2)\\ &+1{\cdot}0606\ (+4+2)\\ =&0+\cdot5303\ (-6)+1{\cdot}0606\ (+6)=-3{\cdot}1818+6.3636=+3.1818\text{ tons.}\end{aligned}$$

Bar 7:

$$\begin{aligned}&\cdot1178\ (+6+4+2-4-2)\\ &+\cdot5303\ (+5+3+1-5-3-1)\\ &+1{\cdot}0606\ (+5+3+1)\\ =&\cdot1178\ (+6)+0+1{\cdot}0606\ (+9)=+\cdot7068+9{\cdot}5454=+10{\cdot}2522\text{ tons}\end{aligned}$$

Bar 8:

$$\begin{aligned}&\cdot1178\ (+7+5+3+1-3-1)\\ &+\cdot5303\ (+6+4+2-4-2)\\ &+1{\cdot}0606\ (+6+4+2)\\ =&\cdot1178\ (+12)+\cdot5303\ (+6)+1{\cdot}0606\ (+12)=+1{\cdot}4136+3{\cdot}1818+\\ &12{\cdot}7272=17{\cdot}3226\text{ tons.}\end{aligned}$$

Bar 9:

·1178 (+8+6+4+2−2)
+·5303 (+7+5+3+1−3−1)
+1·0606 (+7+5+3+1)
=·1178(+ 18)+ ·5303 (+12) + 1·0606 (+ 16) = 2·1204 + 6·3636 + 16·9696= +25·4536 tons.

Bar 10:

·1178 (+9+7+5+3+1−1)
+·5303 (+8+6+4+2−2)
+1·0606 (+8+6+4+2)
=·1178 (+24) +·5303 (+18) +1·0606 (+20) = +2·8272+ 9·5454+ 21·2120= +33·5846 tons.

Bar 11:

·1178 (+10+8+6+4+2)
+·5303 (+9+7+5+3+1−1)
+1·0606 (+9+7+5+3+1)
= ·1178 (+30) + ·5303 (+ 24) +1·0606 (+25) = 3·5340+12·7272+ 26·515=42·7762 tons.

Bar 12:

·1178 (+11+9+7+5+3+1)
+·5303 (+10+8+6+4+2)
+1·0606 (+10+8+6+4+2)
=·1178(+36) + ·5303(+ 30)+ 1·0606(+30) =4·2408 +15·9090 + 32·8180=51·9678 tons.

Now this may be still further simplified, where there is a large number of bars, by the method of differences, thus,

		D_1 Difference.	D_2 Difference.
Compression on bar 6 = 3·1818 tons			
" " 7 = 10·2522 "		+ 7·0704	
" " 8 = 17·3226 "		+ 7·0704	0
" " 9 = 25·4536 "		+ 8·1310	+ 1·0606
" " 10 = 33·5846 "		+ 8·1310	0
" " 11 = 42·7762 "		+ 9·1916	+ 1·0606

i.e., by taking the difference D_1 between a sufficient number of bars, and then the difference D_2 between the first differences D_1, it is found that the second differences D_2 recur in a certain order over and over again, and by writing down these recurring series, adding D_2 to the previous value of D_1, the new value of D_1 is obtained, which in turn added to the compression on the last calculated bar gives the compression on the next bar.

Thus, to determine the compression on bar 12 from the above; the value of D_2 next recurring is 0, and the next value of D_1 is therefore 9·1916 tons, which added to the compression on bar 11 gives 51·9678 tons, which agrees with that previously found.

This may be checked by resolving this compression vertically and horizontally; the vertical component will, if correct, be the reaction at the pier due to the loads at the apices of this set of triangles.

The vertical component in this case $= \frac{51{\cdot}9678}{\sqrt{2}} = 36{\cdot}74$ tons, and the reaction $= 3w_1 + \frac{5}{2}w + \frac{5}{2}W = 3 + 11{\cdot}25 + 22{\cdot}50 = 36{\cdot}75$ tons.

To find the maximum tension on the bars.

Bar 1:

$$\begin{aligned}&{\cdot}1178\,(-10-8-6-4-2)\\ &+{\cdot}5303\,(-11-9-7-5-3-1)\\ &+1{\cdot}0606\,(-11-9-7-5-3-1)\\ =&{\cdot}1178\,(-30)+{\cdot}5303\,(-36)+1{\cdot}0606\,(-36)=-3{\cdot}5340-19{\cdot}0908\\ &-38{\cdot}1816=-60{\cdot}8064 \text{ tons.}\end{aligned}$$

Bar 2:

$$\begin{aligned}&{\cdot}1178\,(-9-7-5-3-1+1)\\ &+{\cdot}5303\,(-10-8-6-4-2)\\ &+1{\cdot}0606\,(-10-8-6-4-2)\\ =&{\cdot}1178\,(-24+{\cdot}5303\,(-30)+1{\cdot}0606\,(-30)=-2{\cdot}8272-15{\cdot}9090\\ &-31{\cdot}8180=-50{\cdot}5542 \text{ tons.}\end{aligned}$$

Bar 3:

$$\begin{aligned}&{\cdot}1178\,(-8-6-4-2+2)\\ &+{\cdot}5303\,(-9-7-5-3-1+1)\\ &+1{\cdot}0606\,(-9-7-5-3-1)\\ =&{\cdot}1178\,(-18)+{\cdot}5303\,(-24)+1{\cdot}0606\,(-25)=2{\cdot}1204-12{\cdot}7272\\ &-26{\cdot}5150=41{\cdot}3626 \text{ tons.}\end{aligned}$$

Bar 4:

$$\begin{aligned}&{\cdot}1178\,(-7-5-3-1+3+1)\\ &+{\cdot}5303\,(-8-6-4-2+2)\\ &+1{\cdot}0606\,(-8-6-4-2)\\ =&{\cdot}1178\,(-12)+{\cdot}5303\,(-18)+1{\cdot}0606\,(-20)=-1{\cdot}4136-9{\cdot}5454\\ &-21{\cdot}2120=-32{\cdot}1710 \text{ tons.}\end{aligned}$$

Bar 5:

$$\begin{aligned}&{\cdot}1178\,(6-4-2+4+2)\\ &+{\cdot}5303\,(-7-5-3-1+3+1)\\ &+1{\cdot}0606\,(-7-5-3-1)\\ =&{\cdot}1178\,(-6)+{\cdot}5303\,(-12)+1{\cdot}0606\,(-16)=-{\cdot}7068-6{\cdot}3636-\\ &16{\cdot}9696=-24{\cdot}0400 \text{ tons.}\end{aligned}$$

From which the remainder of the bars can be determined by the method of differences as already explained.

		D_1	D_2
Tension on bar 1	$=-60{\cdot}8064$ tons		
		$+10{\cdot}2522$	
„ „ bar 2	$=-50{\cdot}5542$ „		$-1{\cdot}0606$
		$+\ 9{\cdot}1916$	
„ „ bar 3	$=-41{\cdot}3626$ „		0
		$+\ 9{\cdot}1916$	
„ „ bar 4	$=-32{\cdot}1710$ „		$-1{\cdot}0606$
		$+\ 8{\cdot}1310$	
„ „ bar 5	$=-24{\cdot}0400$ „		0
		$+\ 8{\cdot}1310$	
„ „ bar 6	$=-15{\cdot}9090$ „		$-1{\cdot}0606$
		$+\ 7{\cdot}0704$	
„ „ bar 7	$=-\ 8{\cdot}8386$ „		0
		$+\ 7{\cdot}0704$	
„ „ bar 8	$=-\ 1{\cdot}7682$ „		$-1{\cdot}0606$
		$+\ 6{\cdot}0098$	
„ „ bar 9	$=+\ 4{\cdot}2416$ „		

It is unnecessary to go any further with the calculations, as bars 10 to

12 must evidently have a greater compression on them than bar 9, and can never be in tension.

The correctness of the calculation for bar 9 may be checked thus,

Maximum tension = ·1178 (−2+8+6+4+2)
+ ·5303 (−3−1+7+5+3+1)
+1·0606 (−3−1)
= ·1178 (+18) + ·5303 (+12) +1·0606 (−4) = +4·2416 tons,

which agrees with that given above, and shows that bar 9 is always a compression bar.

The result of the whole is as follows:

Maximum stress on bar 1 (and also on *fq*) = −60·80 tons
" " " 2 (" " EF) = −50·55 "
" " " 3 (" " *de*) = −41·35 "
" " " 4 (" " CD) = −32·17 "
" " " 5 (" " *bc*) = −24·04 "
" " " 6 (" " AB) = −15·90 " and +3·18 tons
" " " 7 (" " *ia*) = − 8·83 " and +10·25 "
" " " 8 (" " KI) = − 1·76 " and +17·32 "
" " " 9 (" " *lk*) = +25·45 "
" " " 10 (" " ML) = +33·58 "
" " " 11 (" " *nm*) = +42·77 "
" " " 12 (" " HN) = +51·96 "

The compression on G*g* and H*h* are each equal to the vertical component of the tension on bar 1 = −60·80 tons which should be equal to $w_1 \times \frac{5}{8} + w \times 3 + W \times 3 = 2{\cdot}5 + 13{\cdot}5 + 27 = 43$ tons, and which is actually equal to $\frac{60{\cdot}80}{\sqrt{2}} = 42{\cdot}99$ tons. In calculating the dimensions of these two bars, the load borne at *g* and *h* must be added to this.

The bars 6 and AB, if designed as tension bars to carry 15·90 tons tension, would probably be able to take, without special construction, the 3·18 tons compression, to which they are sometimes exposed; but it might be necessary to counterbrace them.

The bars 7 and *ia* being exposed to a compression and tension alternately of nearly equal amount, should be counterbraced, while the bars 8 and KI would evidently be able as compression bars to resist the small tension which sometimes comes on them.

The Flanges or Booms.

An examination of *Table* XI. shows that the stresses in the corresponding parts of the top and bottom flanges are not identical; for example, the compression in the bar *c* B is 17 (*w* + W) *m*, while the tension in the bar C *b* is 16·5 (*w* + W) *m* = 16·5 (*w* + W), *i.e.*, *m* being in this case cot. 45° = 1.

It is nevertheless common in ordinary practice to calculate the stresses in the flanges in the same manner as for continuous web girders, (*Examples* I. and II.) and the result of this in this particular case, viz., that shewn in the tables, at a vertical section half-way between *c* and B, is as follows:

Moment of flexure $= \frac{12\,(w+W)}{84\times 2}\,(c^2-x^2)$ = Stress in either flange × depth of girder (moment of resistance),

where $\frac{12\,(w+W)}{84}$ is the weight per running foot on the girder

whence stress in flange $= \frac{12\,(w+W)\,(42^2-10{\cdot}5^2)}{84\times 2\times 7} = 16{\cdot}87\,(w+W)$ tons.

The method of tables is evidently more troublesome, especially where the number of bars is great, and another method may be employed with advantage.

The greatest stress on the flanges being produced when the girder is fully loaded, it is first necessary to ascertain the stress in the braces in this condition of the load.

Reverting to the case where some of the load is borne at the top apices, this may be obtained from the previous calculations, thus:

				D_1
Bar 1 (and fg)	−60·08			
„ 2 („ FE)	−50·55			+10·25
„ 3 („ de)	− 2·1204	−12·7272	−25·4544 = −40·30	+10·25
„ 4 („ CD)	− 1·4136	− 9·5454	−19·0908 = −30·05	+10·25
„ 5 („ bc)			= −19·80	„
„ 6 („ AB)			= − 9·55	„
„ 7 („ ia)			= + ·70	„
„ 8 („ KI)			= +10·95	„
„ 9 („ ik)			= +21·20	„
„ 10 („ ML)			= +31·45	„
„ 11 („ nm)			= +41·70	„
„ 12 („ HN)			= +51·95	„

To find the stress in G f.

The forces acting at G are the total reaction, the compression in g G, the compression in F G, and the tension in G f (*Fig.* 24), and the horizontal components of these forces acting in one direction must equal the horizontal components acting in the other:

***i.e.*, Tension in f G = Compression in F G cos. i**

$$= \frac{51{\cdot}95}{\sqrt{2}} = 36{\cdot}73 \text{ tons.}$$

Fig. 24.

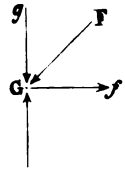
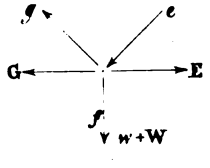


Fig. 25.



Tension in f E;

Here the forces acting are the tension in $g\,f$, the compression in $e\,f$, the tensions in f G and f E, and the weight at f (*Fig.* 25) and

$$\text{Tension in } f \text{ E} = \frac{60{\cdot}80}{\sqrt{2}} + \frac{41{\cdot}70}{\sqrt{2}} + 36{\cdot}73 = 109{\cdot}21 \text{ tons.}$$

$$\text{Tension in E } d = \frac{50{\cdot}55 + 31{\cdot}45}{\sqrt{2}} + 109{\cdot}21 = 167{\cdot}30 \text{ ,,}$$

$$\text{Tension in } d \text{ C} = \frac{40{\cdot}30 + 21{\cdot}20}{\sqrt{2}} + 167{\cdot}30 = 210{\cdot}78 \text{ ,,}$$

$$\text{Tension in C } b = \frac{30{\cdot}05 + 10{\cdot}95}{\sqrt{2}} + 210{\cdot}78 = 239{\cdot}77 \text{ ,,}$$

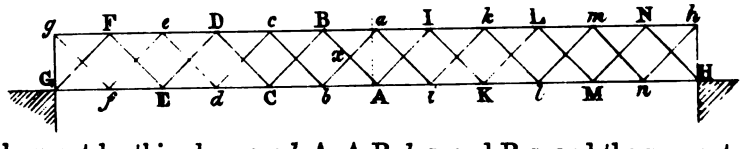
$$\text{Tension in } b \text{ A} = \frac{19{\cdot}80 + {\cdot}70}{\sqrt{2}} + 239{\cdot}77 = 254{\cdot}26 \text{ ,,}$$

It is unnecessary to go further than this, as the girder being symmetrically loaded, when the load is fully on, the stresses on the other half of the girder will be similar to those on this half.

As a check on this calculation, let the stress on b A be calculated by Rankine's method of sections (*R.A.M.*, p. 150, *R.C.E.*, p. 184).

Conceive the girder to be divided into two parts by a vertical plane passing an infinitely small distance to the left of Aa (*Fig.* 26). Then

Fig. 26.



the bars cut by this plane are b A, A B, $b\,a$, and B a, and the moment of their resistances about any point in the vertical plane must be equal to the moment of flexure of the external forces tending to cause rotation about that same point.

Taking moments about a, the moment of resistance is as follows: Stress in b A $\times$ depth of girder + stress in B A $\times$ $a\,x$ + stress in $a\,b \times 0$ + stress in B $a \times 0$, (1)
and the stress in B A is known as well as the distance $a\,x$.

The moment of flexure is

$$\frac{11\,(w_1 + w + \text{W})}{2} \times g\,a - 5\,(w_1 + w + \text{W}) \times a\,\text{D} \qquad (2)$$

Substituting the numerical values, and equating (1) and (2)

$$\text{Stress in } b \text{ A} \times 7' = \frac{11}{2} \times 14{\cdot}5 \text{ tons.} \times 42' - 5 \times 14{\cdot}5 \text{ tons.} \times 21' - 9{\cdot}55 \text{ tons.} \times \frac{7}{\sqrt{2}}$$

$$= 254{\cdot}25 \text{ tons,}$$

which proves the accuracy of the previous method.

In a similar way the stresses in the upper flange are obtained, and the calculation checked.

It is unnecessary to determine the dimensions of the parts in this example, as methods of calculating them and examples have already been given in previous papers (*Tension and Compression Bars, Joints and Fastenings*) and in the first part of this paper.

Remarks on this type of Beam.

(1) The number of system or sets of triangles.

If the number of sets of triangles in any braced girder be large, the stresses on the bars forming each set is proportionately reduced, and the stresses on the central bars may thus be made so small, that it becomes impossible to construct them, even approximately, of the proper theoretical dimensions. This causes a waste of metal, and an unnecessary load on the girder, which again causes increase of dimension in other parts of the girder to carry it; and it interferes with the evenness of the stress through the girder, those bars which are unnecessarily large being under a smaller intensity of stress, and less distorted than other parts.

By reducing the number of sets of triangles, the stresses on each set is increased, and waste of metal on the central bars is avoided. But this may increase the stresses on the bars nearest to the piers so much that they cannot conveniently be made large enough to take them. If it is found, that the end bars, when made as large as they conveniently can be made, cannot nearly resist the stress which will come upon them, the number of sets of triangles must be increased.

Where the end bars are nearly able to meet the stresses, which will come upon them, when inclined at a given angle, their strength may be increased by giving them a smaller inclination to the vertical, and increasing the number of spaces instead of the number of sets of triangles. In effect, the number of sets of triangles is determined by the dimensions which it is possible conveniently to give to the end bars.

(2) It is evident from the foregoing that the angle for the bars must depend upon the actual requirements of the particular structure of which the braced beam forms a part.

The most usual angles in this country are 60° in Warren girders, and 45° in lattice girders.

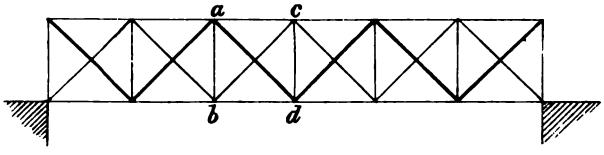
The investigation of the case of a girder having one set of triangles, loaded uniformly on the top boom is given in *Latham's Wrought Iron Bridges*, p. 47 to 65, with this general result that as it is desirable to avoid loading the boom or flange between the apices of the triangles formed by the bars, it is better to use such an angle as will reduce the distance apart of these apices to that which is found to give a stiff platform (p. 127).

(3) It does not seem desirable to use any superfluous bars in these beams, as they must, even with the best workmanship, render the distribution of the stresses over the whole of the bars uncertain.

Thus in a braced girder with two systems of triangles (*Fig.* 27) the vertical bars, *a b*, *c d*, are superfluous, and supposing that *a b* is made slightly shorter than *c d*, a portion of the load at *b* is carried up *a b* to

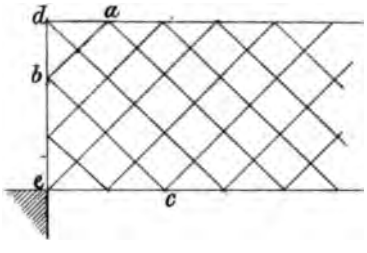
a and so on to the set of bars shewn in dark lines, while very little of the load at *d*, owing to the greater length of *c d*, is carried up to *c*, and consequently the greater part of the load at *b*, is taken by the same set of bars, which receives the load at *b*, and thus one set of bars is overloaded, while the other set is underworked.

Fig. 27.



(4) In lattice girders with more than two sets of triangles, some of the braces finish on the vertical end pillar (*Fig.* 28) and in this case the pillar, in addition to being under a direct compression, is subjected to a transverse stress equal to the difference of the horizontal component of the stresses on the bars which meet upon it, applied at their

Fig. 28.



points of meeting; thus, the bar *a b* is a compression bar tending to bend the pillar *d e* outwards with a force equal to the horizontal component of the compression; the bar *b c* is a tension bar tending to bend the pillar *d e* inward with a force equal to the horizontal component of the tension; and the difference of these two must be taken into consideration in calculating the dimensions of the pillar.

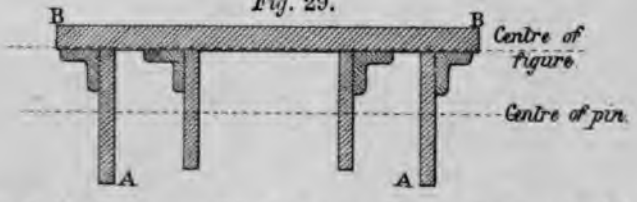
The end pillars of large lattice girders are generally built up plate boxes.

(5.) Rivetted connections for these girders are far cheaper than turned bolts, and they "fix" the ends of the braces, thereby allowing of the compression braces being calculated as "fixed" at the ends.

Theoretically they ought, if the rivetting is well done and especially if the rivet holes are drilled, to be as strong as turned bolts, while if properly arranged (p. 102), they weaken the bars and the booms less. For work to be sent to countries where skilled labour is scarce, the latter have this advantage, that they require fewer skilled workmen to put the work together, than rivetted connections require.

(6) If pins or bolts are used to connect the braces with the flanges, they should theoretically be placed at the centre of figure of the section of the flanges (*Fig.* 29), as this arrangement, if the girder were actually rigid, would distribute the stress uniformly over the section.

Fig. 29.



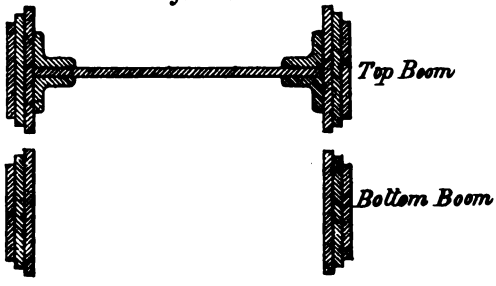
In trough shaped flanges this is not possible, as the centre of figure lies too close to the horizontal flange-plates to allow of the pin being inserted there, and it is of necessity put further from the outsides of the section.

The result of this is that the intensity of the stress is greater on the insides of the section A A, than on the outsides B B.

This inequality in the intensity of the stress is however rudely compensated by the inequality in the opposite direction caused by the actual bending of the girder, which increases the intensity of the stress at B B and relieves it at A A.

A form of flange adopted by Mr. Reilly, C.E., allows of the pins being placed at the centre of figure (*Fig.* 30), (*M.P.I.C.E.*, p. 391, Vol. XXIV., and *U.I.B.R.* p. 75).

Fig. 30.



(7) Where pins are used, the effective depth of the girder is evidently the vertical distance from centre to centre of the pins.

Paragraphs 5, 6 and 7 apply also to the forms of braced girders, described p. 171, *et seq.*

Advantages of Braced Girders.

In two girders of the same depth and span, and equal strength, the braced girder is somewhat lighter, while it does not appear to be less stiff (*U.I.B.R.*, p. 82 and 68; *S.T.S.*, p. 144, Vol. I.).

But from the nature of the construction, the braced girder may have, and usually has a greater depth in proportion to span, and the saving

of metal thus effected in the flanges more than compensates for the increased weight of the web (*U.I.B.R.*, p. 83).

Moreover, it is often possible to employ in the webs of braced girders a higher quality of iron than the plate iron used in continuous web girders, and this again results in lightening the structure (*U.I.B.R.*, p. 82).

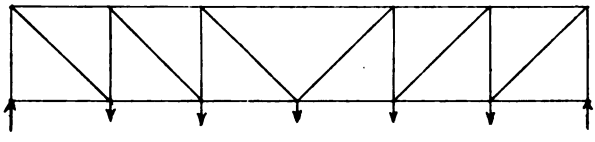
These girders also offer much less obstruction to violent winds than plate girders.

They have been used in this country up to 326 feet in length, but it seems doubtful if any advantage, beyond that of appearance, is gained by using them for less spans than 70 feet.

Braced Girders of the Form Shewn in the Figures.

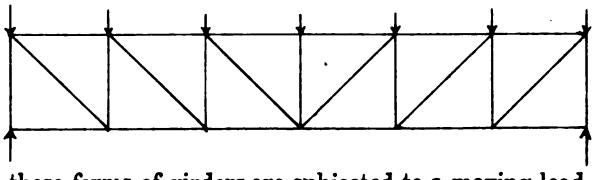
The form of girder shewn in *Fig.* 31 is suitable for a dead load, symmetrically applied on the bottom flange. In this case, the inclined bars are in tension, and the vertical bars in compression, the stress in both being constant.

Fig. 31.



Where the load is on the top, and it is required to apply any part of it at the centre, an additional vertical bar should be inserted to receive the central load, and transmit it to the lower apex at the centre of the girder. In this case, as in the last, all the inclined bars are in tension, and the vertical bars in compression.

Fig. 32.



When these forms of girders are subjected to a moving load, some of the inclined bars come under compression in certain positions of the load, and these must be counterbraced. The chief advantage of this form, viz.: that the compression bars are made as short as they possibly can be made, is then lost, and to retain it, it becomes necessary to insert in each rectangular compartment, except the two at the ends, a second tension brace crossing the first, both being so constructed as to be incapable of resisting any appreciable amount of compression without buckling.

In this case, as the load moves, the vertical bars are either neutral or in compression, one of each pair of cross braces is in tension, and the other neutral, and the end inclined bar is in tension.

Fig. 32.

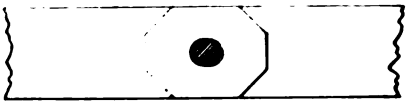
For railway and road purposes, where the platform is to be on the top, the girder may be simplified by carrying the supports up to the top of the boom, and dispensing with the bars A, A, and B, B (*Fig.* 33), the girder then assuming the form shewn in *Fig.* 34.

Whether the construction with cross braces is more economical than that with single counterbraced bars, must be determined by the circumstances of each case.

Fig. 34.

It may be said, that these tension braces, as ordinarily constructed, are competent to take up a considerable compression before buckling, and that, consequently, the distribution of the stresses in the girder is uncertain. This, however, can only be true to an unimportant extent, and even this uncertainty can be got rid of, by jointing the tension braces in the middle, as shewn in *Fig.* 35, the joint being made purposely loose, so that as soon as compression comes on the bar, the joint yields.

Fig. 35.



Probably the simplest way of ascertaining the stresses in the braces of girders of these forms, is by forming tables similar to these already given for the lattice girder, the stresses in the booms being ascertained either by tables, or by the method given for lattice girders (p. 166).

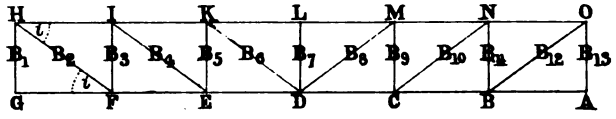
The braces can, however, be dealt with in a way very similar to that given at p. 160, for the lattice girder, known as Latham's method.

EXAMPLE II.

Determine the stresses in a braced girder of the form shewn in *Fig.* 36, the rolling load being on the lower boom.

Taking the load as distributed in a manner similar to that in the lattice girder (Example I.), and making the same supposition as to each

Fig. 36.



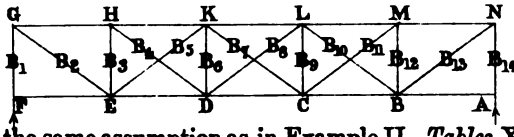
apex being hinged, as well as loaded fully while that in front of it is not loaded at all, *Tables* XII. and XIII. are formed, and from them the maxima stresses in the braces can be ascertained, in the same manner as in the lattice girder.

The stresses in the booms can also be ascertained by similar tables, or, they can be calculated in the manner fully described at page 166.

EXAMPLE III.

Determine the stresses in a braced girder of the form shewn in *Fig.* 37, the rolling load being either on the top or the bottom boom.

Fig. 37.



Making the same assumption as in Example II., *Tables* XIV. and XV. are formed for the braces.

They show that the stress in each brace, though different in amount, remains the same in nature in every condition of the load, and that the maximum stress is produced, when the girder is fully loaded.

The dead and live load may therefore be considered together.

The stresses in the booms can be ascertained by similar tables, or they can be calculated in the manner fully described at p. 166.

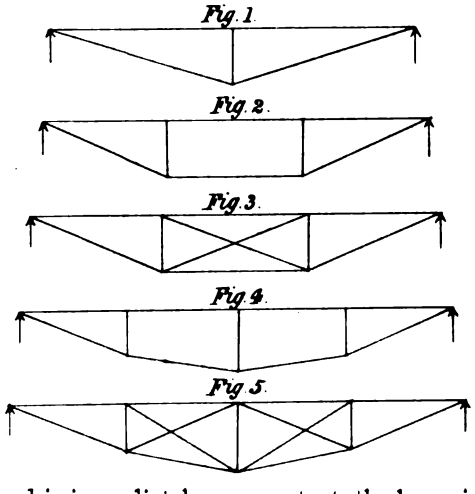
TRUSSED BEAMS, FOR TRAVELLERS, PURLINS, FLOOR GIRDERS, &c.

SMALL braced beams (sometimes imperfectly braced) are frequently used for travellers, and for purlins where the roof trusses are sufficiently far apart to require it, and less frequently for floor girders.

TRAVELLERS.

In this case, the load consists of a crab with a heavy suspended weight capable of being moved along the traveller from end to end, and the moving load bears so large a ratio to the weight of the traveller itself, that the latter may without serious error be omitted in the calculation.

Travellers usually take one of the forms shewn in *Figs.* 1 to 5, those shewn in *Figs.* 2 and 4 being imperfectly braced, and the ordinary construction is to make the beam of a balk of timber, the struts of timber or cast iron, and the tension bars of wrought iron.



When the load is immediately over a strut, the beam is under direct compression only, and to ascertain the values of the different stresses

in this condition of the load, the simplest way for forms 1 to 3 is to form tables similar to those already given for lattice, and other girders, and for forms 4 and 5, which are in fact bowstring beams, to use Professor Clerk Maxwell's method described under the head of "Bowstring Beams" (p. 182).

When the load is between the struts, it produces a transverse stress on the beam, and a compression is produced on the upper fibres of the timber, far exceeding in intensity the compression produced in the first mentioned condition of the load.

It becomes necessary therefore, to ascertain the value of this compression in that condition of the load, which gives it its maximum value, *i.e.*, when the load is at the centre of that length, and to provide for it, as well as for the direct compression transmitted by the braces in that same condition of the load.

EXAMPLE I.

It is required to truss a Memel (Scotch fir) balk, 12″ square, so that it may form one of the beams for a traveller, to carry a single load of 4 tons, *i.e.*, 2 tons on each beam.

The traveller is to be of the form shewn in *Fig.* 6, which gives the dimensions along the lines of stress, or mean fibres of the parts.

Determine:

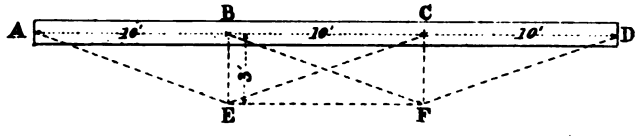
(1) The stresses on the different parts, when the load is over either of the struts.

(2) The stresses on the different parts when the load is at the centre of the beam; and whether the timber is competent to resist the stresses on it in that condition of the load.

(3) The approximate deflection, when the load is at the centre.

(4) The deflection, if the beam had been untrussed and the load at the centre.

Fig. 6.



(1.) The stresses on the different parts, the load W being at B or C.

Table I. Compressions + : Tensions − : n = Cosec. i.

Effect of load	*on* A E	*on* B E	*on* B F	*on* E C	*on* F C	*on* F D
When at B	$-\frac{2}{3}$ Wn	+ W	nil	$-\frac{1}{3}$ Wn	+ $\frac{1}{3}$ W	$-\frac{1}{3}$ Wn
„ „ C	$-\frac{1}{3}$ „	+ $\frac{1}{3}$ W	$-\frac{1}{3}$ Wn	nil	+ W	$-\frac{2}{3}$ Wn

Table II. Compressions + : Tensions − : m = Cot. i.

Effect of load	on A B	on B C	on C D	on E F
When at B	$+\frac{2}{3}Wm$	$+\frac{2}{3}Wm$	$+\frac{1}{3}Wm$	$-\frac{1}{3}Wm$
„ „ C	$+\frac{1}{3}Wm$	$+\frac{2}{3}Wm$	$+\frac{2}{3}Wm$	$-\frac{1}{3}Wm$

(2.) The stresses on the different parts, the load W being at the centre of the beam, one half of it being transmitted by the timber to B and C.

Table III. Compression + : Tension − : n = Cosec. i.

Effect of load	on A E	on B E	on B F	on E C	on F C	on F D
At B	$-\frac{2}{6}Wn$	$+\frac{2}{6}W$	nil	$-\frac{1}{6}Wn$	$+\frac{1}{6}W$	$-\frac{1}{6}Wn$
At C	$-\frac{1}{6}Wn$	$+\frac{1}{6}W$	$-\frac{1}{6}Wn$	nil	$+\frac{1}{2}W$	$-\frac{2}{6}Wn$
Total	$-\frac{1}{2}Wn$	$+\frac{2}{3}W$	$-\frac{1}{6}Wn$	$-\frac{1}{6}Wn$	$+\frac{2}{3}W$	$-\frac{1}{2}Wn$

Table IV. Compressions + : Tensions − : m = Cot. i.

Effect of load	on A B	on B C	on C D	on E F
At B	$+\frac{3}{6}Wm$	$+\frac{2}{6}Wm$	$+\frac{1}{6}Wm$	$-\frac{1}{6}Wm$
At C	$+\frac{1}{6}Wm$	$+\frac{2}{6}Wm$	$+\frac{2}{6}Wm$	$-\frac{1}{6}Wm$
Total	$+\frac{1}{2}Wm$	$+\frac{2}{3}Wm$	$+\frac{1}{2}Wm$	$-\frac{1}{3}Wm$

It is seen on comparing Tables III. and IV. with Tables I. and II., that when the load is at the centre, the stresses in the different parts are either the same or less than they are when the load is either at B or C.

It is obvious that it is unnecessary to consider the case of the load being between A and B or C and D, as the stresses must then be less than in either of the previous cases.

The intensity of the direct compression on the timber per square inch is therefore $\frac{\frac{2}{3}W \text{ cot. } i}{144} = \frac{\frac{2}{3} \times 2 \times 2240 \times \frac{19}{3}}{144} = 69$ lbs.

The intensity of the compression due to the bending, when the load is at the centre of the beam is found by equating the moment of flexure at the centre of BC with the moment of resistance, *i.e.*, $\frac{W \times BC}{4} = \frac{f_o I}{y_o} = \frac{f_o bd^2}{6}$, which, substituting the values, becomes $\frac{2 \times 2240 \times 120}{4} = \frac{f_o \times 12 \times 144}{6}$, whence $f_o = 467$ lbs. per square inch.

Hence the total compression per square inch $= \overset{\text{lbs.}}{69} + \overset{\text{lbs.}}{467} = \overset{\text{lbs.}}{536}$, which gives a factor of safety $= \frac{7200}{536} = 13$, or the timber is amply strong.

It would in fact be stronger than this calculation shews, as the beam is actually continuous, not hinged at the joints.

(3.) The approximate deflection when the load is at the centre.

The deflection of this kind of trussed beam may be approximately arrived at by supposing the timber to be replaced by wrought iron, of such a section that the intensity of stress would compress (or shorten) the iron to the same proportional extent, as the actual direct stress compresses the timber, and then treating the trussed beam as a beam of uniform strength and breadth, having a cross section of equal strength.

The actual direct stress is 69 lbs. on the square inch, which would compress the wood $\frac{69}{E} l = \frac{69 \times l}{1{,}900{,}000}$

This same amount of compression would be produced on a wrought iron bar of 1 inch section by a stress $= \frac{69}{1{,}900{,}000} \times 29{,}000{,}000$ (the value of E for wrought iron) $= 1{,}053$ lbs., say $\frac{1}{2}$ ton.

Substituting this value for f_a in the formula for deflection (p. 37), and limiting the stress in the tension bar to 5 tons on the square inch.

$$V = \frac{n''(f_a + f_b) c^2}{E h_o}$$

$$= \frac{\cdot 5708 (\cdot 5 + 5) 2240 \times 180^2}{29{,}000{,}000 \times 36} = \cdot 22 \text{ inch.}$$

(4). The deflection of the untrussed beam, the load being at the centre.

This is given by the formula (page 36).

$$V = \frac{n''' W c_3}{E I}$$

$$= \frac{2 \times 2240 \times 180^3}{6 \times 1{,}900{,}000 \times \frac{1}{12} \times 12 \times 12^3} = 1 \cdot 33 \text{ inch.}$$

Remarks on other Forms of Travellers.

In the simpler form of traveller, *Fig.* 1, the greatest stress on the timber is produced when the load is half way between either support and the centre of the beam; the greatest stress on all the other parts is when the load is at the centre of the beam.

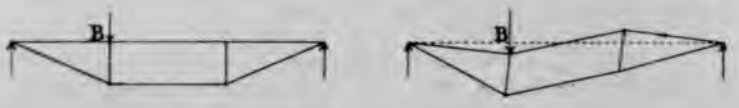
It is not considered necessary to go into the details of the dimensions of the other parts and of the joints of these beams, as they are dealt with on the principles laid down and illustrated in *Tension and Compression Bars*, and *Joints and Fastenings*.

It will be found, however, that the struts, whether of wood or iron, can hardly ever be made as small as their theoretical dimensions.

When these beams are imperfectly braced, as in *Fig.* 2, and loaded with a moving load, the equilibrium depends entirely on the strength of the timber beam, for if the joints were really hinged, the equilibrium could not be maintained without the cross braces, and a load at B,

Fig. 7. would cause the hinged structure to fail as shewn in *Fig.* 8, the traveller being finally pulled off its supports.

Fig. 7. *Fig.* 8.

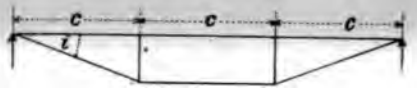


PURLINS.

When trusses, such as *Fig.* 9, are used as purlins, the load is uniformly distributed, and the cross braces may be dispensed with.

In this case the top boom is really a beam fixed at both ends, and supported at two intermediate points, the sections of greatest stress are at the ends, and at the intermediate points of support, where the moments of flexure are equal to one another and to $\frac{w c^2}{3}$; the direct compression is uniform throughout, and $= \frac{3 w c}{4}$ cot. i, from which the dimensions necessary can be ascertained as in Example I.

Fig. 9.



FLOOR GIRDERS.

These beams are seldom used for floor girders as the depth which can be given to the truss is usually so small, and the stress on the iron work consequently so large, that it is better to use wrought or cast iron instead of timber girders.

As a means, however, of stiffening floor girders already in a building, this system of trussing is sometimes found useful; the girder being jacked up from below, the trussing added to the girder, and the tension bars screwed up tight, the trussing being afterwards hidden by plaster or wood work on the ceiling below.

It may also be found useful for stiffening small timber bridges where the headway admits of it.

It is sometimes recommended to set up the tension rods of this class of beam by means of the screws on the ends, until an artificial camber is produced equal to the calculated deflection.

This is radically wrong, because, in order to produce the camber, a heavy stress is put upon every part of the truss before any of the load, which it is intended to carry, comes on; and this stress is simply so much deducted from its ultimate strength.

A naturally cambered beam, if such can be obtained, should be selected, as giving the required camber without any unnecessary stress on the truss.

BOWSTRING BEAMS.

These beams consist of a curved or polygonal bow and a horizontal string, the bow being above or below the string as may be most suitable for the purpose for which the beam is required, and to the proper use of the material of which it is constructed.

Thus, if the bow is of cast iron, it should be above (*i.e.*, in compression), but if of steel, it should, in order to elicit the full value of the material, *i.e*, to use it in the dimensions theoretically necessary, it should be below (*i.e.*, in tension), *Figs.* 1 and 2.

Where the load is uniformly distributed along a horizontal line, as is nearly the case in aqueducts, or generally with dead loads, and conveyed to the bow by a great number of equidistant bars (*Fig.* 3), it is easy to shew (page 180), that the proper curve for the bow, no matter what its height may be, is a parabola.

Fig. 1

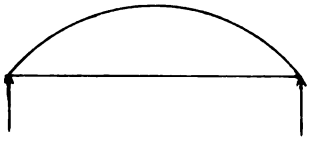


Fig. 2.

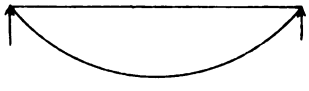
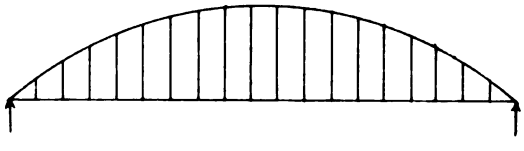


Fig. 3.



In this case the stress in the bow passes through the centre of figure of each section, there is no transverse stress on it, and consequently no bracing is required.

When, however, the girder is subjected to a moving load, no one curve can be suitable for every condition of the load, and transverse stresses are set up in different parts of the bow as the load moves. In this case, the bow must either be made strong enough to resist them in addition to the direct stress upon it, or be braced.

The latter is the simpler and more advantageous method of proceeding, and the bracing generally assumes one of two forms (*Figs.* 4 and 5), the relative advantages of which, as applied to any particular case, can only be determined by a comparative calculation of both.

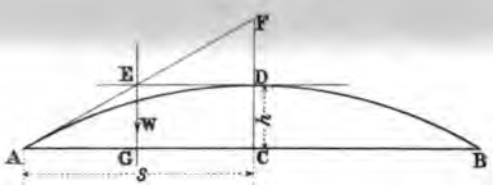
Fig. 4.

Fig. 5.

The method of calculating the stresses in the bow and string of a girder, in which the load is taken as uniformly distributed along a horizontal line, is as follows, it being premised that the load is not, even in aqueducts, owing to the curve of the bow, accurately distributed in this way, though it is sufficiently nearly so to allow of the calculation being made on this assumption without material error.

Let *Fig.* 6 represent a bowstring beam, loaded uniformly along the string, the load being transmitted to the bow by a large number of equidistant vertical rods.

Fig. 6.



Then the equilibrium of the part A D of the bow is maintained by three forces, viz.: the thrust at D, which owing to the load being symmetrical is horizontal, the weight of the load carried between A and C, acting through its centre of gravity, and the reaction at A, tangential to the curve; and the directions of these three forces meet in a point.

Draw D E horizontal, intersecting the vertical through W in E, and join A E.

Produce A E to F.

Then, by similar triangles,

$$A\,G : A\,C :: C\,D : C\,F$$

and $A\,G = \frac{1}{2}\,A\,C$, $\therefore\ C\,D = \frac{1}{2}\,C\,F$, *i.e.*, the subtangent C F of the curve is bisected by the curve in D, which is a property of the parabola only.

To determine the stresses in the bow and string.

Let w be the weight per unit of length.

Let s be the half span.
„ h „ rise of the bow at the centre.

Then, as the forces acting at A are represented by the triangle A E G, in which E G is the load betwen A and C $= w s$,

the tension in the string $: w s :: $ A G : E G : $\frac{s}{2} : h$.

Whence the tension in the string $= \frac{w s^2}{2 h}$ which is constant throughout the string, and is also the value of the thrust in the bow at the crown. Call this thrust H. (1)

The thrust in the bow at A is represented by A E, (*Fig.* 6.)

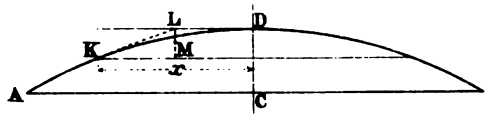
and $\text{A E} = \sqrt{\text{A G}^2 + \text{G E}^2} = \sqrt{\frac{s^2}{4} + h^2}$.

whence thrust at A : $w s :: \sqrt{\frac{s^2}{4} + h^2} : h$.

and thrust at A $= \frac{w s}{2 h} \sqrt{s^2 + 4 h^2}$. (2)

At any point K distant x horizontally from the centre of the beam, the triangle of forces is K L M (*Fig.* 7), in which L M $= w x$, K M $=$ H, and K L $=$ thrust in the bow at K.

Fig. 7.



Whence, as $\text{K L}^2 = \text{K M}^2 + \text{L M}^2$

Thrust at K $= \sqrt{w^2 x^2 + \text{H}^2} = \sqrt{w^2 x^2 + \frac{w^2 s^4}{4 h^2}} =$

$\frac{w}{2 h} \sqrt{4 h^2 x^2 + s^4}$ (3)

It is usual, in designing beams of this form, to make the bow the arc of a circle, which, so long as the rise is not greater than $\frac{1}{8}$ of the span, agrees sufficiently nearly with the curve of a parabola, and this has the advantage of simplifying the work to the workman, and thus avoiding one chance of error.

The formulæ just given apply approximately to chains carrying a load uniformly distributed along a horizontal line, the thrusts becoming tensions, and the tensions becoming thrusts.

EXAMPLE I.

It is required to ascertain the stresses in the bow and string of a bowstring beam of the form shewn in *Fig.* 8, the load being taken

as vertical and distributed uniformly along a horizontal line and conveyed to the bow by a large number of vertical rods, the effective span of the beam being 48 feet, the rise or effective depth at the centre being $\frac{1}{8}$ the span, and the load 2 tons per foot run.

Fig. 8.

Here, $s = 24$ feet.
$w = 2$ tons.
$h = 6$ feet.

Then from Equation (1).

Tension in string = thrust at crown $= H = \frac{w s^2}{2h} = \frac{2 \times 24^2}{2 \times 6} = 96$ tons,

and at any point distant x horizontally from the centre,

$$\text{Thrust} = \sqrt{w^2 x^2 + H^2} = \sqrt{2^2 \times x^2 + 96^2}$$

Let $x = 6$	... ...	then thrust	$= 96{\cdot}7$ tons.
$x = 12$	... ...	„	$= 98{\cdot}9$ „
$x = 18$	... ...	„	$= 102{\cdot}5$ „
$x = 24$, *i.e.*, at support;		„	$= 107{\cdot}3$ „

The increase of stress from the crown to the pier is rather less than 12 per cent., and a uniform section may, if more convenient, be adopted for the bow without serious loss of metal.

The stress in the string is absolutely uniform, and an uniform section is proper for it.

This uniformity of construction avoids the risk of mistake by the workmen.

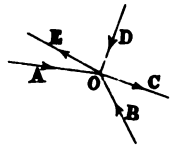
Braced Bowstring Beams.

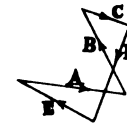
The simplest method of ascertaining the stresses in a braced bowstring beam, to carry a moving load, is by a number of diagrams, constructed on Professor Clerk Maxwell's method of the application of the principle of the polygon of forces, the results obtained being tabulated. In this case, the bow may be of any form.

It is known that if any number of forces act on a point, and are in equilibrium, they may be represented in magnitude and direction by the sides of a polygon drawn parallel to the direction of those forces, taken consecutively. Thus, if A B C D E, *Fig.* 9, represent in magnitude and direction (the directions by means of the arrows) 5 forces acting on the point O, and in equilibrium; they will also be represented by a polygon (*Fig.* 10) in which the sides are in the same order as those forces, and the arrows point all the same way round the polygon. Hence if any number of forces act on a point, and all but two are known both in direction and magnitude, and the directions of those

two are also known, their magnitude and nature can be ascertained by laying down the known forces, and completing the polygon, by two lines parallel to the two forces, whose magnitude and nature are unknown.

Fig. 9. *Fig.* 10.





Supposing, in *Fig.* 9, that A, B, and C are given in magnitude and direction, while D and E are given in direction only, then by laying down A, B, and C in succession as shewn, and drawing D through the end of C, and E through the end of A, until they intersect, the polygon is completed and the magnitudes of D and E ascertained. Putting now the arrows on D and E, taking care that they all point the same way round the polygon, and that the arrows on *Fig.* 9 point the same way as in *Fig.* 10, the nature of the stress in each is also indicated; thus, E points away from the point O and is a tension, while D points towards it and is a compression.

The application of this principle is given in Examples II. and III.

EXAMPLE II.

Bowstring Beams.

Determine the stresses in a bowstring beam of the form shewn in *Fig.* 1, *Plate* I., the loads being distributed, and having the values marked thereon.

The figures above the bow represent the part of the weight of the structure, which is carried at the top apices, as in Example I., *Lattice Girder*. Those below the string represent the dead and live load at the lower apices. As regards the live load, the same assumptions are made as were made in the Example just referred to (p. 155).

It is to be observed that an unusual depth has been given to the beam in this example, for the sake of making the stress diagrams, which of necessity are on a small scale, easier to follow.

The stress diagrams on *Plate* I., have the following significations:

Fig. 2 gives the stresses produced by the loads on the top of the bow, which being permanently imposed, may be dealt with together. It is, perhaps, hardly necessary to observe, that the loads on the top are not of necessity all equal, though they would almost always be symmetrical on each side of the centre. (They would be different in the case referred to at p. 152, *Fig.* 51.)

Figs. 3 to 7, give the stresses produced by each of the loads at the lower apices, which it is more convenient to deal with separately, because the same diagrams, by using a different scale, or by a simple

proportion between the dead and live loads, then answer for both the dead and live loads.

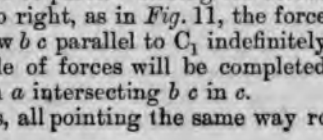
To draw *Fig.* 2, *Plate* I.

The external forces acting on the beam are the forces F to F_7 (of which F and F_7, being carried directly on the supports, produce no effect on the beam, and may be left out of consideration), and the reactions R_1 and R_2, due to the remainder, F_1 to F_6 inclusive, which are equal to each other, and to half the sum of F_1.....F_6.

Commencing at A, there are (omitting F as already explained), at this point, three forces acting, viz., R_1 acting upwards, and two forces in the directions of C_1 and A_1 the values and nature of which are unknown.

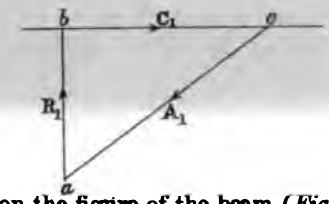
Draw (*Fig.* 12 and *Fig.* 2, *Plate* I.), *a b* parallel to R_1 making it represent R_1 on any scale of parts. Taking the forces in succession from left to right, as in *Fig.* 11, the force next in order is C_1; draw *b c* parallel to C_1 indefinitely through *b*. The triangle of forces will be completed by a line drawn through *a* intersecting *b c* in *c*.

Fig 11.



Putting on the arrows, all pointing the same way round the triangle

Fig. 12.

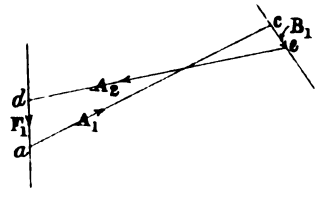


and marking them also on the figure of the beam (*Fig.* 1, *Plate* I.) it is seen that the force C_1 acts away from A, and is, therefore, a tension, while the force A_1 acts towards A, and is a compression.

Next, proceeding to the apex N. At this point there are four forces acting, viz., F_1 acting downwards, A_1 already shewn to be a compression, and acting towards N, and two forces B_1 and A_2, the values and nature of which are unknown.

Lay down (*Fig.* 13) F_1 acting downwards $= d\,a$; draw *a c* parallel to

Fig. 13.



A_1 and in the direction towards which it acts, and make $a\ c = A_1$; through c draw $c\ e$ indefinitely, parallel to B_1, the next force in succession, and through d draw $d\ e$ parallel to A_2, intersecting $c\ e$ in e.

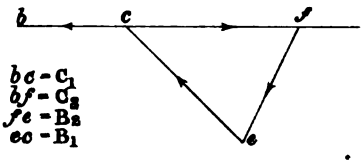
Then the polygon of the forces at N, is $d\ a\ c\ e\ d$, and putting on the arrows, first on the polygon, and then on the figure of the beam, it is seen that B_1 is a tension, and A_2 a compression.

These two figures have been drawn separately, but, it is evident, that *Fig.* 18 may be superimposed on *Fig.* 12, and the same results obtained from one figure, with less chance of error from incorrect drawing. This is done in *Fig.* 2, *Plate* I.

Next, proceeding to the apex B. At this point there are four forces acting, viz., B_1, C_1, C_2, B_2, of which the first two are known.

Dealing with these forces, for the sake of simplicity, in a separate figure, draw (*Fig.* 14) B_1, C_1, both of which are tensions; from b draw $b\ f$ indefinitely, and through e draw $e\ f$, intersecting $b\ f$ in f.

Fig. 14.



Then the polygon of forces is $e\ c\ b\ f\ e$, in which $b\ f = C_2$ is a tension, and $f e = B_2$ a compression. This is also shewn superimposed on *Fig.* 2, *Plate* I.

The same proceeding repeated with F_2, F_3,.....F_6, in succession gives the stresses in the other bars and completes *Fig.* 2.

But it will generally be better to commence the diagram of stress from each pier, and to close upon some central bar, in which case the effect of any small errors in drawing is reduced. As the same result for any bar should be obtained on whichever bar the diagram is completed, a check is afforded of the correctness of the work, for should a different result be obtained for the closing bar working from the two ends, an error must have been made.

When the principle of working out the diagram is thoroughly understood it will be found easier to lay off all the external forces, including all the re-actions at once, taking them in the order in which they are applied to the beam; thus, in this case (*Fig.* 2, *Plate* I.) working round from left to right, R_1 upwards, R_2 upwards, F_6, F_5,.....F_1 downwards; in *Fig.* 3, *Plate* I., R_2 upwards, the load at F downwards, R_1 upwards.

This affords a check on the correct calculation of the re-actions, and on the setting off of the forces, and is necessary, if it is proposed to work from both ends.

Measuring off the stresses by scale, the first line of Table I is obtained.

To draw *Fig.* 3, *Plate* I., for the dead load.

This diagram gives the stresses in the various bars caused by a single load, W, at F.

In this case, the external forces are the force at F, and the two reactions R_3 and R_4, which are respectively $\frac{5}{8}$ W and $\frac{1}{8}$ W.

The diagram is drawn in precisely the same manner as *Fig.* 2, and it may be commenced from each pier, and closed upon any bar.

Figs. 4 to 7 are drawn for the other loads, in a similar manner, and the remaining lines of Table I. are obtained.

As this load is always on the beam, it is only necessary to provide for the difference of any tensions and compressions produced: and the resultant stresses due to the dead load, are thus obtained (Table I.).

The live load.

To ascertain the stresses due to the live load, Table II. is formed, in which the stresses produced are obtained by a direct proportion from those in Table I. In this case, for the sake of simplicity, the rolling load at each lower apex has been taken as twice the dead load at the same apex; and the stresses caused by the loads from F to B, are simply double those in Table I. These stresses may also be obtained by direct measurement on the scale for live loads.

As the loads may be on the beam, at different times, in such a way as to produce the maximum stress of each kind, it is necessary to provide for both; and the maxima stresses of each kind are accordingly obtained in Table II.

Putting Tables I. and II. together, Table III. is obtained, in which, by adding together the stresses due to the dead load, and the maxima stresses due to the live load, when the stresses are of the same kind; and by deducting those due to the dead load, from the maxima stresses due to the live load, when they are of different kinds, the stresses of each kind which it is necessary to provide for are ascertained.

This table shews;

(1.) That there is a difference in this case of only 9 per cent. in the stress at the centre and end of the bow, which would allow of the bow being constructed of uniform section, without serious waste of metal.

(2.) That the stresses on the braces are small in comparison with those on the bow and string.

(3.) That it is necessary to counterbrace most of the braces.

EXAMPLE III.

Determine the stresses in a bowstring beam, of the form shewn in *Fig.* 1, *Plate* II., the loads being distributed, and having the values shewn thereon.

The figures at the top represent the weight of the structure itself, distributed on the different apices, as in Example I., *Lattice Girder*, those below represent the dead and live load on the bottom apices. As regards the live load, the same assumptions are made as were made in the Example just referred to (p. 156).

Of the internal bars or braces those which are vertical are to be constructed to resist either tension or compression; those which cross each other are to be constructed to resist tension only, and to yield to a comparatively slight compression.

The stress diagrams on *Plate* II. have the following significations:

Fig. 2 gives the stresses produced by the loads on the top of the bow, which, being permanently imposed, may be dealt with together.

Figs. 3 to 7 give the stresses produced by each of the loads, on the lower apices, which it is more convenient to deal with separately, because the same diagrams (by using a different scale, or by a simple proportion between the dead and live loads) then answer for both the dead and live loads.

In each of these latter diagrams some of the braces disappear, and these are the braces which, in the position of the load shewn in that particular diagram, are superfluous.

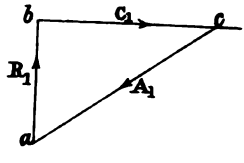
To draw *Fig.* 2.

The external forces acting on the beam are the forces F_1 to F_6 (of which F_1 and F_6 being carried directly on the supports, produce no effect on the beam, and may be left out of consideration), and the reactions R_1 and R_2, due to the remainder F_1 to F_5 inclusive, which are equal to each other, and to half the sum of $F_1 \ldots\ldots F_5$.

Commencing at A there are (omitting F as already explained), at this point three forces acting, viz., R_1 acting upwards, and two forces in the direction of C_1 and A_1, the values and nature of which are unknown.

On the diagram (*Fig.* 15 and *Fig.* 2, *Plate* II) draw *a b* parallel to R_1, and make it represent R_1 on any scale. Taking the forces in succession from left to right draw *b c* parallel to C_1 indefinitely through *b*. The triangle of forces will be completed by a line drawn through *a*, intersecting *b c* in *c*.

Fig. 15.



Putting on the arrows all pointing the same way round the triangle, and marking them also on the figure of the beam, it is seen that the force C_1 acts away from the point A, and is, therefore, a tension, while the force A_1 acts towards A, and is, therefore, a compression.

Proceeding either to M, or to B, it is seen that there are, at each, three unknown forces, *i.e.*, at M, the forces in the directions B_1, B_2, and A_2; and at B, the forces in the directions C_2, B_3, and B_1, and if all these bars were capable of acting both as tension and compression bars, the distribution of the stress in them would be indeterminate.

As, however, the bars B_2 and B_3 are purposely constructed to take tension only, and therefore yield, if called upon to act as compression

bars, they become incompetent, and consequently superfluous in that case; and if it be determined, in each position of the load, which of each pair of cross braces is in tension, and which would, if it acted at all, be in compression, the superfluous bars are known.

Let it be supposed, first, that the bar B_3 is the superfluous bar; then at the apex B there are three forces, viz. C_1 which is known, and C_2, B_1, which are unknown.

Fig. 16.

Make $c\,b = C_1$ (*Fig.* 16 and *Fig.* 2, *Plate* II.), and draw $b\,d$ indefinitely through b, parallel to C_2. As this line actually covers $c\,b$, and passes through c, it is evident that a line drawn through c parallel to B_1 and intersecting cd can have no value, and, therefore, if B_3 is superfluous, B_1 is superfluous also, and $C_1 = C_2$.

On this supposition proceed to the point M. Here there are four forces, viz., F_1, A_1, which are known, and B_2, A_2, which are unknown, (B_1 having been shewn to be superfluous).

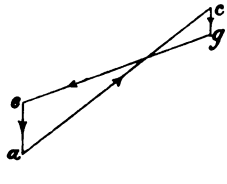
Fig. 17.

Make (*Fig.* 17) $e\,a = F_1$, acting downwards, and $a\,c = A_1$, already determined in *Fig.* 15, draw $c\,f$ through c parallel to B_2, and through e draw $e\,f$ parallel to A_2, intersecting $c\,f$ in f.

Putting on the arrows, it is seen that $c\,f = B_2$ is a tension bar, and $e\,f = A_2$ a compression bar, so that, if the bars B_1 and B_3 were removed entirely, equilibrium would be maintained with this load.

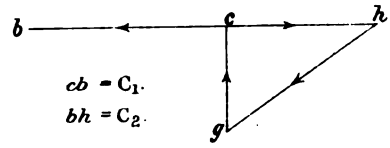
For the sake of demonstrating more completely that B_2 and B_3 are superfluous with this load, let it now be supposed that B_2 is removed: then at M the forces acting are F_1, A_1, known, and B_1, A_2, unknown; and proceeding as before, the polygon of forces at M is $e\,a\,c\,g\,e$ (*Fig.* 18), in which $c\,g = B_1$ is a tension bar, and $g\,e = A_2$ a compression bar.

Fig. 18.



At B the forces acting are B_1, C_1 known, and C_2, B_2 unknown; and proceeding as before, the polygon is $g\ c\ b\ h\ g$ (*Fig.* 19), in which $h\ g = B_2$ is a compression bar, so that if B_2 were left out, B_2 would be called upon to resist compression, and the beam would fail.

Fig. 19.



Proceeding similarly for the other bars, and for each condition of the load, the rest of the stress diagram (*Fig.* 2, *Plate* II.) is obtained, and subsequently the stress diagrams for the loads at the lower apices (*Figs.* 3 to 7), as in Example II., from which the Tables are formed as in that Example.

Advantages of Bowstring Bridges.

The following is extracted from *Unwin's Iron Bridges and Roofs*, p. 77.

"The great practical advantage of the bowstring over other forms of girder results from the fact that the stress on its members is nearly uniform from the centre to the abutment. There is a difficulty in girders of uniform depth in realising the best distribution of the material, owing to the great variation in the magnitude of the stresses. But in the bowstring bridge there is no great variation in the stress, and consequently there need be little variation in the section of the parts. With a uniform load the tension on the horizontal boom is uniform, that on the ties is uniform, and that on the arched boom increases from the centre to the springing by only 8 per cent. if the ratio of span to depth is 10; and by 12 per cent. if the ratio is 8. Hence if the section of all the parts were made uniform throughout, the excess of material above that theoretically necessary need not be more than 2 or 3 per cent. of the weight of the girder. In such a bridge, therefore, the bars and plates might all be of uniform dimensions from end to end of the span. The advantage of this from a practical point of view can scarcely be overrated. Lastly, as this form of bridge easily admits a depth of 1-8th of the span, we shall be prepared to find that it is a very economical construction."

ROOFS.

LOAD ON ROOFS.

THE load on all roofs is partly permanent, and partly occasional.

The permanent part of the load consists of the roof covering, and the framework supporting it; and, where there is a ceiling or other weight such as heavy lamps carried by the framework, that weight also forms part of the permanent load.

The occasional part of the load consists in some countries of the weight of a certain amount of snow, and of the pressure of the wind; and in other countries of the pressure of the wind only.

PERMANENT LOAD ON ROOFS.

The weight of different kinds of roof covering, and framework, is given with sufficient accuracy for the purposes of calculation in the following tables, which are compiled partly from various works and partly arrived at by an analysis of the weight of different kinds of covering.

TABLE I.

	Per Square Foot.
Lead covering, including laps, but not boarding or rolls	5½ lbs. to 8½ lbs.
Zinc covering. do. 14 to 16 zinc gauge	1½ ,, 1¾
Corrugated iron, galvanised, 16 W.G.	3½ ,,
18 W.G.	2½ ,,
20 W.G.	2 ,,
Sheet iron, 16 W.G.	2½ ,,
,, ,, 20 W.G.	1½ ,,
Slating laid with a 3 inch lap including nails, but not battens or iron laths:	
,, Doubles, 13 inches × 9 inches, at 18 cwt. per 1,200...	8½ ,,
,, Ladies, 16 inches × 8 inches, at 31·5 ,, ...	8½ ,,
,, Countesses, 20 inches × 10 inches, at 50 ,, ...	8 ,,
,, Duchesses, 24 inches × 12 inches, at 77 ,, ...	8½ ,,
Tiles, plain, 11 inches × 7 inches, laid with a 3 inch lap and pointed with ... mortar, including laths and absorbed rain	18 ,,
Tiles, pan, 13½ inches × 9½ inches, laid with a 3 inch lap and pointed with mortar, including laths and absorbed rain	12 ,,
Tiles, Italian (ridge and furrow) not including the boarding, but including mortar and absorbed rain	14 ,,
Slate Battens 3½ inches × 1 inch:	
For Doubles	2 ,,
For Countesses	1½ ,,
Slate Boarding ¾ inch thick	2½ ,,
1 inch ,, ...	3½ ,,
1¼ inches ,, ...	4½ ,,
Wrought Iron Laths, angle irons,	
For Duchess Slates	2 ,,
For Countess ,, ...	1¼ ,,
Cast Iron plates, ¾ inch thick	15 ,,
Thatch, including battens	6½ ,,
Lath and Plaster Ceiling	8 ,,
Ceiling Joists	3 ,,
Common Rafters	3 ,,
Collar Beams	2 ,,
Framing for wooden roofs, including purlins and ridge boards, but exclusive of tie beams:	
20 feet span, king post, rise ⅓th span	2 lbs. per feet super.
30 ,, ,, ,, ...	2 ,,
40 ,, queen post ,, ...	2 ,,
50 ,, ,, ,, ...	3 ,,
60 ,, ,, ,, ...	4 ,,
Tie Beams:	
20 feet span, king post	11lbs. per foot run of tie,
30 ,, ,, ...	20 ,, ,,
40 ,, queen post...	18 ,, ,,
50 ,, ,, ...	20 ,, ,,
60 ,, ,, ...	30 ,, ,,

The following table of the weight of roof framing of iron roofs is taken from *Unwin's Bridges and Roofs:*

TABLE II.

	Clear Span. Feet.	Distance apart of Principals. Feet.	Weight per sq. foot of covered area in lbs. Of Purlins, &c.	Of Principal.	Total Ironwork.	Total with covering.
Trussed Roofs.						
Pent	15	...	...	...	3·5	...
Common Truss ..	37	5	1·1	3·5	4·6	6·9
,,	40	12	2·0	3·5	5·5	...
,,	54	14	6·5	3·0	9·5	...
,,	55	6½	4·6	7·0	11·6	...
,,	72	20	4·2	2·8	7·0	...
,,	84	9	2·6	5·9	8·5	...
,,	50	10	...	...	3·0	5·2
,,	100	14	...	...	7·0	9·0
,,	130	26	0·8	5·6	6·4	8·0
,,	140	12	...	4·5	...	...
Bowstring Roofs.						
Manchester	50	11	...	...	9·6	...
Lime Street	154	26	...	4·9	...	...
Birmingham	211	24	...	7·3	11·0	...
Arched Roofs.						
Small Corr. Iron...	40	...	...	...	...	2·5
,,	60	...	...	...	...	3·5
Strasburg Rail	97	13	...	...	12·0	...
Paris Ex. Large ...	153	26	9·5	5·5	15·0	...
Dublin	41	16	3·4	7·3	10·7	...
Derby	81½	24	10·8	6·0	16·8	...
Sydenham	120	...	7·9	3·9	11·8	...
,,	72	...	8·4	2·9	11·3	...
St. Pancras	240	29⅓	7·4	17·1	24·5	...
Cremorne	45	14½	6·2	5·3	11·5	...

Occasional Load on Roofs.

Snow.

The weight of snow per cubic foot varies very much both with the temperature at which it falls and with the time during which it is allowed to remain undisturbed.

Snow, weighed at Chatham at different times in 1867, 1868, and 1869, varied in weight from 5½ lbs. to 10¾ lbs. per cubic foot.

In Canada it was found to weigh 14 lbs. 4 ozs. on falling, 21 lbs. 4 ozs., 24 hours after falling, temperature 8° Fahr. 28 lbs. 10 ozs. 72 hours after falling, temperature 30° Fahr. (*Sir H. James' Instructions for taking Meteorological Observations*, Appendix, p. 30.)

As snow in this country is not likely when there is a strong wind to accumulate on a roof to a greater depth than 6 inches, it does not seem necessary to make a greater allowance on this account than 5 lbs. per superficial foot of horizontal surface covered. It is necessary to increase this amount in countries like Canada where the snow storms continue for a longer period than they do in this country, and it must be remembered also, that in such countries snow is frequently very unequally distributed over the roof.

Wind.

The action of the wind on roofs is not accurately known.

The action of a horizontal wind on inclined surfaces may be deduced with considerable accuracy from Hutton's experiments (*Hutton's Tracts*, No. 36, *Whirling Machine*, Vol. III., p. 163), and the maximum horizontal force of the wind in this country during a number of years is known,* so that if the wind were always horizontal the Engineer would have precise data for designing his roof.

While, however, it is certain that the wind does not always blow horizontally, the limits of its deviation from the horizontal are not known, nor is it known to what extent its force when deflected from the horizontal, differs (if it does differ), from its measured horizontal force.

Under these circumstances the Engineer can only follow one of two courses.

1st. He may copy roofs which have been in existence a sufficiently long time to justify a belief that they are safe.

2nd. He may from the information available draw his own conclusions as to the amount of the wind force which it is advisable to provide against, and base his calculations upon them.

The assumption most commonly made by Engineers in this country, until comparatively lately, has been that suggested by Tredgold (*T.C.*, footnote, p. 98, edition 1840), which is, that the wind and snow together may be taken as equal to a uniformly distributed vertical load of 40 lbs. per square foot of horizontal surface covered.

This assumption not only neglects altogether the horizontal effect of the wind, but makes the vertical effect the same, whatever the pitch of the roof.

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* The following is an extract of a letter from the Astronomer to the Mersey Docks Estate to the Dock Secretary, in reply to a letter from myself requesting information as to the maximum force of the wind recorded at the Mersey Docks Observatory:

"*Bidston,*
"*Birkenhead, January 8th,* 1870.

"Our Anemometer records extend from the commencement of 1852.

"The largest pressures have occurred since we have been at Bidston. You will find by my Weekly Reports that on the 27th December, 1868, the pressure was 80 pounds on the square foot, and during one hour on the same day the horizontal motion of the air was 93 miles."

The greatest pressure in pounds per square foot recorded at the Royal Observatory, Greenwich, during the 5 years ending December, 1867, was 41 pounds.—H. W.

It is so manifestly erroneous that it is useless to base any mathematical calculations upon it; and if no more correct assumption could be made, it would be better to leave such calculations alone, and copy existing roofs.

For any span likely in the present day to be executed in timber, there are sufficient examples of roofs which have been in existence long enough to justify a belief in their permanent security, and it may be inferred that Tredgold's assumption, and the factor of safety allowed for the material have together been, in their case at any rate, sufficient to cover the stresses actually produced.

The same may perhaps be said of the smaller iron roofs.

The Engineer may safely imitate or copy the timber roofs and probably the smaller iron roofs.

With iron roofs of the largest size this is not the case. The introduction of large iron roofs is of comparatively recent date, the conditions of stress in them, owing to one end of the roof being left free to move as the metal expands or contracts, are different from the condition of stress in timber roofs, and it is not safe to assume, that, because Tredgold's assumption has been sufficient to give security to timber and small iron roofs, it will do so equally for large iron roofs, which may in no case as yet have been subjected to the maximum measured horizontal force of the wind.

For the smaller timber roofs, such as Couple and Collar beam roofs, it is unnecessary to make any calculation, as no material economy is likely to be effected by them (Tables, p. 197, and 199.) For the larger timber and small iron roofs of the ordinary form the Engineer may also adopt the dimensions already proved so far to be sufficient. (*H.A.S.H.*, p. 26, 27).

But even for the larger timber and small iron roofs, and certainly for large iron roofs it is worth while to consider the effect of the horizontal pressure of the wind.

It has been shown already that, during a period of 18 years in a part of England notoriously liable to heavy gales, the maximum horizontal wind force has reached 80 lbs. on the square foot, and that during one hour its horizontal motion was 92 miles, which by the ordinary rule for reducing velocity to pressure* is equivalent to a presure of $42\frac{1}{4}$ lbs. It has also been shown that at the Royal Observatory, Greenwich, the maximum horizontal pressure reached, on one occasion in 5 years, 41 lbs. on the square foot.

It may then fairly be supposed that the maximum horizontal wind pressure to which any roof is likely to be exposed for any length of time, does not exceed $42\frac{1}{4}$ lbs., and that the absolute maximum horizontal wind pressure to which a roof may be exposed momentarily and at long intervals of time does not even in the most exposed situations exceed in this country 80 lbs.

If in order to allow for the possibility of the wind being deflected into a direction normal to the roof, without losing any of its force, the horizontal wind force be taken at 50 lbs. per square foot, and if the

* The square of the velocity in miles per hour multiplied by ·005, gives the pressure in lbs. per square foot, or $v^2 \times .005 = P$.

intensity of the stress in the parts of the frame be limited in timber to $\frac{1}{5}$th of the ultimate strength, and in wrought iron to 6 and 5 tons per square inch in tension and compression respectively, the effect of the maximum pressure acting for a very short period of time, and very occasionally, will be in most instances to increase the stresses in the main bars from about 20 to 30 per cent., an increase which the material can bear without injury for a very short period, while the increase in the smaller bars, though much larger in proportion, is still absolutely small, and may be met by increased quantity of metal in them.

In some forms of roofs the effect may be to alter the nature of the stress, and bring tension bars into compression, but even here the compression will be small in comparison with the tensions which these bars are intended to bear, and the bars will be strong enough to resist it. This is illustrated in the example of a bowstring roof.

Action of a Horizontal Wind on an Inclined Surface.

The following is taken from *Unwin's Iron Bridges and Roofs*, p. 120, with the exception that 50 lbs. horizontal pressure is substituted for 40 lbs. which he considered sufficient to provide against.

Let P be the intensity of the wind pressure in lbs. per square foot, on a surface perpendicularly opposed to it; i the inclination of any plane surface to the wind's direction. Then the intensity of the pressure normal to the surface will be:

$$P_n = P \sin i^{1\cdot84 \cos i.-1}.$$

Thus

$$P_n = P_h \operatorname{cosec} i. = P\ (\sin. i)^{-1}.$$

Hutton's Tract gives

$$P_h = P \sin. i^{1\cdot84 \cos i.}$$

$$\therefore P_n = P \sin. i^{1\cdot84 \cos i.-1}.$$

The component of that pressure, parallel to the wind's direction, will have the intensity;

$$P_h = P \sin. i^{1\cdot84 \cos i.}$$

The component perpendicular to the wind's direction, the intensity

$$P_v = P \cot. i \sin. i^{1\cdot84 \cos i.}$$

That is, if the wind blows horizontally, P_h is the horizontal and P_v the vertical component of the pressure on the roof. Putting $P = 50$, the following values of the normal pressure and its components, for various inclinations of the roof surface to the wind, are obtained:

	Lbs. per square foot of surface.		
Angle of Roof.	P_n	P_v	P_h
5°	6·3	6·1	0·5
10	12·1	12·0	2·1
20	22·6	21·3	7·8
30	33·0	28·5	16·5
40	41·6	31·9	26·8
50	47·6	30·6	36·5
60	50·0	25·0	42·5
70	51·3	17·5	48·1
80	50·5	8·8	49·8
90	50·0	0	50·0

The inaccuracy in the values of P_n for 50°, 60°, and 70°, is caused by their being derived from an approximate formula.

Nature of the Wind Pressure.

The wind pressure is essentially a live load, varying from moment to moment both in force and direction, and often constituting absolutely a suddenly applied load. But it acts in precisely the same manner upon the anemometer by which its force is measured, and the effect of its sudden application is therefore included in the recorded pressure, that recorded pressure being the effect which would be produced by a dead weight of that amount.

Load to be Provided for.

In designing a roof it is recommended that the following loads should be provided for in this country.

1st. Weight of roof covering, framing, ceiling, or other weight.

2nd. 5 lbs. of snow per square foot of horizontal surface covered.

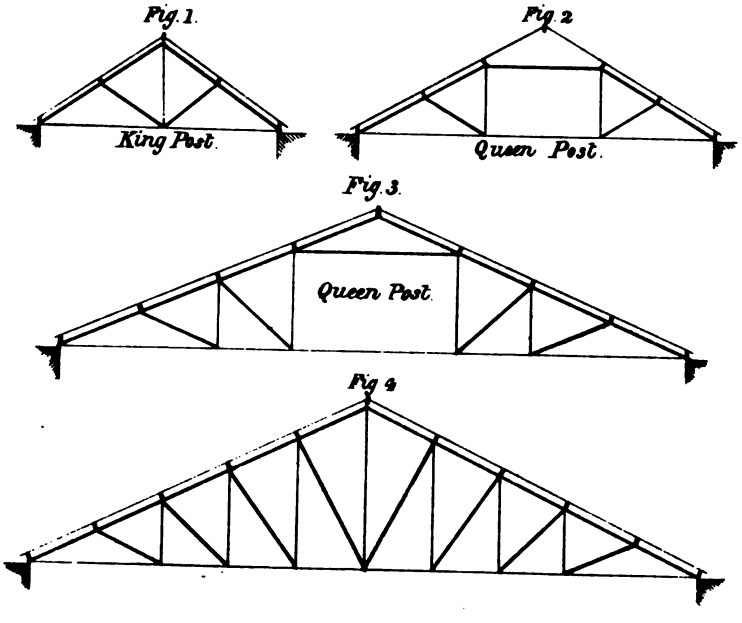
3rd. 50 lbs. of horizontal wind pressure per square foot of a vertical surface directly opposed to it, acting on either side of the roof.

In other countries the snow and wind pressure should be taken from local observations, and in sheltered situations the wind pressure may perhaps be reduced.

Distribution of the Loading.

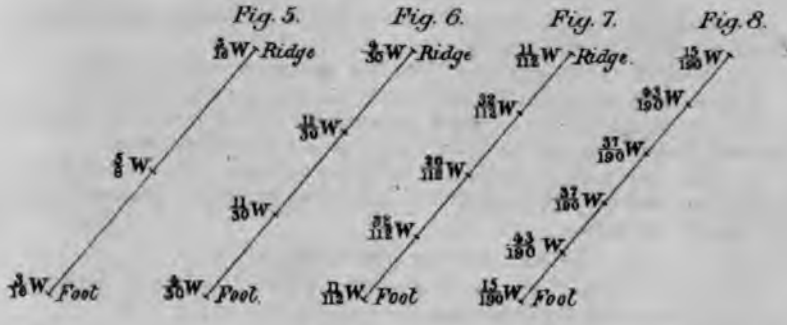
The distribution of the loading on the different points of the supporting framework depends upon the construction of the roof.

Figs. 1 to 4.



In roofs with two plane sides where the supporting framework consists of trusses at intervals (*Figs.* 1 to 4) connected by horizontal beams (purlins, pole-plates, ridge boards), resting on or very close to the joints of the trusses, and on which the common rafters immediately supporting the roof covering rest, the distribution of the load on the framework varies with the number of horizontal beams.

The common rafters being nearly in the condition of continuous beams, the part of the load borne on the points of support may be taken as shewn in *Figs.* 5 to 8, where W is the load on one side of the roof, and the distances between the points of support are equal.

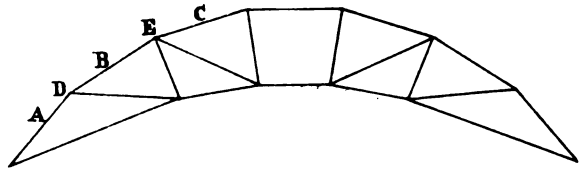


Or, each point of support may, without serious error be taken as carrying the load on a part of the roof, from centre to centre of two adjoining bays.

Where the roof covering rests directly on the horizontal beams, each such beam carries the load on a part of the roof from centre to centre of two adjoining bays, and where there are a number of purlins close together the same distribution may be taken.

In roofs of the bowstring form, the horizontal beams being supported at the joints of the truss, and the common rafters, where there are any, not being continuous, each beam supports the load on a part of the roof from centre to centre of two adjoining bays, *i.e.*, from A to B is supported at D, from B to C is supported at E (*Fig.* 9.)

Fig. 9.

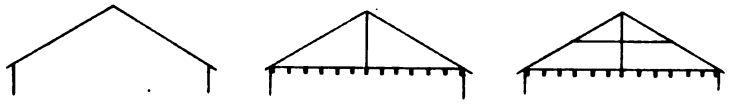


Consideration of Different Kinds of Roofs, Single Span, or Couple Roof.

This kind of roof (*Fig.* 10) may be used for permanent buildings where the span does not exceed 12 feet, the feet of the rafters being usually tied together by the joists which carry the ceiling. Where there are no ceiling joists or other tie, there is in every condition of the loading an inclined thrust on the walls. Where there are ceiling joists or other tie there is an inclined thrust on the walls, only when the wind is blowing on one side of the roof.

For spans of 12 feet and over, it is better construction, with a view to ensure an undeflected ceiling, to insert binders or transverse beams, tying the wall plates together at intervals of 10 feet to carry ceiling joists running longitudinally, the binders being supported in the middle by a light iron king rod (*Fig.* 11), and it is also better construction for spans of 14 feet up to 18 feet to insert a collar half-way up the rafter to stiffen it (*Fig.* 12).

Fig. 10. *Fig.* 11. *Fig.* 12.



The scantlings proper for this kind of roof, the timber being good Baltic fir, *i.e.*, the timber of the Scotch fir (*Pinus Sylvestris*), are given in Table III.

TABLE III.

Single span or couple roof, rise $\frac{1}{4}$th span, slated with countess slates on boarding.

Span from centre to centre of wall plate.	Rafters.	Ridge Board.	Ceiling Joists.	Remarks.
8 feet.	$2\frac{3}{4}$ × 2	7 × $1\frac{1}{2}$	4 × 2	
10 „	$3\frac{1}{4}$ × $2\frac{1}{4}$	7 × $1\frac{1}{2}$	5 × 2	
12 „	4 × $2\frac{1}{2}$	7 × $1\frac{1}{2}$	6 × 2	
14 „	$4\frac{1}{2}$ × $2\frac{1}{4}$	7 × $1\frac{1}{2}$	7 × 2	These spans should have a collar, but in that case the dimensions given in Table IV. for them may be used.
16 „	5 × $2\frac{1}{4}$	8 × $1\frac{1}{2}$	8 × 2	
18 „	$5\frac{1}{2}$ × $2\frac{1}{4}$	8 × $1\frac{1}{2}$	9 × 2	

When a binder is used, its deflection between the points at which it is supported should not exceed $\frac{1}{60}$ inch per foot run of the length between the points of support, *i.e.*, $\frac{1}{60}$ inch multiplied by half the span in feet, and it may be calculated as a beam supported at both ends, having a clear length equal to half the span, loaded with a uniformly distributed load, consisting of a length of the ceiling and ceiling joists 10 feet long and half the span broad, added to half of its own weight.

The thickness of walls of properly constructed houses is necessarily determined by other considerations than that of mere stability, such as keeping out the rain, heat or cold, and they are always more than sufficiently strong to resist the thrusts above mentioned with the given limits of span. It is therefore not considered necessary to give methods for determining the thickness of the walls.

If the load or strength of the timber is known to vary from those referred to in the tables, the Engineer can have no difficulty in allowing for the different circumstances, and for any special case it will be sufficient to treat the rafters as a beam supported at both ends, loaded with a uniformly distributed load consisting of the component, normal to the roof, of the weight of the roof covering and framework, snow and wind pressure, and having a deflection not exceeding $\frac{1}{60}$ inch per foot run of length of rafter.

Collar Beam Roof.

This kind of roof (*Fig.* 13) is used for spans over 12 feet and not exceeding 18 feet, the ceiling being carried on the underside of the rafters and collar, and the collar being sometimes suspended from the ridge board.

Fig. 13.

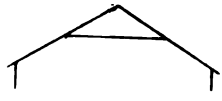


Fig. 14.

The ordinary position of the collar is half way up the rafter, it being intended to act as a support to the middle of the rafter as well as to carry the ceiling. So long as the wall does not yield to the thrust, it actually performs this duty, but if the wall yields, the collar is brought into tension, and the rafter, in addition to being deflected by the distributed load, is further deflected by the tension of the collar applied at the centre of its length.

The result is that the roof gets out of shape and assumes the form shown in *Fig.* 14.

It is not advisable therefore to use this form of roof on wooden huts, unless the plates are prevented spreading by a tie, or on any building where the walls cannot be depended upon to resist the thrust without yielding.

Where the walls are of the ordinary thickness, they are able to resist the inclined thrusts without a tie, and this roof has then the advantage of economy by lessening the height of the wall necessary for a given height of room.

The scantlings for the ordinary conditions may be taken from the following table, the timber being good yellow fir, *i.e.*, the timber of the *Pinus Sylvestris*.

TABLE IV.

Collar beam roof, collar half way up, without a tie, rise $\frac{1}{4}$th span slated with countess slates on boarding.

Span from centre to centre of wall plate.	Rafter.	Ridge.	Collar.	Remarks.
8 feet.	3 × 2	7 × $1\frac{1}{2}$	2 × 2	In these spans it is not necessary to use the collar except to carry the ceiling.
10 „	$3\frac{1}{2}$ × 2	7 × $1\frac{1}{2}$	$2\frac{1}{2}$ × 2	
12 „	4 × 2	7 × $1\frac{1}{2}$	$3\frac{1}{2}$ × 2	
14 „	3 × $2\frac{1}{4}$	7 × $1\frac{1}{2}$	$4\frac{1}{2}$ × 2	
16 „	$3\frac{1}{2}$ × $2\frac{1}{4}$	7 × $1\frac{1}{2}$	5 × 2	
18 „	4 × $2\frac{1}{4}$	7 × $1\frac{1}{2}$	$5\frac{1}{2}$ × 2	

If the load or strength of the timber is known to vary materially from those referred to in the Table, the Engineer can have no difficulty in allowing for the different circumstances, and for any special case the rafter may be treated as a beam supported at both ends, having a length equal to half that of the rafter between ridge and wall plate, loaded with a uniformly distributed load consisting of the component, normal to the roof, of the roof covering and framework, snow and wind pressure, and having a deflection not exceeding $\frac{1}{10}$th inch per foot run of half of the length of the rafter.

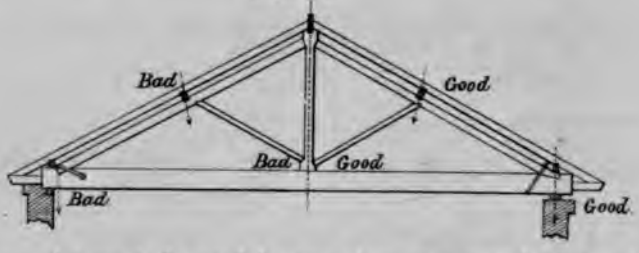
Framed Timber Roofs.

Framed roofs are used for spans exceeding 18 feet. In framed roofs of the best form, each part of the framing with the exception of the tie beam, where there is a ceiling, is under compression or tension only, and not under any transverse stress, except what is due to the weight of the part itself.

To ensure this, it is necessary that the purlins should be placed over the joints of the principal rafters with the braces or struts (*Fig.* 15), and it is better also, in order to avoid unnecessary transverse stress on the tie beam, that the junction of the principal and tie beam should rest on the wall (*Fig.* 15). The braces or struts should also be in-

clined downwards as much, and rest on the king post as low, as possible (*Fig.* 15). The ends of the tie beam should not be built into the walls, and ventilation should be secured all through the roof.

Fig. 15.



The form of truss to be used for any given span may be determined by considering, 1st. That it is not desirable to have the parts of the principals between the points of support so long that there is any tendency to buckle under the thrust. 2nd. That the distance apart of the purlins shall not be so great as to make the purlins and common rafters uneconomically large. 3rd. That the points of supports of the tie beam shall not be so far apart, as to render it uneconomically large. Experience shows that these three objects are attained by limiting the distance between the points of support on the principal rafter to about 8 feet.

In designing a roof for any span, it is only necessary to determine the length of the principal rafter, divide it into lengths not exceeding 8 feet each, and put in a strut at each of these divisions.

The king post roof would thus be applicable for a roof, with a principal rafter 16 feet long, *i.e.*, for a span of about 30 feet. The queen post would be applicable for a roof with a principal rafter 24 feet long, or about 45 feet span, after which roofs of the form given in *Figs.* 3 and 4 must be used.

The scantlings of the different parts in ordinary use are given in *Hurst's Tredgold's Carpentry*, *Hurst's Handbook*, p. 26, and in books of Engineering Formulæ.

The stresses on the different parts of a perfectly braced roof frame can be ascertained by diagram by Professor Clerk Maxwell's Method, already explained at p. 182, and applied in the following examples, a perfectly braced frame being one in which the bars composing the frame divide it completely into triangles. Thus *Figs.* 1 and 4 are instances of perfectly braced frames, while *Figs.* 2 and 3 are instances of imperfectly braced frames.

The only limit to the application of the method by which the following examples are dealt with is that there should be no superfluous braces, but it may be extended to cases where there are superfluous braces, by considering in each case which braces are superfluous and supposing them to be actually absent, as already done in Example III., *Bowstring Beams.*

EXAMPLE I.

Determine the maxima stresses, and the dimensions of the timbers in a king post truss of Scotch fir, having a ceiling on the tie, under the following conditions:

Clear span from centre to centre of wall plates, 28 feet.

Rise of Roof = $\frac{1}{4}$ span.

Distance apart of trusses, 10 feet from centre to centre.

Purlins, pole plate and ridge board placed directly over the joints of the truss; and the common rafters continuous from ridge to pole plate.

Roof covering, countess slates on $\frac{3}{4}$" boarding, on fir common rafters.

Snow, 5lbs. per square foot of horizontal surface covered.

Wind, horizontal, and having a force of 50 lbs. on the square foot of vertical surface directly opposed to it.

Remarks.

The distance apart of the trusses is about that in ordinary use for timber framed roofs, this distance allowing the trusses to rest directly on the piers between the windows of an ordinary building. They should not be placed over the window openings.

The rise or pitch of the roof is the usual rise given in this country to slated roofs for dwellings in ordinary situations covered with countess slates.

In very exposed situations this pitch is not enough to prevent the rain drifting up under the slates, and it is then better to give a greater pitch, or use larger and heavier slates.

Iron roofs frequently have a pitch of one-fifth of the span, but it is evident that the pitch should be governed by the covering, and the purpose for which the roof is required.

For railway stations and other roofs, where a slight occasional leak is of little importance, the smaller pitch may be used for the sake of economy.

For the pitch of various kinds of roof covering, see *Hurst's Tredgold's Carpentry*, p. 148.

Preliminaries.

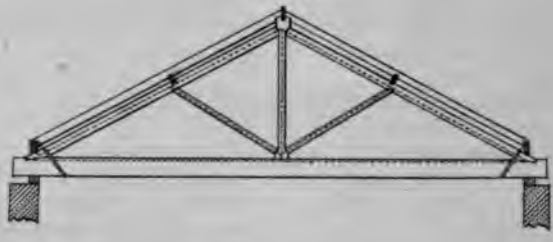
Weight of slating, boarding, common rafters and framing (p. 190), = $15\frac{1}{2}$ lbs.

Wind pressure, horizontal, 50 lbs. per square foot of vertical surface, which produces a normal pressure = 30 lbs. per square foot. (Table, p. 194).

Then constructing an approximate drawing of the roof truss (*Fig.* 16) the length of rafter is 16 feet, and the total weight on one truss is as follows:

Roof covering, &c. ...	2 × 16′ × 10′ × $15\frac{1}{2}$ lbs.	=	4,960 lbs.
Tie, ceiling, &c.	28′ × 10′ × $12\frac{3}{4}$ „	=	3,570 „
Snow	2 × 16′ × 10′ × $4\frac{1}{2}$ „	=	1,440 „
Wind (normal on one side)	16′ × 10′ × 30 „	=	4,800 „

Fig. 16.



The common rafters being continuous from pole plate to ridge, these forces are distributed as follows:

Roof covering, &c., $\frac{3}{32}$ = 465 lbs. at A and B; $\frac{5}{16}$ = 1,550 lbs. at C and D; $\frac{6}{32}$ = 930 lbs. at E.

Tie, ceiling, &c, $\frac{3}{16}$ = 669 lbs. at A and B; $\frac{5}{8}$ = 2,231 lbs. at F,

Snow, $\frac{3}{32}$ = 135 lbs. at A and B; $\frac{5}{16}$ = 450 lbs. at C and D; $\frac{6}{32}$ = 270 lbs at E.

Wind, $\frac{3}{16}$ = 900 lbs. at A or B; $\frac{5}{8}$ = 3,000 lbs. at C or D; $\frac{3}{16}$ = 900 lbs. at E, according as the wind is from the left or the right.

Construct a diagram to any convenient scale, consisting of a line joining the centres of the abutment of the principal rafters, and of other lines drawn along the centres of the other parts of the truss, as shown on *Fig.* 16, and on *Plate* I., *Figs.* 1 and 2. These lines will represent the lines of stress in the different parts of the truss.

The ordinary king post truss is not a perfectly braced frame, owing to the braces meeting above the tie, and when there is an unsymmetrical load, as when the wind is blowing on one side of the roof, a certain amount of transverse stress is produced on the lower end of the king post.

To simplify the diagram the braces may, however, be drawn as meeting on the tie without serious error, and in fact the braces should be brought lower down than they usually are in practice.

1st. To ascertain the stresses produced by the permanent load, and snow, all of which act vertically.

This load is symmetrically distributed, and therefore produces a reaction on each point of support equal to one half of its amount, *i.e.*, 4,985 lbs.

On any convenient scale of parts set off (*Plate* I., *Fig.* 3) these reactions R_1 and R_2, and the forces acting at the different points, which are as follows:

		Roof covering.		Tie, &c.		Snow.		
F_1	=	465	+	669	+	135	=	1,269 lbs.
F_2	=	1,550			+	450	=	2,000 „
F_3	=	930			+	270	=	1,200 „
F_4	=	1,550			+	450	=	2,000 „
F_5	=	465	+	669	+	135	=	1,269 „
F_6	=			2,231			=	2,231 „

It is known that if any number of forces act on a point, and are in equilibrium, they may be represented in magnitude and direction by the sides of a polygon drawn parallel to the direction of these forces taken consecutively. Thus if A, B, C, D, E represent in magnitude and direction (the direction by means of the arrows) 5 forces acting on the point O and in equilibrium, they will also be represented by a polygon (*Fig.* 18) in which the sides are in the same order as those forces, and the arrows point all the same way round the polygon. Hence if any number of forces act on a point, and all but two are known both in direction and magnitude, and the directions of those two are also known, their magnitude and nature can be ascertained by laying down the known forces, and completing the polygon by two lines parallel to the forces whose magnitude and nature are unknown.

Fig. 17.

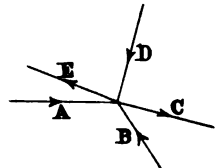
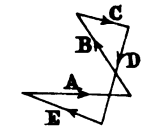


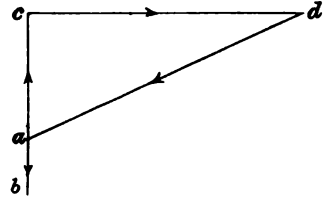
Fig. 18.



In the case now under consideration the forces acting at A (*Fig.* 1, *Plate* I.) are as follows: F_1 acting downwards, R_1 acting upwards, and two stresses, one along A F, the other along A C, the magnitudes and nature of which are unknown.

Make (*Fig.* 19 and *Fig.* 3, *Plate* I.) *a b* on any scale of parts to represent $F_1 = 1{,}269$ lbs., and make *b c* to represent $R_1 = 4{,}285$ lbs., from *c* draw *c d* parallel to A B (this being the force next in order to R_1) and from *a* draw *a d*, meeting *c d* in *d*. Mark the arrows on *c d*, *d a* as

Fig. 19.



shewn, *i.e.*, pointing the same way round the polygon as the forces F_1, R_1. Then the polygon of the forces acting at A is *a b c d a*, and as in this polygon the stress in A F represented by *c d* acts in the direction *c d*, *i.e.*, away from the point A, it must be a tension; while as the stress in A C represented by *d a* acts in the direction *d a*, *i.e.*, towards the point A, it must be a compression. Call these stresses A_1 and C_1 in *Fig.* 3, *Plate* I., and letter the parts of the truss to which they correspond, also A_1, C_1 (*Fig.* 1, *Plate* I).

Proceeding to the next point C. The forces acting at this point are F_2 acting downwards, the compression in A C just determined, acting upwards towards C and the two stresses in C F and E C. Make *e a* (*Fig.* 3, *Plate* I.) to represent F_2. From *d* draw *d f* parallel to C F, and from *e* draw *e f* parallel to E C, intersecting *d f* in *f*.

Then the polygon of the forces acting at C is *e a d f e*, in which the stress in C F, represented by *d f*, acts towards the point C, and is a compression; and the stress in E C, represented by *f e*, acts towards the point C, and is also a compression.

Call the stresses in C F and C E, B_1 and A_2 respectively, and letter the corresponding parts of the truss with the same letters.

Similarly at E. the forces are F_3, A_2, B_2, and A_3, and the polygon formed by them is *g e f h g*, in which B_2, represented by *f h*, is a tension, and A_3, represented by *g h*, is a compression.

Similarly at F, the forces are B_2, B_1, C_1, F_6, C_2, and B_3, the polygon is *h f d c k l h*; C_2 is a tension, and B_3 a compression.

Also similarly at D, the forces are F_4, A_3, B_3, and A_4; the polygon is *m g h l m*, A_4 is a compression, and if the diagram be rightly constructed, A_4 passes through the end of B_3, *i.e.*, through *l*.

Finally at B, the forces are F_5, A_4, C_2 and R_2, and the polygon is *n m l k n*.

It is evident, that all the external forces may be laid down at once, and the diagram may be commenced from each end, and be closed upon any bar in the truss

2nd. To ascertain the stresses produced by the wind blowing from the left.

This load is uniformly distributed over one side of the roof. It amounts on one truss to 4,800 lbs., and is distributed, supposing the wind to be from left, 900 lbs., at A and E (F_7 and F_9) and 3,000 lbs. at C, (F_8) (*Fig.* 2, *Plate* I.).

Laying these down to scale (*Fig.* 2, *Plate* I.) the force at A (F_7) is borne directly at A, and produces a reaction equal and opposite to itself. The force at C produces reactions parallel and opposite to its direction at A and B, equal respectively to $3{,}000 \times \frac{B\,x}{AB} = 2{,}000$ lbs. and $3{,}000 \times \frac{A\,x}{AB} = 1{,}000$ lbs.

The force at E produces reactions parallel and opposite to its direction at A and B, equal respectively to $900 \times \frac{B\,y}{AB} = 339$ lbs., and 900 lbs. $\times \frac{A\,y}{AB} = 561$ lbs.

The reaction due to the force F_8 at B may be found by construction (*U.I.B.lt*, p. 6).

Take (*Fig.* 20) two axes, making an angle with each other = C *x* A, (*Fig.* 2, *Plate* I.), O being the origin of co-ordinates. On any scale, make O*a* = A*x*, and O*y* = A B. On any scale of parts make O *b* = F_8. Complete the parallelogram *a b*.

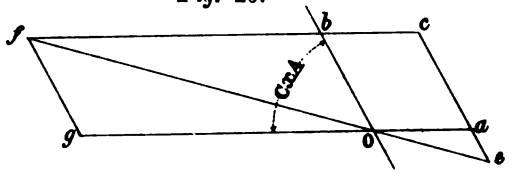
Produce *c b* to meet in *f* a line drawn through *g* parallel to O *b*.

Join *f* O, and produce it to meet *c a* produced in *e*.

Then *a e* represents the reaction at B, on the same scale that O *b* represents F_8, and deducted from F_8 gives the reaction at A.

Similarly for any other forces.

Fig. 20.



The sum of these reactions at A and B are respectively 3,239 lbs. (R_3) and 1,561 lbs. (R_4).

Then the forces acting at A are F_7 acting downwards, R_3 acting upwards, and the stresses in A F and A C.

On the principle already explained make *o p* (*Fig.* 4, *Plate* I.) to represent F_7, and *p q* to represent R_3, draw *q r* parallel to A F, and *o r* parallel to A C. Then *q r* represents the stress in A F, and is a tension; *o r* represents the stress in A C, and is a compression. Call these stresses C_1 and A_1, and mark the corresponding parts of the truss also C_1, A_1.

Similarly the forces at C are F_8, A_1, B_1, and A_2; the polygon is *s o r t s*: B_1 and A_2 are both compressions.

Similarly, the forces at E are F_9, A_2, B_2, and A_3; the polygon is *u s t v u*; B_2 is a tension, and A_3 a compression.

Similarly, the forces at F are B_2, B_1, C_1, C_2, and B_3; the polygon is *v t r q v*, whence C_2 is a tension, and B_3 having no value, because C_2 meets the end of B_2, is shewn to be a superfluous bar for this pressure.

Similarly, the forces at D are A_3, B_3 and A_4; but B_3 has been shewn to be under no stress; therefore A_4 must be equal and opposite to A_3, and is a compression, represented by *u v*.

Finally, the forces at B are A_4, C_2, and R_4 and the polygon (in this case a triangle) is *u v q u*, in which, if the diagram has been drawn correctly, *u q* accurately represents R_4.

Here again, the diagram might have been worked from each end, and closed on any bar in the truss.

When the wind is blowing from the right, the stresses just ascertained are inverted, similar stresses being produced on the right half of the truss to those just determined for the left, and *vice versâ*.

These stresses may now be tabulated, and the maxima stresses obtained, as given in the following table.

Table of Stresses. Compressions +. Tensions −.

Stresses due to	Roof covering, &c., and snow.	Wind on left.	Wind on right.	Maxima stresses.	Increase of stress with 80lbs. wind force.
	lbs.	lbs.	lbs.	lbs.	per cent.
on A C	+8,400	+4,700	+3,200	+13,100	21·5
„ C E	+6,200	+2,800	+3,200	+ 9,400	20·4
„ E D	+6,200	+3,200	+2,800	+ 9,400	20·4
„ D B	+8,400	+3,200	+2,700	+13,100	21·5
„ C F	+2,300	+3,600	Nil.	+ 5,900	36·6
„ F E	−4,400	−1,800	−1,800	− 6,200	17·4
„ F D	+2,300	Nil.	+3,600	+ 5,900	36·6
„ A F	−7,600	−5,200	−2,200	−12,800	24·3
„ F B	−7,600	−2,200	−5,200	−12,800	24·3

To calculate the dimensions.

1st. The principal (A C).

The length between the supports is about 90 inches, the width, in order to guard against buckling, should be $4\frac{1}{2}$ inches. (*Tension and Compression Bars*, p. 83.)

Let d=the depth in inches, and let $\frac{1}{4}$ of the bearing surface of the section be lost by the tenon.

Then, as in Example II. (*Tension and Compression Bars*):

$$\tfrac{3}{4} \times 4{\cdot}5 \text{ inches} \times d \times 1{,}000 \text{ lbs.} = 13{,}100 \text{ lbs.}$$

$$d = 4 \text{ inches nearly.}$$

and in order that it may not buckle downwards it should be made 4·5 inches square.

2nd. The strut (C F).

The length of this piece is about 90 inches, its width is not enough to prevent its having a tendency to buckle, and it will, from the way in which it is fixed, be more likely to buckle in a vertical plane than transversely to the plane of the truss. It will be better therefore to make its depth a little greater than its breadth.

Let d be the depth and 3 inches the breadth.

Then, as in Example V. (*Tension and Compression Bars*),

$$\frac{5{,}900}{3\,d} = 1{,}440 \div \left(1 + \frac{1}{250}\,\frac{90^2}{d^2}\right)$$

$$d = 4 \text{ inches nearly.}$$

3rd. The tie beam.

The length of the unsupported part of the tie beam is 14 feet, and it will be the same width as the principal.

In order that the ceiling may not crack, the tie beam should not deflect more than $\frac{1}{240}$ inch per foot run in length, *i.e*, $\frac{14}{240}$ inch in the centre of the length between supports.

Each half of the beam is in the condition of a beam fixed at one end, and supported at the other, stiffened by being under a direct tension, but it may be judicious to treat it as a beam supported only at both ends.

Then (*Deflection of Beams*, p. 36),

$$V = \frac{n''' W c^3}{E I}$$

where $n''' = \frac{5}{48}$.

W = total load on the length between supports = $\frac{3{,}570}{2}$ lbs.

c = half the length between supports = 84 inches.

E = Modulus of Elasticity of average fir = 168,000 lbs.

I = Moment of Inertia = $\frac{1}{12} \times \frac{2}{3} \times d^3$ (d being the depth).

V = $\frac{14}{240}$ inch,

whence $d^3 = 499{\cdot}2$

$d = 7{\cdot}9$ inches,

or adding on 3 inches for the parts cut away in framing, the total depth = 11 inches.

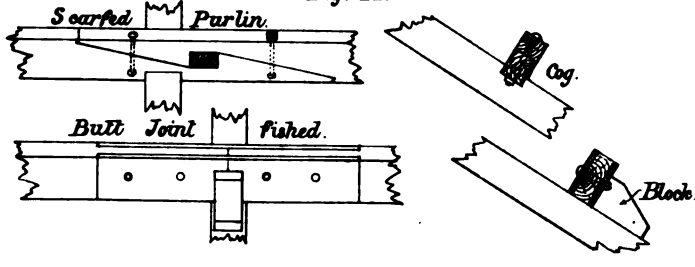
Even these dimensions are considerably less than those in ordinary use, yet it can hardly be doubted that they are sufficient.

4th. The King post. The tension on this is only 6,200 lbs., while the working resistance to tension of average fir is $\frac{1800}{6}$ lbs. per square inch, so that a section of 2·4 square inches would be enough. But this would not allow of the joints at the ridge and tie beam being properly made, and its dimensions must be determined by what is necessary to make a sound connection.

It may be advantageously replaced by an iron king rod.

If the purlins are properly fixed and jointed, so as to make them continuous without diminishing their section, they may be calculated as beams fixed at both ends, and loaded with a uniform load, consisting of the roof covering, their own weight, snow, and the normal pressure

Fig. 21.



of the wind; having a deflection limited to $\frac{1}{60}$ inch per foot run of their length. To secure this the purlins should not be scarfed, but butted on one to another and secured by a fish plate on each side, and it is better to block them than to cog them down on the principals.

The Pole plate, if properly fixed, not scarfed, may, if not blocked up

from the wall be dealt with in the same way as the purlins. If blocked up from the wall, the length of each part being reduced, the dimensions may be reduced accordingly.

The Ridge board need not be calculated, its dimensions being determined by what is necessary to make a sound connection at the head of the common rafters.

The Common rafters may be dealt with in the same way as the purlins.

EXAMPLE II.

Determine the maxima stresses in a king post truss of fir, having a ceiling on the tie, under the following conditions.

Clear span from centre to centre of wall plates, 28 feet.

Rise of roof = $\frac{1}{4}$th span.

Distance apart of trusses, 10 feet from centre to centre.

Purlins at distances of 2 feet along the principals, and the slate boarding carried directly upon them.

Roof covering, countess slates on $1\frac{1}{2}$ inch boarding.

Snow, 5 lbs. per square foot of horizontal surface covered.

Wind horizontal, and having a force of 50 lbs. on the square foot of vertical surface directly opposed to it.

Remarks.

The difference between this example and the last consists in the different mode of applying the load, the principal rafters now being under a transverse stress, as well as under a direct thrust.

Preliminaries.

Weight of roof covering, boarding and framing exclusive of tie = $14\frac{1}{4}$ lbs. per square foot of roofing surface.

Weight of tie, ceiling joists, and ceiling = $12\frac{3}{4}$ lbs. per square foot of horizontal suface covered.

Weight of snow = 5 lbs. per square foot of horizontal surface covered = 4·5lbs. per square foot of roofing surface.

Wind pressure, horizontal = 50 lbs. per square foot of vertical surface, which produces a normal pressure of 30 lbs. per square foot (Table, p. 194).

Then taking the diagram already constructed for Example I., the length of the rafter is 16 feet, and the total weight on one truss is:

Roof covering, &c. ...	2 × 16′ × 10′ × $14\frac{1}{4}$ lbs.	=	4,560 lbs.
Tie, ceiling, &c. ...	28′ × 10′ × $12\frac{3}{4}$,,	=	3,570 ,,
Snow	2 × 16′ × 10′ × $4\frac{1}{2}$,,	=	1,440 ,,
Wind (normal on one side)	16′ × 10′ × 30 ,,	=	4,800 ,,

As the purlins are distributed at equal distances along the rafter, and are not at the joints of the framing, these forces are distributed as follows:

Roof covering, &c. $\frac{1}{8}$ = 570 lbs. at A and B; $\frac{1}{4}$ = 1,140 lbs. at C, D, and E.

Tie, ceiling, &c. $\frac{3}{16}$ = 669 lbs. at A and B; $\frac{5}{8}$ = 2,231 lbs. at F.

Snow, $\frac{1}{8}$ = 180 lbs. at A and B; $\frac{1}{4}$ = 360 lbs. at C, D and E.

Wind, $\frac{1}{4}$ = 1,200 lbs. at A or B; $\frac{1}{2}$ = 2,400 lbs. at C or D; and 1,200 lbs. at E.

according as the wind is from the left or the right.

As in Example I, construct a diagram of the roof (*Plate* II., *Fig.* 1).

1st. To ascertain the stresses produced by the permanent load, and snow.

The reactions at the points of support are each 4,785 lbs.

The forces acting at the different points are as follows:

	Roof Covering, &c.		Tie, &c.		Snow.		
F_1 =	570	+	669	+	180	=	1,419 lbs.
F_2 =	1,140			+	360	=	1,500 „
F_3 =	1,140			+	360	=	1,500 „
F_4 =	1,140			+	360	=	1,500 „
F_5 =	570	+	669	+	180	=	1,419 „
F_6 =			2,231			=	2,231 „

And constructing the diagram as in the last case, the stresses in the different bars are obtained. (*Plate* II., *Figs.* 1 and 3.)

2nd. To ascertain the stresses produced by the wind blowing from the left.

The forces are distributed as shown in *Plate* II., *Fig.* 2, viz.:

F_7 = 1,200 lbs.
F_8 = 2,400 „
F_9 = 1,200 „

and in the same way as in Example I., the reactions they produce are

R_3 at A = 1,200 + 1,600 + 400 = 3,200 lbs.
R_4 at B = + 800 + 800 = 1,600 „

Constructing the diagram, as before, the stresses in the different bars are obtained, (*Plate* II., *Figs.* 2 and 4).

When the wind is blowing from the right, these last stresses are inverted, similar stresses being produced on the right half of the truss to those just determined for the left and *vice versâ*.

The stresses being tabulated, the maxima stresses are arrived at.

Stresses due to.	Roof covering, &c., and snow.	Wind on left.	Wind on right.	Maxima stresses.	Increase of stress with 60 lbs. wind force.
	lbs.	lbs.	lbs.	lbs.	per cent.
on A_1	+7,600	+4,000	+3,100	+11,600	20·7
„ A_2	+6,000	+2,400	+3,100	+ 9,100	20·4
„ A_3	+6,000	+3,100	+2,400	+ 9,100	20·4
„ A_4	+7,600	+3,100	+4,000	+11,600	20·7
„ B_1	+1,700	+3,000	Nil.	+ 4,700	38·3
„ B_2	−3,800	−1,400	−1,400	− 5,200	16·1
„ B_3	+1,700	Nil.	+3,000	+ 4,700	38·3
„ C_1	−6,800	−4,500	+2,000	−11,300	23·9
„ C_2	−6,800	−2,000	−4,500	−11,300	23·9

To calculate the dimensions.

1st. The principal.

The calculation for this differs from that in the last case, in that the distribution of the load causes the thrust on the principal to be unequally distributed over its section.

The total thrust produced is the same as if the loading had been collected at the joints of the truss; but the bending of the rafter under the distributed load creates a tension on one side and a compression on the other side of the neutral axis, the first of which relieves the compression caused by the direct thrust while the second increases it.

The greatest direct thrust is at the foot of rafter, but the greatest stress produced by the bending under the uniformly distributed load, (taking the foot of the rafter as supported only), as well as the greatest total intensity of stress, is at the point of junction of the principal with the first strut, and it is sufficient to determine the section at this point.

The thrust here (P) is 9,100 lbs.

The effect of the bending is to relieve the compression due to P on the upper side, and to increase it on the lower side, the rafter being in the condition of a beam uniformly loaded, fixed at the centre, and supported at the ends.

Let b be the breadth of the principal, and let d be the depth.

Let P be the thrust = 9,100 lbs. as just explained.

Let M = Moment of Flexure at the section of greatest stress, which is at the junction of the principal with the first strut and which in this case is $\frac{w c^2}{2}$ (*Fixed Beams*, p. 57).

Let c = half the length of the unsupported portion $= \frac{90''}{2} = 45''$.

Let w = weight per inch run in length of the principal = 16 lbs. (roof covering and snow) resolved normally to the roof + 30 lbs. wind = 15 + 30 = 45 lbs.

Let f_s = ultimate strength of fir in compression = 7,200 lbs. per square inch. (*Tension and Compression Bars*, p. 86.)

Let 5 = Factor of Safety.

Then the intensity of the compression produced by the bending will be obtained by equating the Moment of Flexure with the Moment of Resistance.

$$i.e.\ \frac{w c^2}{2} = \frac{f_o}{y_o} \mathrm{I} = \frac{f_o}{6} b d^2 \ (\textit{Stress in Beams}, \text{p. } 17).$$

where f_o = intensity of the stress at the extreme fibre on that side of the section where the elastic limit is first reached.

I = Moment of Inertia of the section.

y_o = distance from neutral axis to the outside of the section on the same side as f_o.

$$\text{whence } f_o = \frac{w c^2 \times 6}{2 b d^2} = \frac{45 \times 45^2 \times 6}{2 b d^2} = \frac{3 \times 45^3}{b d^2}$$

The intensity of the compression produced by the direct thrust $= \frac{9{,}100 \text{ lbs.}}{b d}$

These two together must not be greater than $\frac{f_s}{5} = \frac{7{,}200}{5}$

Whence, when condition is just fulfilled,

$$\frac{3 \times 45^3}{b\,d^2} + \frac{9{,}100}{b\,d} = \frac{7{,}200}{5}$$

As the principal cannot bend sideways owing to its being stiffened by the purlins, it may be made as narrow as 4 inches; whence $d = 7{\cdot}8$ inches.

If the principal were regarded as "fixed" at the lower end, which, for the reasons given at p. 60 (*Fixed Beams*), viz, that the timber is sure to shrink and become "imperfectly fixed," is not proper, the timber could be reduced in section, as the stress produced by the bending would be reduced, although at the same time the direct thrust would be larger, as the section of greatest total stress would then be at the foot of the rafter.

The principal would, therefore, be $7\frac{3}{4}$ or 8 inches by 4 inches in this case.

The other parts of the truss would be treated in precisely the same manner as in Example I.

EXAMPLE III.

Determine the maxima stresses, and the dimensions of the timber in a queen post truss of fir, having a ceiling on the tie, under the following conditions:

Clear span from centre to centre of wall plates, 40 feet.

Rise of roof = $\frac{1}{4}$ span.

Distance apart of trusses, 10 feet from centre to centre.

Purlins, pole plate, and ridge board placed directly over the joints of the truss, and the common rafters continuous from ridge to pole plate.

Roof covering, countess slates on $\frac{3}{4}$ inch boarding on fir common rafters.

Snow, 5 lbs. per square foot of horizontal surface covered.

Wind, horizontal and having a force of 50 lbs. on the square foot of vertical surface directly opposed to it.

Remarks.

The same remarks apply as in the last Example.

Preliminaries.

Weight of roof covering, boarding, common rafters, and framing, exclusive of tie beam (p. 190) = $15\frac{1}{2}$ lbs. per square foot of roofing surface.

Weight of snow = 5 lbs. per square foot of horizontal surface covered = $4{\cdot}5$ lbs. per square foot of roofing surface.

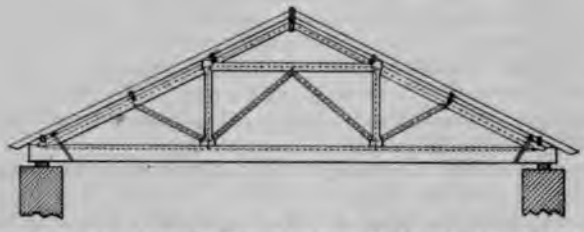
Wind pressure = 50 lbs. per square foot of vertical surface which produces a normal pressure = 30 lbs. (Table, p. 194).

Then constructing an approximate drawing of the roof truss the length of rafter is 23 feet, and the total weights on one truss are as follows:

Roof covering, &c. ...	... $2 \times 23' \times 10' \times 15\frac{1}{2}$ lbs.	$=7,130$ lbs.
Tie, ceiling, &c. ...	... $40' \times 10' \times 13$ „	$=5,200$ „
Snow	... $2 \times 23' \times 10' \times 4\frac{1}{2}$ „	$=2,070$ „
Wind, normal to one side	... $23' \times 10' \times 30$ „	$=6,900$ „

The common rafters in properly constructed roofs being continuous from pole plate to ridge these forces are distributed as follows (p. 196):

Fig. 22.



Roof covering, &c., $\frac{4}{60}=475$ lbs. at A and B, $\frac{11}{60}=1,307$ lbs. at C and D, $\frac{11}{60}=1,307$ lbs. at E and F, $\frac{8}{60}=951$ lbs. at G.

Tie, ceiling, &c., $\frac{4}{30}=693$ lbs. at A and B, $\frac{11}{30}=1,907$ lbs. at H and K.

Snow, $\frac{4}{60}=138$ lbs. at A and B, $\frac{11}{60}=380$ lbs. at C and D, $\frac{11}{60}=380$ lbs. at E and F, $\frac{8}{60}=276$ lbs. at G.

Wind, $\frac{4}{30}=920$ lbs. at A or B, $\frac{11}{30}=2,530$ lbs. at C or D, $\frac{11}{30}=2,530$ lbs. at E or F, $\frac{4}{30}=920$ lbs. at G, according as the wind is from the left or from the right.

Construct a diagram to any scale. (*Plate* III., *Fig.* 1.)

The ordinary queen post truss is an imperfectly braced frame, partly owing to the centre line of the braces meeting the centre line of the queen above the line of stress in the tie in most cases, and partly owing to the want of two braces such as B_3 and B_6, in the absence of which, when the wind is blowing, transverse stresses are produced on the principal rafters.

To simplify the diagram the braces may be drawn as meeting the queen on the tie without serious error, and in practice they should be brought down lower on the queen than they are.

1st. To ascertain the stresses produced by the permanent load and snow, all of which act vertically.

The load is symmetrically distributed, and, therefore, produces a reaction on each point of support equal to one half its total amount, *i.e.*, 7,200 lbs, $=R_1$ and R_2.

The forces are :

	Roof covering, &c.		Tie, &c.		Snow.		
F_1	= 475	+	693	+	138	=	1,306 lbs.
F_2	= 1,307			+	380	=	1,687 „
F_3	= 1,307			+	380	=	1,687 „
F_4	= 951			+	276	=	1,227 „
F_5	= 1,307			+	380	=	1,687 „
F_6	= 1,307			+	380	=	1,687 „

		Roof covering, &c.		Tie, &c.		Snow.		
F_7	=	475	+	693	+	138	=	1,306 lbs.
F_8	=			1,907			=	1,907 ,,
F_9	=			1,907			=	1,907 ,,

Then, by the principles laid down in Example I., the following polygons are formed, beginning from each end and closing on *l*, the junction of B_4, B_2, and B_7.

In laying down the external forces, it is convenient to begin at F_7 and follow the forces round to the left (*Plate* III., *Fig.* 3).

F_7 to F_1 all act downwards, and form one continuous line, R_1, the next external force, acting upwards is measured back on these forces, F_9 and F_8 acting downwards are again measured downwards in succession, and R_2 again upwards, completing the polygon of external forces.

The polygons are as follows:

at A, *a b c d a*, in which $A_1 = a\,d$ is a compression.
$C_1 = c\,d$ is a tension.
at C, *e a d f e*, in which $B_1 = f\,d$ is a compression.
$A_2 = e\,f$ is a compression.

As at E and H there are three unknown forces, it is necessary to pass them by and go to the point U, where there is only one known force, F_4, and two others unknown, and where the polygon becomes a triangle, *i.e.*, *g h k* in which $A_3 = h\,k$ and $A_4 = g\,k$ are compressions.

Returning to E, the polygon is *k h e f l k*, in which $B_2 = f\,l$ is a tension, $B_4 = k\,l$ is a compression.

At H, *l f d c m l*, in which $C_2 = m\,l$ is a tension, and B_3 is superfluous, C_3 running into the end of B_2 already determined.

Beginning at B, the polygons are as follows:

At B, *s r q p s*, in which $A_6 = q\,p$ is a compression, $C_3 = p\,s$ a tension.

At D, *p q n o p*, in which $A_5 = n\,o$, and $B_8 = o\,p$ are compressions.

At F, *o n g k l o*, in which $B_5 = k\,l$ is a compression, and $B_7 = o\,l$ is a tension.

This closes the polygon, and shows that in this condition of the loading B_6 is also unnecessary.

2nd. To ascertain the stresses produced by the wind blowing from the left.

The load is distributed over one side of the roof, and normal to it, in the proportions already stated, *i.e.*, when the wind is on the left, 920 lbs. at A and G (F_{10} and F_{13}) and 2,530 lbs. at C and E (F_{11} and F_{12}).

Laying these down to scale, the force at A (F_{10}) is borne directly at A, and produces a reaction equal and opposite to itself. The force at C produces reactions, parallel and opposite to its direction, at A and B, equal respectively to

$$2{,}530 \times \frac{B\,x}{A\,B} = 2{,}008 \text{ lbs., and } 2{,}530 \times \frac{A\,x}{A\,B} = 522 \text{ lbs.}$$

That at E produces reactions $= 2{,}530 \times \frac{B\,y}{A\,B} = 1{,}471$ lbs. at A,

and $2{,}530 \times \frac{A\,y}{A\,B} = 1{,}059$ lbs. at B.

That at G produces reactions $= 920 \times \frac{B\,z}{A\,B} = 345$ lbs. at A,

and $920 \times \frac{A\,z}{A\,B} = 575$ lbs. at B.

Then the total reactions are,

at A, 920 + 2,008 + 1,471 + 345 = 4,744 lbs.
B, 522 + 1,059 + 575 = 2,156 lbs.

Laying these down to scale, and working the diagram in the manner already described, the stresses are ascertained as shewn on *Plate* III., *Figs.* 2 and 4.

When the wind is blowing from the right, the stresses just ascertained are inverted, similar stresses being produced on the right half of the truss to those just determined for the left, and *vice versâ*.

The stresses may now be tabulated, and the maxima stresses obtained, as given in the following table:

Table of Stresses. Compressions +. *Tensions* −.

Stresses due to	Roof covering, etc., and snow.	Wind on left.	Wind on right.	Maxima stresses.	Increase of stress with 80lbs wind force.
	lbs.	lbs.	lbs.,	lbs.	per cent.
on A C	+13,200	+ 7,400	+ 4,300	+20,600	21·5
„ C E	+11,300	+ 5,600	+ 4,300	+16,900	19·8
„ E G	+ 1,400	+ 700	+ 1,100	+ 2,500	26·4
„ G F	+ 1,400	+ 1,100	+ 700	+ 2,500	26·4
„ F D	+11,300	+ 4,300	+ 5,600	+16,900	19·8
„ D B	+13,200	+ 4,300	+ 7,400	+20,600	21·5
„ C H	+ 1,800	+ 3,100	Nil.	+ 4,900	37·9
„ H E	− 2,700	Nil.	− 1,400	− 4,100	20·5
„ H L	Nil.	− 2,000	+ 1,900	+1,900 / −2,000	60·0
„ E L	+ 8,900	+ 5,500	+ 2,800	+14,400	22·9
„ L F	+ 8,900	+ 2,800	+ 5,500	+14,400	22·9
„ L K	Nil.	+ 1,900	− 2,000	+1,900 / −2,000	60·0
„ K F	− 2,700	− 1,400	Nil.	− 4,100	20·5
„ K D	+ 1,800	Nil.	+ 3,100	+ 4,900	37·9
„ A H	−11,700	− 8,300	− 2,900	−20,000	25·9
„ H K	−10,100	− 4,200	− 4,200	−14.300	17·6
„ K B	−11,700	− 2,900	− 8,300	−20,000	25·9

To calculate the dimensions.

1st. The principal.

The length between the supports is about 90 inches, and the width to guard against buckling should not be less than $4\frac{1}{2}$ inches.

Supposing the principal to be square, and calculating as in Example I., the scantling for the lower part AC, where the stress is greatest, is thus obtained,

$$\tfrac{3}{4} \times d^2 \times 1{,}000 = 20{,}600 \text{ lbs., whence } d = 5{\cdot}2 \text{ inches.}$$

It will not generally be worth while to reduce the principal in the part C E, but the stress in the part E G is so very much smaller that it is worth while to reduce the timber there.

The length of this part will then certainly be more than 20 times its least dimension, and it will have to be treated as a long beam under compression as in Example V., *Tension and Compression Bars*. Make it square, and let its side be d inches.

The stress is 2,500 lbs.

$$\text{Then } \frac{2{,}500}{d^2} = \frac{1{,}440}{1 + \frac{1}{250}\,\frac{90^2}{d^2}}$$

whence $d = 3$ inches nearly.

2nd. The strut (C H).

The length of this piece is about 90 inches, and its width will not be enough to prevent its having a tendency to buckle rather than to crush.

It will, as in the king post truss, be better to make the depth, d, a little larger than the breadth, b, bending being more likely to take place vertically than horizontally from the way in which the strut is fixed.

$$\text{Then } \frac{4{,}900}{bd} = \frac{1{,}440}{1 + \frac{1}{250}\,\frac{90^2}{d^2}}$$

Let $b = 3$ inches, then $d = 4{\cdot}6$ inches nearly.

The other struts are treated in a similar manner.

The dimensions obtained are as follows:

For the straining beam E L, if the breadth be the same as that of the principal, *i.e.*, 5·2 inches, the length being about 80 inches, and the stress 14,400 lbs., the depth by the same formula as the last is 5 inches nearly or the straining beam should be square.

3rd. The strut (H L).

If this be taken as square in section, the length being 110 inches, and the stress in compression being 1,900 lbs., the side of the square by the same formula will be 3 inches. This strut is sometimes in tension (*see* Table), so that its connection with the other parts must be of a kind to enable it to act as a tension bar, and it would be well to make it capable of taking up 30 per cent. of the increased stress with 80 lbs. wind pressure.

This strut and the strut L K may both be replaced by iron rods.

4th. The tie beam.

The tie beam is supported at four points, and the length of each part is about 13·5 feet. (*See* tie beam in Ex. I.) The deflection must not exceed $\frac{13{\cdot}5}{40}$, say $\frac{7}{20}$ inch.

The weight supported on each length is $\frac{5{,}200}{3}$ lbs.

The breadth will be the same as that of the principal.

Then (*Deflection of Beams*, p. 36)

$$V = \frac{n''' W c^3}{EI}$$

$$\text{or } \frac{1}{40} = \frac{5}{48} \frac{5{,}200 \times 80^3}{3 \times 1{,}680{,}000 \times \frac{1}{12} \times 5.2'' \times d^3}$$

$$d = 7{\cdot}6''.$$

and adding 3 inches for the parts cut away in the framing, the total depth is about 11 inches.

5th. The queen posts.

The tension on each of these is only 4,100 lbs., while the working resistance to tension of average fir is $\frac{13000}{5}$ lbs. per square inch, so that a section of less than 2 square inches would suffice.

This would not allow of the joints being properly made, and its dimensions must be determined by what is necessary for sound connections, and in most cases it might be advantageously replaced by an iron rod.

The Purlins, Pole Plate, Ridge Board, and Common Rafters, are to be dealt with as already described in Example I.

Composite and Iron Roofs.

The different forms of roofs constructed, either entirely of iron or of a mixture of timber, cast iron, and wrought iron, are shewn in *Plate* IV., and the spans for which they are ordinarily used are stated against them.

The distance apart of the trusses in such roof varies.

In the smaller roofs, *i.e.*, up to about 60 feet span, the distance apart is generally between 5 and 7 feet, though it is sometimes greater than this (Table, p. 191).

When the load is collected over the points of junction of the principal rafter and the struts, the principal rafter is subjected to compression only, and each length of it is in the condition of a long compression bar.

As the strength of a long compression bar decreases rapidly as its length increases, it is manifestly desirable that the distance between the struts should not be so great as to render it necessary to put in a large extra quantity of material in order to prevent the rafter bending under the thrust.

When the load is uniformly distributed along the principal rafter by

means of purlins, or iron laths at short intervals, which is the more usual construction in the smaller roofs, the principal rafter is under a transverse load as well as a thrust; and as the intensity of the stresses, produced by the transverse loads in beams of the same section uniformly loaded, increases as the square of the length, it is evident in this case, as in the last, that the distance between the struts should not be so great as to make the principal rafter uneconomically large.

The struts are always in the condition of long compression bars, and their length should, therefore, be kept down as much as possible, but as they have, in comparison with the principal, a small thrust to withstand, economy is more likely to be attained by reducing the length of each unsupported part of the principal than by reducing the length of the struts or their number.

It may also be more economical sometimes to put the trusses further apart so as to bring more load on each, because, the sections of the compression bars being thereby increased while their length remains the same, the tendency to bend under the thrust is reduced, and more resistance per square inch of section is got out of the iron.

In designing an iron roof, the Engineer should aim at having as many of his braces as possible in tension, and his compression bars as short as possible.

Forms of principal rafter.

In the smaller roofs these are usually either of timber, T iron, two angle irons back to back ⅂⌜ or two channel irons back to back][.

Timber principals have the serious objection of not expanding or contracting appreciably with changes of temperature, while the iron parts of the roof contract and expand considerably.

The effect of this is, that when the metal expands, the tension on the tie is relieved, and when it contracts it is increased. It may even happen, in the first case, that the tie rod ceases to act as such, and the feet of the principals spread until tension is again produced; while in the second case, the tension may be so intensified as either to bend the principal upwards or to overstrain the tie rod.

T iron is a convenient form for principals, the top being made as small as it conveniently can, and the vertical part comparatively thick. The vertical member affords a means of making easily a sound connection with the other parts of the truss.

Bulb T iron with a thin web, and a bulb, somewhat larger in section than the top table, gives a greater resistance with the same quantity of metal than T iron; but its cost is considerably greater, and it is not quite so easy to connect with the other parts of the truss.

The double angle ⅂⌜ and double channel][form afford great strength laterally, and having an intermediate space between the angles or channels, the connection with the other parts is as easy and at least as sound as with T iron. The angles or channels may be kept apart by timber filling, or if there is no other wood in the roof, with a view to affording extra security against fire, they may be kept together and

apart by small bolts passing through cast iron distance pipes, rivetted at the ends, thus :

Fig. 23.

Channel iron is, however, more costly than ⊤ iron.

It is to be observed that it is not possible with ⌐ and ⊤ iron to make the connections so as to produce uniform stress on the sections of the bar, and allowance should be made for this (p. 107).

Forms of struts.

In the smaller roofs these are either ⌐ or ⊤ irons, the latter form affording the greater resistance to bending.

A good and simple form is that of two flat bars swelled out in the centre, and kept together and apart by rivetted bolts passing through distance pipes. (*Fig.* 24.)

Fig. 24.

This form allows of the bolts or rivets being placed on the mean fibre of the bars composing the strut. (*Joints and Fastenings*, p. 102 to 107).

Forms of tension bars.

These are either single round bar, or one or more flat bars, fixed with the greatest dimension vertical. The latter have the advantage of deflecting less by their own weight.

Tables of dimensions of different parts of wrought-iron roofs of the ordinary forms are given in Books of Engineering Formulæ (see *H.A.S.H.*, p. 30 and 31), and similar tables are given in the *Engineers' and Contractors' Pocket Book* for 1866, p. 76.

It must not be forgotten that these roofs have been calculated for a smaller force of wind than it has been thought in this Paper advisable to provide against, and for a given distance apart of the trusses.

For larger iron roofs, the different parts being built up in the same manner as girders assume a great variety of forms.

In them, the load is collected at the points of junction of the principal rafter and struts by means of purlins, and the trusses are put at much greater distances apart (Table, p. 191).

These roofs are usually crossbraced (*Fig.* 29) the braces which cross each other being made to resist tension only, and the uncrossed braces to act both as compression and tension bars.

In drawing the stress diagram in these cases, it is necessary to

consider at each point which of the cross braces is superfluous, and to regard it as absent. This is illustrated in Example V.

For good construction all iron trusses should be fixed only at one end, and it is better to fix that end from which the heaviest gales are most likely to blow.

It is desirable to strengthen iron roofs against the effect of wind blowing at an angle with the axis of the roof by iron ties, passing obliquely up from the springing to the ridge, and secured on the under side of the battens or purlins.

There are no means of calculating these ties, but they need not be large.

EXAMPLE IV.

Determine the maxima stresses, and the dimensions of the iron in a wrought-iron roof truss of the form shewn in *Fig.* 25 under the following conditions:

Clear span	... 30 feet.
Rise of Roof	... $\frac{1}{4}$th span.
Distance apart of trusses ...	... 6 feet.

Roof covering, countess slates on $\frac{3}{4}$" boarding, resting on angle-iron purlins filled in with 3" × $1\frac{1}{2}$" fir.

Snow, 5 lbs. per square foot of horizontal surface covered.

Wind, horizontal, and having a force of 50 lbs. per square foot of vertical surface directly opposed to it.

Preliminaries.

Weight of covering, boarding, battens, laths, and framing=16 lbs. per square foot of roofing surface.

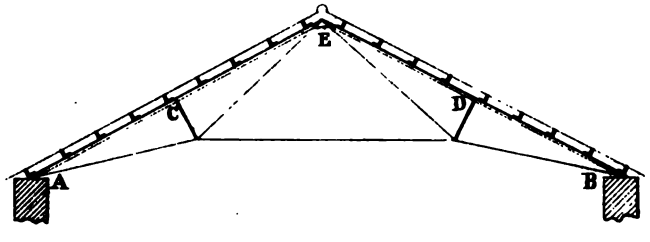
Weight of snow=5 lbs. per square foot of horizontal surface=$4\frac{3}{4}$ lbs. (nearly) per square foot of roof surface.

Wind pressure, horizontal=50 lbs. per square foot of vertical surface, which produces a normal pressure on a roof of this inclination=25 lbs. (nearly). (Table, p. 194.)

Then constructing an approximate diagram of the roof truss, the length of rafter is $16\frac{1}{4}$ feet, and the total weights on one truss are as follows:

Roof covering, etc. ...	$2 \times 16\frac{1}{4}' \times 6' \times 16$ lbs.	=3,120 lbs.
Snow	$2 \times 16\frac{1}{4}' \times 6' \times 4\frac{3}{4}$ „	= 926·25 „
Wind normal (on one side),	$16\frac{1}{4}' \times 6' \times 25$ „	=2,437·5 „

Fig. 25.



As the weight is not collected at the joints of the truss, the distribution of the load is as follows:

Roof covering, etc., $\frac{1}{8}$=390 lbs. at A and B; $\frac{1}{4}$=780 lbs. at C, D, and E.

Snow, $\frac{1}{8}$=116 lbs. at A and B; $\frac{1}{4}$= 232 lbs. at C, D, and E.
Wind, $\frac{1}{4}$=609 „ A or B; $\frac{1}{2}$=1,218 „ C or D.
$\frac{1}{4}$= 609 „ at E, according as the wind is blowing on one side or other of the roof.

1st. To ascertain the stresses produced by the permanent load only.

It is better not to add the weight of the snow to that of the roofing, &c., in iron roofs, because in some forms of iron roofs the vertical load produces in some of the bars stresses of a different nature from those produced by the wind, which stresses neutralize one another and relieve the stress on those particular bars, and as the snow is only an occasional load, it must not be counted upon as affording this relief, although, when the stresses produced by it and by the wind are of the same nature, provision must be made for the sum of the two.

Construct a diagram of the centre lines of the bars, *Plate* V., *Fig.* 1, lay down the forces as in previous cases, from which draw the stress diagram, *Plate* V., *Fig.* 1 in the manner already described.

2nd. To ascertain the stresses produced by the snow.

The stresses produced by the snow are obtained by a direct proportion between the weight per square foot of the roof covering, &c., and that of the snow, in this case as 16 lbs. is to 4·75 lbs.

3rd. To ascertain the stresses produced by the wind.

Let the left end of the roof truss be the fixed end, the other end being free to move so soon as the expansive or contractive force is sufficient to overcome the friction.

Then when these two are just in equilibrium, the support at the right end supplies only a vertical reaction, and in that case the whole of the horizontal component of the wind pressure has to be taken up by the point of support at the left end.

It becomes necessary, therefore, to consider four cases.

1st. Wind on left, temperature the same as at the time of last movement of the roof.
2nd. Wind on right, do.
3rd. Wind on left, expansive or contractive force just equalling the friction.
4th. Wind on right, do.

Case 1. The reactions at A and B, calculated as in previous cases, are respectively

at A=609+863+254=1,726 lbs.
B= 355+355= 710 „

Construct a diagram *Plate* V., *Fig.* 2, and lay down the forces as in previous cases, from which draw the stress diagram *Plate* V., *Fig.* 6.

Case 2. Here a similar diagram to the last is obtained, but inverted.

Case 3. In this case the whole of the horizontal component of the reactions has to be supplied by the left pier, and the forces and reactions are as shewn in *Plate* V., *Fig.* 3, from which the stress diagram, *Plate* V., *Fig.* 7, is obtained.

Case 4. Here the whole of the horizontal component of the reactions has to be supplied by the left pier, in the opposite direction to the last case; and the forces and reactions are as shewn in *Plate* V., *Fig.* 4, from which the stress diagram, *Plate* V., *Fig.* 8, is obtained.

These stresses may now be tabulated as follows, and the maxima stresses obtained:

Table of Stresses. (*Compressions* +. *Tensions* —.

Stresses due to.	Roof Covering, &c., only.	Snow.	Wind on left, roof in normal condition.	Wind on right, roof in normal condition.	Wind on left, end on point of motion.	Wind on right, end on point of motion.	Maxima stresses.	Increase of stress with 30lbs. wind force.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	per cent.
on A_1	+4,460	+1,390	+4,400	+2,440	+4,530	+2,530	+10,380	26·2
„ A_2	+4,200	+1,200	+4,400	+2,440	+4,530	+2,530	+ 9,930	27·3
„ A_3	+4,200	+1,200	+2,440	+4,400	+2,700	+4,440	+ 9,840	27·1
„ A_4	+4,460	+1,390	+2,440	+4,400	+2,700	+4,440	+10,290	25 9
„ B_1	+ 700	+ 210	+1,210	Nil.	+1,210	Nil.	+ 2,120	34·2
„ B_2	−1,900	− 620	−2,880	− 400	−3,000	− 300	− 5,520	32·6
„ B_3	−1,900	− 620	− 400	−2,880	− 610	−2,900	− 5,420	32·1
„ B_4	+ 700	+ 210	Nil.	+1,210	Nil.	+1,210	+ 2,120	34·2
„ C_1	−4,200	−1,290	−4,540	−2,050	−4,900	−1,300	−10,390	26·3
„ C_2	−2,380	− 700	−1,750	−1,750	−2,000	−1,100	− 5,080	23·6
„ C_3	−4,200	−1,290	−2,050	−4,540	−2,500	−3,900	−10,030	27·1

To calculate the dimensions.

1st. The principal rafter.

This would be made of uniform section throughout; the greatest thrust is at the foot of the rafter, but the greatest stress produced by the bending under the uniformly distributed load, and the greatest total intensity of stress is at the point of junction of the principal with the first strut; and it is sufficient to determine the section at this point.

The thrust here (P) is 9,930 lbs.

The effect of the bending is to relieve the compression due to P, on the upper side of the section at this point, and to increase it on the lower side, the rafter being in the condition of a beam uniformly loaded, fixed at the centre and supported at the ends.

The Moment of Flexure* at the section under consideration is $\frac{w c^2}{2}$, w being the normal pressure per inch run in length of the rafter, c half the length of the unsupported part of the rafter, *i.e.*, in this case, one-fourth of the whole length.

Equating this with the Moment of Resistance of the section,

$$\frac{w c^2}{2} = \frac{f_o I}{y_o}$$

$$\text{and } f_o = \frac{w c^2 y_o}{2 I}$$

Then on the tension (upper) side of the section of the principal,

$$\frac{w c^2 y_o}{2 I} - \frac{P}{\text{area of the section (S)}} \text{ must not exceed 6 tons} \quad (1)$$

and on the compression (lower) side of the section,

$$\frac{w c^2 y_o}{2 I} + \frac{P}{\text{area of section (S)}} \text{ must not exceed 5 tons} \quad (2)$$

Let it be determined to use a ⊤ iron for the principal.

As the lower part of the web is under compression over the strut, the web should not be made too thin.

As the rafter cannot buckle sideways, owing to its being stiffened by the purlins, the table of the ⊤ may be made as narrow as it can be, provided there is room for the angle irons to be rivetted on.

A width of $2\frac{1}{2}$ inches is sufficient for this, and this table should be thin, the neutral axis being then thrown further down the web of the ⊤, causing the iron to act with greater advantage.

Let the section shewn in *Fig.* 26 be selected, and for the sake of simplifying the calculations, let it be supposed to consist of rectangles.

Fig. 26.

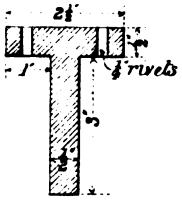
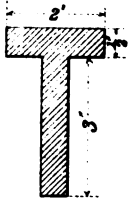


Fig. 27.



* The value of the Moment of Flexure at the junction of the principal with the first strut varies with the number of struts, and is as follows:

With one strut (*Fig.* 5, *p.* 196) $\frac{w c^2}{2}$

,, two struts (*Fig.* 6, *p.* 196) $\frac{2}{5} w c^2$

,, three struts (*Fig.* 7, *p.* 196) $\frac{3}{7} w c^2$

,, four struts (*Fig.* 8, *p.* 196) $\frac{8}{19} w c^2$

Then the section for purposes of calculation is as in *Fig.* 27, the rivet holes being deducted in the tension flange.

The bolt hole in the web, if properly filled, does not affect the strength of the section and may be left out of consideration.

The centre of gravity of this nett section is $\frac{13}{16}$ inch from the top edge of the table.

The Moment of Inertia of the section about a horizontal axis passing through its Centre of Gravity = 2·984.

Here w = normal wind pressure + component, normal to the roof, of weight of roof and snow, per inch run of rafter = 12·5 lbs. + 10·37 lbs. × cos. 21° 48′ (angle of inclination of roof) = 12·5 + 9·6 = 22·1 lbs.

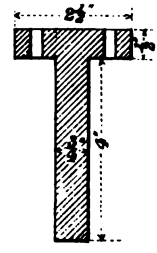
„ $c = \frac{196}{4} = 49$ inches.

Then (eq. 2) $\frac{22{\cdot}1 \times 49^2 \times \frac{13}{16}}{2 \times 3} + \frac{9{,}930}{2{\cdot}5} = 19{,}456 + 3{,}972$

= 23,428 which is much greater than 11,200 lbs. so that this section is far too small.

Try the section in *Fig.* 28.

Fig. 28.



Then the neutral axis is at $\frac{11}{16}$ inches from the top, I=7, and y_0 on the compression (lower) side = $\frac{11}{16}$ inches,

and $\frac{22{\cdot}1 \times 49^2 \times \frac{11}{16}}{2 \times 7} + \frac{9{,}930}{4} = 9{,}712 + 2{,}482 = 12{,}194$, which is 1,000 lbs. more than 11,200 lbs. so that this section is too small, and another should be tried.

2nd. The struts (B_1 and B_2).

These are to be treated as long compression bars hinged at both ends and subjected to a compression of 2,120 lbs. (*Tension and Compression Bars* and Example III).

The Tension Braces (C_1, C_2, C_3, B_2 and B_3).

These are to be treated in the manner already described in *Tension and Compression Bars.*

N.B. The joints must be calculated by the methods given in *Joints and Fastenings*, the bolts being put as nearly as possible on the intersection of the mean fibres of the different bars.

EXAMPLE V.

Determine the maxima stresses in a wrought-iron roof truss of the form shewn in *Fig.* 29, under the following conditions:

Clear span, 100 feet.

Rise of rib, 20 feet.

„ tie, 7 feet.

Distance apart of trusses, 16 feet.

Roof covering, half, zinc on $1\frac{1}{2}$" boarding, half, glass, carried on purlins 8 feet apart, each alternate purlin resting on the joints of the principal with the braces.

Snow, 5 lbs. per square foot of horizontal surface covered.

Wind, horizontal and having a force of 50 lbs. per square foot of vertical surface directly opposed to it.

The vertical braces are to be constructed to act either as tension or compression bars; the inclined internal braces are to be constructed as tension bars only, incapable of resisting any material compression without bending; the parts of the tie to be constructed as tension bars only, unless on investigating the stresses, it is found that some parts actually come into compression.

Fig. 29.

Preliminaries.

Construct a diagram of the lines of resistance of the bars composing the truss (*Plate* VI., *Fig.* 1).

Weight of roof covering and framing = 13 lbs. per square foot of roofing surface (Tables, pp. 190 and 191).

Weight of snow = 5 lbs. per square foot of horizontal surface covered

= on 1st and 7th parts of rib, 3·9 lbs. per sq. foot of roofing surface

= on 2nd and 6th „ 4·5 lbs. „ „

= on 3rd, 4th, and 5th „ 5 lbs. „ „

Wind pressure, horizontal = 50 lbs. per square foot of vertical surface, which produces a normal pressure varying on the different parts of the rib.

Then the length of each part of the rib being 16 feet, the total weights on one truss are as follows:

Roof covering, &c.	... 7 × 16′ × 16′ × 13 lbs.	=	23,296 lbs.	
Snow	100′ × 16′ × 5 „	=	8,000	„
Wind (normal on one side):				
at foot of rib	8′ × 16′ × 43 „	=	5,504	„
at junction of first strut	16′ × 16′ × 34 „	=	8,704	„
„ second „	16′ × 16′ × 22 „	=	5,632	„
„ third „	16′ × 16′ × 7 „	=	1,792	„

these pressures being obtained by considering that the wind acts normally to a tangent to the general curve of the bow, *i.e.*, in the direction of the radius, at the junctions of the rib with the other parts of the truss.

As the weight is collected at the joints of the truss, and as the rafters are not continuous over more than one part of the rib, the distribution of the load is as follows:

Roof covering, &c., 1,664 lbs. at A and H.; 3,328 lbs. at B, C, D, E, F, and G.

Snow, 500 lbs. at A and H; 1,080 lbs. at B and G; 1,220 lbs. at C and F; 1,280 lbs. at D and E, nearly.

Wind, as already given.

It is to be observed that the whole weight of the framing is taken as applied at the top of the truss. Strictly, half the weight of the braces, and the whole weight of the tie rod, ought to be collected at the points of junction of the tie and braces, but the error thus introduced is very small, while the construction of the diagrams is much simplified. In large roofs in actual practice, however, it may be well to distribute the weight accurately.

1st. To ascertain the stresses produced by the permanent load only; not adding the snow for the reasons given in Example IV.

Lay down the external forces (*Plate* VI., *Figs.* 1 and 6).

Determine the stresses on A_1, C_1 (*Fig.* 6).

As either B_2 or B_3 is a superfluous bar, and both are by the conditions to be capable of resisting tension only, it is necessary to determine which of the two, supposing the other to be omitted, would be a compression bar; and this bar being constructed only as a tension bar must be the superfluous bar. For if a compression comes on it, it bends, and the tension comes upon the other bar, on which, therefore, the safety of the truss depends.

If the stress diagram be constructed first with B_2 in and B_3 omitted, and then with B_3 in and B_2 omitted, it will be seen that B_3 is in compression with this load, and B_2 in tension. B_3 therefore is the superfluous bar.

In the same way it will be found that B_6, B_8, B_9, B_{11}, and B_{14} are all superfluous bars for the permanent load.

2nd. To ascertain the stresses produced by the snow.

Owing to the form of the roof, the snow is not a uniformly distributed load, and a diagram is required.

Constructing a diagram (*Plate* VI., *Figs.* 2 and 7), the stresses are determined, and the same bars are found to be superfluous as with the permanent load.

3rd. To ascertain the stresses produced by the wind on the left, roof in its normal condition as to temperature (see remarks on this subject, p. 222).

The forces are F_{17} =5,504 lbs.
" F_{18} =8,704 "
" F_{19} =5,632 "
" F_{20} =1,792 "

and the reactions are;

At A, 5,504 lbs., 7,137 lbs., 3,830 lbs., and 1,004 lbs., each acting parallel and opposite to the force which produces it.

At H, 1,567 lbs., 1,802 lbs., and 788 lbs., each acting parallel and opposite to the force which produces it.

Compound these reactions into two reactions, R_5 and R_6 (*Plate* VI., *Fig.* 3), and construct the stress diagram (*Plate* VI., *Fig.* 8).

The superfluous bars in this condition of the load are B_2, B_5, B_8, B_{11}, B_{14}.

4th. To ascertain the stresses produced by the wind on the right, roof in its normal condition.

These are ascertained by inverting the last diagram. The superfluous bars B_3, B_6, B_9, B_{12}, B_{15}.

5th. To ascertain the stresses produced by the wind on the left, expansive or contractive force just equalling the friction.

When this occurs, the whole of the horizontal component of the wind pressure has to be taken up by the left point of support; the forces and re-actions are as shewn in *Plate* VI., *Fig.* 4, from which the stress diagram, *Fig.* 9, is obtained.

The superfluous bars are B_2, B_5, B_8, B_{11}, B_{14}.

6th. To ascertain the stresses produced by the wind on the right, expansive or contractive force just equalling the friction.

In this case the forces and reactions are as shewn in *Plate* VI., *Fig.* 5, and the stress diagram, *Fig.* 10, is obtained.

The superfluous bars are B_3, B_6, B_9, B_{12}, B_{15}.

The stresses may now be tabulated, and the maxima stresses obtained as given in the following table:

Table of Stresses. Compressions +. Tensions —.

Stresses due to 1	Roof covering, &c. 2	Snow. 3	Wind on left, roof in normal condition. 4	Wind on right, roof in normal condition. 5	Wind on left, end on point of motion. 6	Wind on right, end on point of motion. 7	Maxima stresses. 8	Increase of stress with 80lbs. wind force. 9
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	per cent.
A_1	+ 24,200	+ 9,600	+ 29,000	+ 9,600	+ 30,400	+ 3,000	+ 64,200	28·4
A_2	+ 22,400	+ 9,000	+ 30,600	+ 10,800	+ 31,800	+ 4,600	+ 63,200	30·1
A_3	+ 23,000	+ 8,700	+ 23,800	+ 13,400	+ 25,000	+ 7,200	+ 55,700	26·7
A_4	+ 21,400	+ 8,400	+ 18,300	+ 18,300	+ 18,000	+ 12,200	+ 48,100	22·6
A_5	+ 22,000	+ 8,700	+ 13,400	+ 23,800	+ 13,400	+ 19,400	+ 54,500	26·2
A_6	+ 23,400	+ 9,000	+ 10,800	+ 30,600	+ 10,600	+ 26,400	+ 62,000	29·6
A_7	+ 24,200	+ 9,600	+ 9,600	+ 29,000	+ 9,400	+ 24,400	+ 62,800	27·7
B_1	— 1,400	— 700	+ 2,800	— 700	+ 2,400	+ 600	— 2,800 / + 1,400	15 / 120
B_2	— 1,200	— 600	Nil.	— 2,200	Nil.	— 1,800	— 4,000	33·0
B_3	Nil.	Nil.	— 8,800	Nil.	— 8,700	Nil.	— 8,800	60
B_4	— 800	— 600	+ 3,400	+ 300	+ 3,800	+ 1,400	— 1,400 / + 3,000	0 / 78
B_5	— 1,400	— 400	Nil.	— 3,600	Nil.	— 3,400	— 5,400	40
B_6	Nil.	Nil.	— 7,400	Nil.	— 8,800	Nil.	— 8,800	60
B_7	— 1,200	— 500	+ 2,400	+ 1,000	+ 2,200	+ 2,200	— 1,700 / + 1,200	0 / 120
B_8	Nil.	Nil.	Nil.	— 6,600	Nil.	— 6,500	— 6,600	60
B_9	Nil.	Nil.	— 6,600	Nil.	— 6,300	Nil.	— 6,600	60
B_{10}	— 1,200	— 500	+ 1,000	+ 2,400	+ 1,000	+ 3,800	— 1,700 / + 2,600	0 / 87·7
B_{11}	Nil.	Nil.	Nil.	— 7,400	Nil.	— 9,200	— 9,200	60
B_{12}	— 1,400	— 400	— 3,600	Nil.	— 4,000	Nil.	— 5,800	37·2
B_{13}	— 800	— 600	+ 300	+ 3,400	+ 300	+ 4,700	— 1,400 / + 3,900	0 / 72·3
B_{14}	Nil.	Nil.	Nil.	— 8,800	Nil.	— 9,400	— 9,400	60
B_{15}	— 1,200	— 600	— 2,200	Nil.	— 2,200	Nil.	— 4,000	33
B_{16}	— 1,400	— 700	— 700	+ 2,800	— 600	+ 4,000	— 2,800 / + 2,600	15 / 92·3
C_1	— 18,800	— 7,900	— 29,000	— 6,400	— 31,800	+ 8,000	— 58,500	32·6
C_2	— 19,500	— 7,800	— 22,200	— 6,300	— 24,800	+ 7,800	— 52,100	28·5
C_3	— 20,400	— 8,300	— 16,800	— 8,400	— 18,200	+ 6,000	— 46,900	23·2
C_4	— 21,400	— 8,400	— 11,400	— 11,400	— 13,100	+ 3,000	— 42,900	18·3
C_5	— 20,400	— 8,300	— 8,400	— 16,800	— 9,800	— 2,200	— 45,500	22·1
C_6	— 19,500	— 7,800	— 6,300	— 22,200	— 7,600	— 9,200	— 49,500	26·9
C_7	— 18,800	— 7,900	— 6,400	— 29,000	— 7,700	— 16,400	— 55,700	31·2

On examining this table, it is seen that in every condition of the load, the bars A_1 to A_7 are in compression, and therefore that condition of the loading, which produces the absolute maximum stress in each, must be provided for.

This will be produced when the snow is on the roof, and the wind

blowing with its maximum force, in one of the four possible cases. The maximum stress in each case is shewn in column 8.

Column 9 shews that, even when the wind blows with 80 lbs. horizontal force, the increase of the stress does not exceed 30 per cent. in any of these bars. They are in the condition of bars fixed at both ends, loaded with a central weight, as well as their own weight uniformly loaded, and under a thrust also.

It is to be observed, that the wall or other support at the fixed end of the roof must be able to resist the thrust produced on it, when the wind is blowing either from the left or the right with its full force of 80 lbs., and the rollers at the other end are on the point of motion, while the support at the roller end must be able to resist the thrust produced on it, when the wind is blowing either from the left or the right with its full force of 80 lbs., and the roof is in its normal condition as to temperature.

As regards the internal braces, the table shews that B_1, B_4, B_7, B_{10}, B_{13}, and B_{16}, *i.e.*, all the vertical bars, are sometimes in tension and sometimes in compression, and they must be designed accordingly.

These bars, as well as the other internal braces, would be subjected to a large increase of stress with an increase of the wind force, and it would be wise to provide against this by making the bars, especially those which have to act as compression bars, larger than is necessary for the 50 lbs. force.

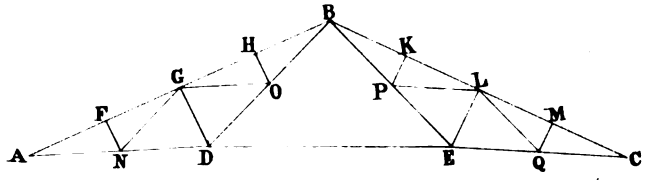
This can be done without any considerable extra cost, as the stresses are comparatively small in amount.

Taking the case of bar B_{13}, the compression on it with 50 lbs. wind force is 3,900 lbs., while with 80 lbs. wind force it becomes 6,720 lbs., which is still a comparatively small force to provide against; or taking the case of bar B_{14}, the tension, 9,400 lbs., is liable to an increase of 60 per cent., and if 30 per cent. of this is provided for, the total stress is still comparatively small, and the bar will never be strained more than the rest of the truss.

As regards the tie rods, C_1 to C_7, the table shows that in every condition of the load, the tension produced by the weight of the roof itself is more than sufficient to overcome any compression produced in any of them (see bars C_1 to C_4, Col. 7), and they are therefore always in tension.

In such a case as *Fig.* 8, *Plate* IV., which is an example of secondary trussing (*R.C.E.*, p. 546), *i.e.*, a frame having small secondary trusses,

Fig. 30.



ANG, GOB, BPL, LQC, forming part of larger primary trusses, ADB, CEB, the stress diagram cannot be drawn by Maxwell's method alone, because in arriving at the points G or D, working from A, or at the points L or E, working from C, there are three forces, the magnitudes of which are unknown. But the difficulty is easily overcome by employing Rankine's method of sections to determine the stress on one of them. In this particular case the stress on DE can be determined by supposing a vertical section to pass infinitely close to B, taking moments about B, and equating the moment of flexure of the forces acting on one side of that section, with the moment of resistance of the bars cut by the section.

The stress on DE being determined in this manner, and inserted on the stress diagram, the unknown forces at D and E, are reduced to two in number, and the diagram can be drawn.

BUTTRESSES, CHIMNEYS, WALLS.

There are two methods of designing structures, which are to be composed of a number of pieces or blocks having plane surfaces of contact, such as bricks or blocks of stone, and to be put together either with mortar or without it.

In the first method, supposing mortar to be used, its tenacity and adhesion are neglected, and it is regarded simply as a means of filling the interstices between the rough surfaces of the bricks or stones, so as to give them a true bearing on one another. In this case the structure depends for its stability solely on its weight and form.

In the second method the tenacity and adhesion of the mortar are taken into account, the bricks or stones being bound together in one piece, capable of exerting, independently of its weight, a greater or less resistance to a force applied transversely. In this case, the weight and form of the structure are not such as to make it stable, and the deficient stability has to be furnished by the resisting power of the material.

The first method is that usually adopted in calculating the dimensions of heavy structures, such as abutments for arches, or roof frames, retaining walls for earth or water, and tall chimneys, in which even with the greatest care unequal settlements are liable to occur, dislocating some of the joints, or fracturing the blocks themselves, and so destroying the tenacity or adhesion of the material and its power of resisting, independently of its weight, a transverse stress. In such structures failure would be very serious, perhaps causing loss of life.

The second method may be applied with safety in lighter structures, and especially in long enclosure walls, where the probability of unequal settlement is small, and the consequences of failure comparatively trifling.

It is shewn further on that unless the action of the wind on long walls is very much overrated, the majority of long enclosure walls actually constructed and standing, must frequently depend for their stability on the tenacity and adhesion of their material, their weight and form alone being insufficient to give them power to resist the pressure of the storms which occur every year in this country, and which must act sometimes at right angles to the face of the walls.

Stability of a Structure of Uncemented Blocks.

The theory of the stability of a structure of uncemented blocks is given by the Rev. Canon Moseley (*Engineering and Architecture*, p. 406, *Edition*, 1855), and also by Professor Rankine (*Applied Mechanics*,

p. 226, *et seq.* ; *Civil Engineering*, p. 218, 396, *et seq.*), who takes into consideration a question not investigated by the Rev. Canon Moseley, viz., the power of the material to resist crushing, and gives certain approximate rules for providing against this source of failure.

This theory may be stated as follows:

If the material of the blocks were not liable to crush, such a structure could fail, under any pressures that might be applied to it, only in two ways.

1st. By the overturning of some one or more of the blocks on their edges.

2nd. By the sliding of some one or more of the blocks on their surfaces of contact with the adjoining blocks, *i.e.*, on their bed joints.

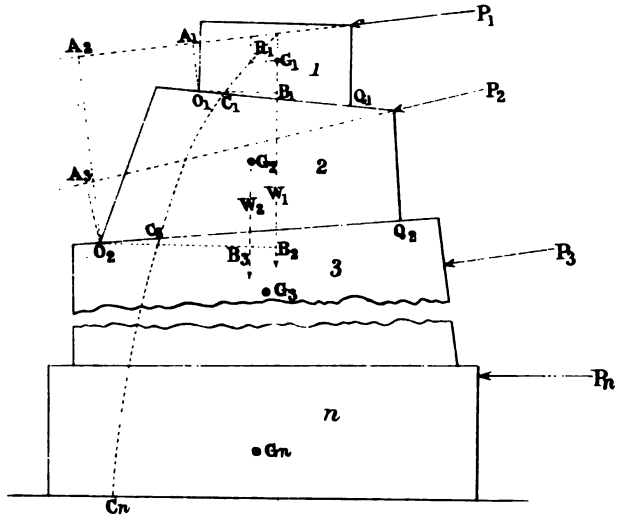
But as all material is liable to crush, a third way of failure, viz., by crushing, is possible, if the pressure be great enough and distributed over too small a surface, and this must always, except in very light structures of strong materials, occur when overturning is about to take place.

Failure by Overturning or Rotation.

Let *Fig.* 1 represent a structure of uncemented blocks, subjected to any number of pressures, P_1, P_2, P_3.....P_n.

Let G_1, G_2, G_3,.....G_n, be the centres of gravity of the blocks, 1, 2, 3,.....*n*, and let W_1, W_2, W_3.....W_n, be their weights.

Fig. 1.



Then commencing at block 1, the forces acting are P_1 tending to overturn the block about O_1, with a leverage O_1 A_1, and W_1 resisting the overturning with a leverage O_1 B_1.

Then if $P_1 \times O_1A_1$ be greater than $W_1 \times O_1B_1$, the block will overturn about O_1.

If $P_1 \times O_1A_1$ be equal to $W_1 \times O_1B_1$, the block is on the point of overturning.

If $P_1 \times O_1A_1$ be less than $W_1 \times O_1B_1$, the block is stable, its stability being greater in proportion as $W_1 \times O_1B_1$ is greater than $P_1 \times O_1A_1$.

It is easily shewn in the first case, that the resultant R_1 of P_1 and W_1 does not intersect the base $O_1 Q_1$ of the block, but falls outside of that base; in the second case, that R_1 passes through O_1; in the third case, that R_1 intersects the surface $O_1 Q_1$ in some point C_1 inside of the point O_1.

The first condition of stability of a single block is then, that the resultant of all the forces acting upon it must intersect its bed joint.

Considering now blocks 1 and 2 together, the forces acting are P_1, P_2, tending to overturn the blocks about O_2, with leverages O_2A_2, O_2A_3 respectively, and W_1, W_2, resisting the overturn with leverages O_2B_2, O_2B_3, respectively.

Then if $P_1 \times O_2A_2 + P_2 \times O_2A_3$ be greater than $W_1 \times O_2B_2 + W_2 \times O_2B_3$, the blocks will overturn about O_2.

If $P_1 \times O_2A_2 + P_2 \times O_2A_3$ be equal to $W_1 \times O_2B_2 + W_2 \times O_2B_3$, the blocks are upon the point of overturning about O_2.

If $P_1 \times O_2A_2 + P_2 \times O_2A_3$ be less than $W_1 \times O_2B_2 + W_2 \times O_2B_3$, the blocks are stable.

Here as before, it can be shewn that in order that the two blocks may not overturn about O_2, the resultant R_2 of P_1, P_2, W_1, W_2 must intersect $O_2 Q_2$ in some point C_2.

In the same manner the successive resultants, R_3.....R_n of all the forces acting above any bed joint must intersect that bed joint.

Joining all these points of intersection C_1, C_2.....C_n, a polygon is obtained which, if the joints were infinitely close together would be a curve, the intersection of which with any bed joint would give the position of the resultant pressure at that joint.

The points of intersection are called Centres of Pressure, or Centres of Resistance, and the curve is Moseley's Line of Resistance.

Now if the centre of pressure at any bed joint were to be actually at the edge of that joint, the whole pressure would be concentrated on that edge, which would in any ordinarily heavy structure be unable to bear the pressure, and would be crushed.

If the centre of pressure were even too near to the edge of the joint, there might be sufficient concentration of the pressure on the edge to crush the material, and failure might ensue.

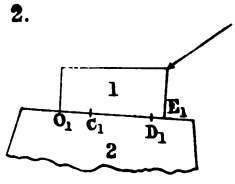
It becomes necessary therefore, to consider how far inside the outer edge the centre of pressure must be, in order to eliminate the risk of crushing.

Now it is commonly admitted as true that when two bodies having plane surfaces of contact, are pressed together, as the blocks 1 and 2 are pressed together in *Fig.* 1, the greatest pressure is at the outer edge O_1, and it decreases uniformly inwards.

The pressure, therefore, vanishes at a point D_1, *Fig.* 2, such that $O_1D_1 = 3\ O_1C_1$, and any breadth D_1E_1 beyond D_1 is under no pressure.

The total pressure is therefore distributed over the distance C_1D_1, and its maximum intensity, *i.e.*, at O_1, is equal to twice the intensity it would have, if it were uniformly distributed over O_1D_1.

Fig. 2.



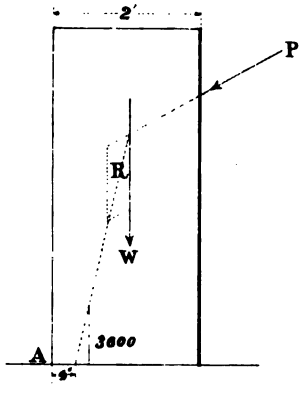
This maximum intensity must not exceed what the material can safely bear (*U.R.T.*, p. 197), *i.e.*, in ordinary structures from $\frac{1}{4}$ to $\frac{1}{8}$ (*U.R.T.*, p. 205) of the ultimate crushing strength of the material, according as the load to be borne is dead, or live. This is the second condition of stability.

In actual practice the factor of safety is seldom as low as 8.

Hence comes the following rule:

The resultant of all the forces, applied above any bed joint of a structure of uncemented blocks, including the weight of the structure above that bed joint, must intersect that bed joint sufficiently far inside its outside edge to ensure that the maximum intensity of the pressure shall not exceed what the material can safely bear; and this maximum intensity is equal to twice the normal component of the whole resultant pressure divided by the number of units in three times the distance from the outside edge to the centre of pressure.

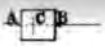
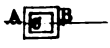
Fig. 3.



To illustrate this, let *Fig.* 3 represent the section, 1 foot long, of a wall 2 feet thick, subjected to a pressure P; let the resultant R of P and

W, the weight of the wall, intersect the base at a distance of 4" from its outside edge, A, and let its normal component be equal to 3,600 lbs.; then this normal pressure will be distributed over the outer 12" of the base of the wall, the intensity of the normal pressure per square inch at A will be $\frac{\text{lbs. } 3{,}600}{144''} \times 2 = 50$ lbs. per square inch, and the inner half of the base of the wall is free from pressure. Yet the wall is perfectly safe both against overturning and crushing.

Professor Rankine (*R.A.M.*, p. 227), says, "it appears that the exact determination of the limiting position of the centre of pressure at a plane joint is, strictly speaking, a question relating to the strength of materials (*i.e.*, to their resistance to crushing). Nevertheless, an approximation to that position can be deduced from an examination of the examples which occur in practice, without having recourse to an investigation founded on the theory of the strength of materials." And he then gives (*R.A.M.*, p. 229) the following approximate extreme positions for the centre of pressure C, when the pressure is distributed over the whole of the bed joint in which C has this position, decreasing uniformly from the side to which C is nearest, to nothing at the opposite side:

Figure of base, or bed joint under consideration.		Distance from outside edge.
Rectangle (solid)		$AC = \frac{1}{3}AB$.
Square (solid)		$AC = \frac{1}{3}AB$.
Eclipse (solid)		$AC = \frac{3}{8}AB$.
Circle (solid)		$AC = \frac{3}{8}AB$.
Hollow square (factory chimneys)		$AC = \frac{1}{6}AB$.
Circular ring (factory chimneys)		$AC = \frac{1}{4}AB$.

The simplest method of applying these principles in any particular case is to draw on a large scale the proposed structure, and to test it for stability, altering the dimensions and form until the required degree of stability is obtained.

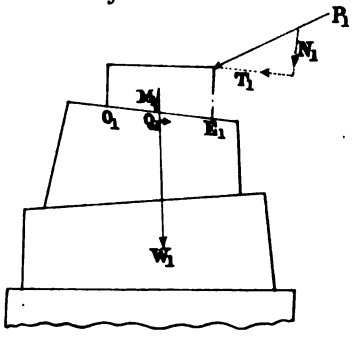
When the pressures applied to the structure, and the bed joints of the structure are horizontal, this can be done very easily by equations, as in the examples of chimneys given further on.

When either the pressures or the bed joints are inclined, it is generally simpler to work the problem by construction as in Example I.

FAILURE BY SLIDING.

Taking the same structure and same forces as in *Fig.* 1, and commencing as before with block 1, the force, P_1 (*Fig.* 4) may be resolved into two forces, one, N_1, normal to the bed joint, the other, T_1, parallel to the bed joint.

Fig. 4.



The force, T_1, acts to make the block slide along the block on which it rests in the direction $E_1 O_1$, while the force, N_1, acts to press the blocks together.

The weight, W_1, of the block may be resolved in a similar manner into two forces, M_1 and Q_1, Q_1 acting to make the block slide in direction $O_1 E_1$, and M_1 to press the blocks together.

Now by the laws of friction of solid bodies it is known that the force required to make one body resting on another commence to move along that other is a constant proportional to the force pressing the two bodies together, its value depending on the nature of the surfaces in contact, and not on their extent. The value of this constant (commonly called the "co-efficient of friction") has been determined experimentally for different materials (chiefly by General Morin), and it is given in Rankine's *Civil Engineering*, p. 172; Moseley's *Engineering and Architecture*, p. 155; *Useful Rules and Tables*, p. 180 and 186.

It is easy to show that the co-efficient of friction (f) is the tangent of the angle at which, if the lower body were slowly tilted up, the upper body would commence to move down it.

This angle is called the "angle of repose" of the two bodies.

It is further easy to shew that when motion is on the point of occurring, the angle, which the resultant of any forces acting upon the block, including its own weight, makes with a normal to the surface of contact, is equal to the angle of repose.

The forces acting to cause the block 1 to move along its base are therefore $T_1 - Q_1$, while the forces acting to resist the motion are $(N_1 + M_1) f$.

If $T_1 - Q_1$, be greater than $(N_1 + M_1) f$, the block will slide; if $T_1 - Q_1 = (N_1 + M_1) f$, the block is on the point of sliding; while if $T_1 - Q_1$ be less than $(N_1 + M_1) f$, the block is stable against sliding,

and the greater the excess of $(N_1+M_1)f$, the greater the stability of the block.

The same reasoning applies to every bed joint, and the second condition of mere stability is that the sum of the components parallel to the bed joint of all the forces acting above that bed joint, must not be greater than the sum of their components normal to that bed joint, multiplied by the co-efficient of friction of the material. In other words, the angle which the resultant of all the forces acting above any bed joint makes with the normal to that joint must not be greater than the angle of repose of the material.

It would appear desirable to allow a margin of security, and not to consider a structure safe, in which the direction of the resultant forces on any bed joint makes with the normal to that joint an angle, greater than that of which a certain fraction of the co-efficient of friction is the tangent, and it is proposed to fix this fraction at $\frac{2}{3}$.

There is seldom any difficulty in doing this, as in cases where sliding is at all likely to occur the bed joints can be inclined so as to reduce the angle.

In structures subjected to a single external pressure applied at or near the top, as in the case of house walls and abutments of arches, it is evident that the direction of the thrust makes a greater angle with the normal to the bed joint (supposing them to be parallel with one another) at the joint next below the point of application of the thrust, than at any other, and it is therefore only necessary to inquire into the stability of one joint.

Where, however, more than one pressure is applied, it may be necessary to inquire into the stability of several of the bed joints in this respect.

The conditions of stability of a structure of uncemented blocks are, then, as follows:

1st. The line of resistance must intersect every bed joint.

2nd. The angle which the resultant pressure at any bed joint makes with a normal to that joint must not be greater than the angle of which $\frac{2}{3}$ of the co-efficient of friction of the material is the tangent.

3rd. The point of intersection must fall sufficiently far within the outer edge to prevent any risk of crushing at that edge.

EXAMPLES.

It is considered unnecessary to give an example of a house wall, because the thickness of such walls is necessarily determined by other considerations than that of mere stability, such as keeping out the rain, heat, or cold, and they are therefore not calculated in practice, their thickness being determined by experience.

The cases of retaining walls and arch abutments, are dealt with separately, and no example is therefore necessary here.

The examples are consequently confined to buttresses, tall chimneys, and enclosure walls, the last being dealt with under the second method, *i.e.*, taking the tenacity and adhesion of the mortar into consideration.

EXAMPLE I.

A buttress of strong red brickwork weighing 112 lbs. per cubic foot, of the form and dimensions given in *Fig.* 5, and of a uniform width of 5 feet, has to sustain two inclined thrusts in the positions, and having the directions and magnitudes shown in the same figure; the tenacity and adhesion of the mortar to be neglected.

Inquire into its stability by determining:

1st. Whether the conditions of stability, laid down by Professor Rankine as regards overturning, are fulfilled at the joints AB, CD.

2nd. Whether the maximum intensity of the pressure at A and C is limited to what is necessary to the security of the buttress.

3rd. Whether the buttress is secure against sliding on the joints GH and EF.

Remarks.

The pressures P and P_1, are the thrusts due to a roof and an arch, and are intended to illustrate the method of working by construction.

As regards failure by overturning, the buttress is evidently not liable to overturn at any point above K, the pressure P having no leverage above that point.

It is also evident that the pressure P can never be deflected by any vertical weight, however great, into a vertical direction, and consequently the resultant of P, and of the weight must approach continually nearer to the outside of the buttress as the height below K increases, so that the buttress is more likely to overturn at the bed joint AB, than at any joint above it.

Owing to the increase of width at EF, it is not so liable to overturn about E as about A, and by similar reasoning to that just given, it is more liable to overturn about C than about any joint between C and E.

From the increase of width below CD, the buttress is not so liable to overturn about any point below C, as about C.

It is therefore only necessary to inquire into the stability as regards overturning and crushing at the joints AB and CD.

As regards sliding, the bed joints most liable to slide are GH, where the upper pressure is applied, and the joint immediately below the lower pressure, which may be supposed to be at EF, and if these joints are secure, all others are so.

Preliminaries.

Let P = upper pressure = 100 cwt.
„ P_1 = lower pressure = 160 cwt.
„ 1 cwt. = weight of brickwork per cubic foot.

1st. As regards overturning.

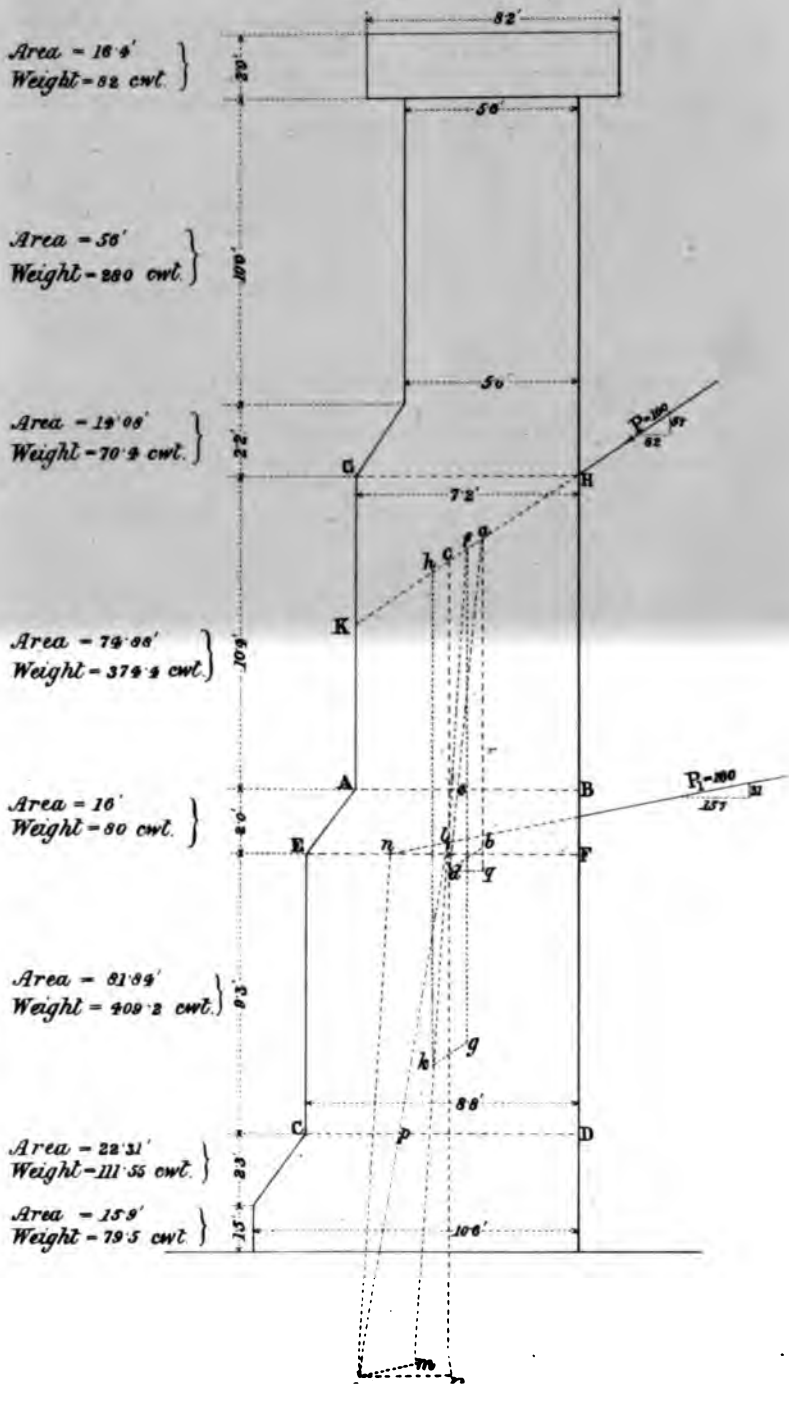
About AB.

The weight of the buttress above AB = 806·8 cwt.

The centre of gravity of this part of the buttress is in the vertical line *ab*.

Produce P to K, intersecting *ab* in *a*.

Fig. 5.


82'
Area = 16·4'
Weight = 82 cwt.
20'
56'
Area = 56'
Weight = 280 cwt.
100'
50'
Area = 19·08'
Weight = 70·2 cwt.
22'
P=180
57
52
G
H
7·2'
K
Area = 74·88'
Weight = 374·4 cwt.
109'
A
B
Area = 16'
Weight = 80 cwt.
20'
E
F
R=200
31
257
Area = 81·84'
Weight = 409·2 cwt.
83'
88'
C
D
Area = 22·31'
Weight = 111·55 cwt.
23'
Area = 15·9'
Weight = 79·5 cwt.
15'
106'

On any scale of parts make $ac = P = 100$ cwt., and on this same scale make $ad = 806{\cdot}8$ cwt.

Complete the parallelogram *ad*.

Then *ad* on the same scale is the resultant pressure on AB, and it intersects AB in *e*. A*e* being more than $\frac{1}{3}$rd AB, the condition of stability is fulfilled at this joint.

About CD.

The weight of the buttress above CD is 1,296 cwt., and the centre of gravity of this part of the buttress is in the vertical line *fg*.

On the same scale as before, make $fh = P = 100$ cwt., and $fg = 1{,}296$ cwt.

Complete the parallelogram *fk*. Then *fk* on the same scale is the resultant of P and of the weight of the buttress above CD.

Produce P_1, intersecting *fk* in *l*.

Make $lm = fk$, and on the same scale as before make $ln = P_1 = 160$ cwt.

Complete the parallelogram *lo*. Then *lo*, on the same scale, is the resultant of all the forces above CD, and it intersects CD in *p*, so that $Cp = \frac{1}{3}$rd CD.

Hence the condition of stability is just fulfilled. If it were not fulfilled the dimensions would be increased, and if it were more than fulfilled they might be diminished.

2nd. As regards the greatest intensity of the pressure.

At A.

Resolve the resultant *ad*, normal to and parallel to AB.

Then *aq* is the normal pressure on AB, and it equals 864 cwt. = 96,768 lbs.

It is distributed over a surface 5′ by 7·2′ = 36′ × 144 = 5,184 square inches.

It is very nearly uniformly distributed, the centre of pressure being nearly at the centre of AB.

The intensity of the pressure at A is therefore very little in excess of the mean pressure, *i.e.*, nearly 19 lbs. on the square inch.

The ultimate crushing resistance of strong bricks being 1,100 lbs. per square inch, there is a factor of safety of 57.

To determine "accurately" the intensity of the pressure at A.

Let N lbs. be the total normal pressure on one inch in width of the buttress.

„ *d* inches be the distance of the centre of pressure from A = AC, (*Fig.* 6).

„ *l* inches be the length of AB.

„ *n* lbs. be the pressure per square inch at A.

„ n_1 lbs. „ „ „ B.

The pressure being uniformly varying, decreasing from A, may be represented by a trapezoid, ABDE, in which AE = *n*, BD = n_1, AB = *l*, AC = *d*, and C is the centre of pressure.

Then the vertical through C passes through the centre of gravity of the trapezoid.

Fig. 6.

Let the verticals through F and G pass through the centres of gravity of the triangles EAB and EDB.

Then CF : CG :: Δ BDE : Δ EAB :: DB : AE :: $n_1 : n$.

and FG : CG :: $n_1 + n : n$.

Now $FG = \frac{1}{3} AB = \frac{1}{3} l$.

$CG = AG - AC = \frac{2}{3} l - d$.

$$N = \frac{n_1 + n}{2} \times l, \text{ and } n_1 + n = \frac{2N}{l}.$$

Whence $\frac{1}{3} l : \frac{2}{3} l - d :: \frac{2N}{l} : n$

$$\text{and } n_1 = \frac{2l - 3d}{l} \quad \frac{2N}{l},$$

$$\text{also } n_1 = \frac{3d - l}{l} \quad \frac{2N}{l}.$$

In this case $N = \frac{96{,}768}{60} = 1{,}612{\cdot}8$ lbs.

$l = 86{\cdot}4''$

$d = 38{\cdot}4''$

whence $n = 24{\cdot}9$ lbs., or about 6lbs. per square inch more than was given by the less accurate method.

This gives a factor of satety of about 44.

At C.

Resolve the resultant *lo*, normal to and parallel to C D.

Then *lr* is the normal pressure, and it equals 1,384 cwt. = 155,008 lbs.

It is distributed over a surface of 5 feet $\times$ 8·8 = 44 sq. feet = 6,336 sq. inches.

It is a uniformly varying pressure, increasing from nothing at D, to a maximum at C, and its intensity at C therefore is (p. 233), $\frac{155{,}008}{6{,}336} \times 2 = 48{\cdot}8$ lbs., which gives a factor of safety of more than 22.

The buttress is amply safe against failure by crushing.

3rd. As regards failure by sliding.

At GH.

Resolve the pressure P, normal to and parallel to GH.

Its components are 57 cwt. and 82 cwt.

The weight of the buttress above GH = 432·4 cwt.

The total force tending to produce sliding is therefore 82 cwt.

The total force resisting it is (57 + 432·4=) 489·4 cwt. × co-efficient of friction (which may be safely taken at ·5) = 244·7 cwt., or 3 times the first. The joint is therefore safe.

At EF.

Resolve the pressure P_1 normal to and parallel to EF.

Its components are 31 cwt. and 157 cwt.

The weight of the buttress above E F = 886·8 cwt.

The total force tending to produce sliding is 82 cwt. + 157 cwt. = 239 cwt.

The total force resisting it is (886·8 + 31 + 57) × ·5=587·4 cwt., or more than twice the first.

This joint is therefore safe.

Summary. The buttress is secure in every respect.

EXAMPLE II.

It is proposed to erect a round brick chimney, of which the diametral vertical section (without projections of cap, and without fire-brick lining) is shown in *Fig.* 7.

Enquire into its stability by determining:

1st. What pressure of wind the chimney is capable of bearing at each of the bed joints AB, CD, EF, when the position of the centre of pressure is at the limit given by Professor Rankine's rule (p. 234) *i.e.*, at a distance equal to $\frac{1}{4}$ of the diameter from the leeward edge of the diameter.

2nd. Whether the maximum intensity of the pressure at the leeward side of any bed joint is limited to what is necessary to the security of the chimney.

3rd. Whether the maximum intensity of wind in this country, taken at 55lbs. per square foot (see p. 242) normal to a vertical plane, is sufficient to make the chimney slide on any of these bed joints.

The axis of the chimney is assumed to be vertical.

The brickwork is supposed to be built with moderately quick setting mortar, such as blue lias lime mortar, so that its co-efficient of friction is that of dry brickwork (*U.R.T.*, p. 186; *R.A.M.*, p. 211). The tenacity and adhesion of the mortar is not to be taken into account.

Remarks.

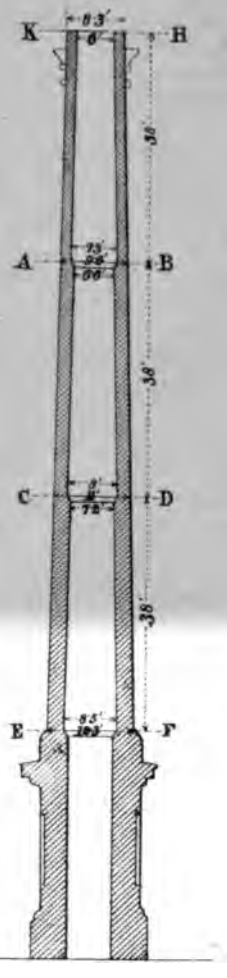
The only force tending to overturn a chimney, or to make it slide on any bed joint is the pressure of the wind.

It is usual to assume that the wind blows horizontally, and with the same force at all heights above the ground.

The effect of the slight batter on the outside of most chimneys may be left out of consideration without material error, the more especially as any such error is on the side of security.

As to the force of the wind, for which it is necessary to provide, there is some difference of opinion. In *Applied Mechanics*, p. 240, Professor Rankine says, "the greatest intensity of the pressure of the

Fig. 7.



wind against a flat surface directly opposed to it, hitherto" (1864) observed in Great Britain, has been 55 lbs per square foot, but at p. 149 it has been shown that the wind force measured in Great Britain has reached 80 lbs. on the square foot, and it would seem prudent to provide for this, at any rate in very exposed situations.

It will be seen that the chimney in this Example at its weakest joint is capable of resisting 47·6 lbs. wind pressure; that in Example III. is capable of resisting 44·3 lbs.; while the great chimney of St. Rollox quoted by Professor Rankine, is capable of resisting 55 lbs. at its weakest joint (*R.A.M.*, p. 640).

As regards failure by overturning; the chimney is more likely to fail by overturning at the bed joint AB than at any joint above it, because, considering the chimney as a cylinder, the moment of the wind pressure increases as the square of the depth from the top of the chimney, while the moment of stability of the chimney increases only as the weight, *i.e.*, as the depth from the top.

In the same way it is more likely to fail at CD than at any bed joint between it and AB, and similarly, it is more likely to fail at EF than at any bed joint between it and CD.

The additional breadth and thickness of the base render it self-evident that failure is not likely to occur below EF, if the chimney is safe above EF.

It is therefore only necessary to enquire into the stability of the three joints AB, CD, EF.

As regards the maximum intensity of the pressure at the outer edge of any bed joint, it is manifest that if the joints be considered horizontal, which they sensibly are, and the wind as horizontal, the total normal pressure on any bed joint is the weight of the chimney above it, and this is greatest for each section of the chimney at the joints where the thickness of the brickwork alters. It is therefore only necessary to examine into the liability of the material to crush at AB, CD, and EF.

As regards failure by sliding, the force tending to produce sliding at any bed joint, *i.e.*, the wind pressure, is simply as the depth of that joint below the top of the chimney, and so long as the thickness of the brickwork is uniform, the resistance to sliding is simply as that same depth. Therefore, above AB, one joint is as likely to slide as another, and if the joint AB is secure they all are.

Below AB, the weight increases faster than the wind pressure, and if AB is secure, all the bed joints below are also secure.

It is therefore only necessary to enquire into the stability of AB as regards failure by sliding.

The total pressure of wind against the side of a cylinder (*R.A.M.*, p. 240) is about one-half of the total pressure against a diametral plane of that cylinder.

This gives the cylindrical form a great advantage over the rectangular.

Octagonal chimneys may be treated nearly as cylindrical.

The chimneys given in Examples I. and II. are chimneys which have been actually constructed a few years since, and are now standing.

For the rules regulating the internal dimensions of furnace chimneys, see *Useful Rules and Tables*, p. 297; also, *Molesworth's Engineering Formulæ*, p. 91, 92.

NOTE.—The ordinary rule for the proportion of tall chimneys is

$$A = \frac{112\ HP}{\sqrt{h}} = \frac{12\ F}{\sqrt{h}}$$

where A is the least area of the chimney in square inches;
HP is the horse power of the boiler, *i.e.*, the number of cubic feet of water which it can raise from 212° into steam per hour.
h is the height of the chimney in feet, measured from the level of the fire grate.
F is the number of lbs. of coal consumed under the boiler per hour.
112 is a constant.

The constant 112 is Tredgold's, and is considered to be too small, and as a steam engine chimney cannot be too large, if it is provided with a damper, it is better to use a larger constant.

Mr. Robert Armstrong, C.E., in his *Chimneys, Furnaces and Fireplaces*, 1866, gives the following rule, which is obtained by using a constant varying from 280 to 210, and reducing the area to square feet;

$$\frac{\text{Area} \times \sqrt{h}}{2} = \text{HP},$$

which is sufficient for the worst kind of coal.

The constant may be reduced to 1·5 for better qualities of coal, and for the best coal somewhat lower.

There being two unknown quantities A and h in this formula, it is necessary to fix some relation between the two, and in *Molesworth's Engineering Formulæ*, p. 92, this is fixed, viz., that the diameter at the base should not be less than $\frac{1}{10}$ of the height.

It is simpler to work this Example by equations than by construction.

Preliminaries.

Let x lbs. = horizontal pressure of wind per square foot of flat surface.
„ 100 lbs. = weight of brickwork in mortar.

1st. As regards failure by overturning.

At the bed joint A B;

Mean external diameter = 8·95′
„ internal „ = 6·65′
Mean area of annulus = 28·18 square feet.

Weight of chimney above AB = $28{\cdot}18' \times 38' \times 100$ lbs. = 107,084 lbs.

Moment of stability at AB = $107{,}084 \times \frac{9.6'}{4}$ = 257,002 ft. lbs. (1)

Pressure of wind above A B = $x \times 8{\cdot}95' \times 38' = 340{\cdot}1\,x$ lbs.

The position of the centre of pressure is at 18·5 feet above A B, *i.e*, at the centre of figure of A B H K.

Moment of Pressure = $340{\cdot}1\,x \times 18{\cdot}5' = 6{,}292\,x$ ft. lbs. (2)

Equating (1) and (2)

$$6{,}292\,x = 257{,}002$$
$$x = 40{\cdot}8 \text{ lbs. per square foot.}$$

As this is the value of x for a wind acting on a flat surface directly opposed to it, while the chimney under consideration is cylindrical, the pressure which the chimney can bear at A B is 81·6 lbs. per square foot.

At the bed joint C D;

Mean external diameter between A B and C D = 10·3′
„ internal „ „ = 7·3′
„ area of annulus „ = 41·47 square feet.

Weight of chimney above CD = $107{,}084 + 41{\cdot}47' \times 38' \times 100$ lbs. = 264,670 lbs.

Moment of stability at C D = $264{,}670 \times \frac{11'}{4}$ = 727,843 ft. lbs. (3)

Pressure of wind above C D = $9{\cdot}65' \times 76' \times x = 733{\cdot}4\,x$ lbs.

The position of the centre of pressure is at 36·2 feet above CD.

Moment of Pressure = $733{\cdot}4\,x \times 36{\cdot}2' = 26{,}549\,x$ ft. lbs. (4)

Equating (3) and (4)

$$26{,}549\,x = 727{,}843$$
$$x = 27{\cdot}4 \text{ lbs. per square foot.}$$

Hence, as before, the chimney can resist 54·8 lbs. on the square foot above CD.

At the bed joint EF.

Proceeding as above;

$$\text{Moment of Stability at EF} = 458{,}530 \text{ lbs.} \times \frac{12{\cdot}3'}{4} = 1{,}493{,}927 \text{ ft. lbs.} \quad (5)$$

$$\text{„ Pressure „} = 1{,}174{\cdot}2\,x \text{ lbs.} \times 53{\cdot}3' = 62{,}585\,x \text{ ft. lbs.} \quad (6)$$

$$\text{and } 62{,}585\,x = 1{,}493{,}927$$

$$= 23{\cdot}9 \text{ lbs.}$$

and the chimney can resist 47·8 lbs. on the square foot above EF.

This chimney therefore falls a little short of the amount of stability considered necessary by Professor Rankine.

2nd. As regards the greatest intensity of the pressure.

At the bed joint AB.

If the bed joints be regarded as horizontal, which may be done without sensible error, the normal pressure is the weight of the chimney above the joint, *i.e.*, 107,084 lbs., and it decreases uniformly from one side to the other across the diameter of the chimney.

The area of the ring at AB is 30·3 square feet.

The mean intensity of pressure is therefore $\frac{107{,}084}{30{\cdot}3' \times 144} = 24{\cdot}5$ lbs.

and the maximum intensity, being twice this, is 49 lbs.

As strong red brick can bear 1,100 lbs. per square inch, there is a factor of safety of between 22 and 23.

At the bed joint CD;

The weight of the chimney above CD = 264,670 lbs.

The area of the ring at CD = 44·5 square feet.

The mean intensity of pressure $= \frac{264{,}670}{44{\cdot}5' \times 144} = 41{\cdot}3$ lbs.

The maximum intensity of pressure = 82·6 lbs.; and the factor of safety is between 13 and 14.

At the bed joint EF;

The weight of the chimney above EF = 485,830 lbs.

The area of the ring at EF = 61·6 square feet.

The mean intensity of pressure $= \frac{485{,}830}{61{\cdot}6' \times 144} = 54{\cdot}7$ lbs.

The maximum intensity = 109·4 lbs., and the factor of safety is 10 nearly.

There is no fear therefore of the bricks crushing at any of the joints.

3rd. As regards failure by sliding;

It is only necessary to enquire into the stability of the joint AB.

The maximum wind pressure above AB $= 340{\cdot}1' \times \frac{55}{2}$ lbs. $= 9{,}353$ lbs.

The weight of the chimney above AB = 107,084 lbs.

Taking the co-efficient of friction at ·5, the force to resist sliding = 107,984 × ·5 = 53,542 lbs. or 6 times as much as the force tending to produce it.

It has already been shown that if the joint AB is secure against sliding all others are at least equally so.

Summary.

It appears that this chimney, supposing that no reliance is placed on the tenacity and adhesion of the mortar, is safe against a wind pressure of 47·8 lbs. per square foot of a flat surface directly opposed to it.

If the wind pressure were to exceed 47·8 lbs. per square foot, the centre of pressure would approach somewhat nearer to the leeward side of the chimney, and the maximum intensity of the pressure would be increased. There is, however, even at the weakest joint, a factor of safety of 10, so that there would be little danger of the bricks crushing under the increased intensity of pressure due to the excess of the wind pressure.

To fulfil the conditions laid down by Professor Rankine, the diameter and thickness of brickwork at the bottom of the chimney shaft, should be somewhat increased, and still more for 80 lbs. wind pressure.

In regard to sliding the chimney is perfectly safe.

EXAMPLE III.

It is proposed to erect a square brick chimney, of which the side elevation and central vertical section (omitting projections of cap, and without the fire brick lining) are as shown in *Figs.* 8 and 9.

Enquire into its stability by determining,

1st. What pressure of wind the chimney is capable of bearing at each of the bed joints AB, CD, EF and GH, when the position of the centre of pressure is at the limit given by Professor Rankine's rule, (p. 234), *i.e.*, at a distance equal to $\frac{1}{6}$th of the width of the joint from the leeward edge of that joint.

2nd. Whether the maximum intensity of the pressure at the leeward side of any bed joint is limited to what is necessary to the security of the chimney.

3rd. Whether the maximum pressure of the wind, taken at 55 lbs. (see p. 242) per square foot normal to a vertical plane, is sufficient to make the chimney slide on any of the bed joints.

The same assumptions are made as in Example I.

Remarks.

The remarks as to pressure of wind, already made on Example I. apply equally to this Example.

As regards failure by overturning, the stability of the bed joints AB, CD, EF, GH, must be enquired into. Below GH the projecting buttresses will prevent overturning, if the conditions of stability are fulfilled at that joint.

As regards failure by crushing, it is necessary to enquire into the maximum intensity of pressure at the same joints.

As regards failure by sliding, it is sufficient to enquire into the stability of the joint AB.

Preliminaries.

Let x lbs. = horizontal pressure of wind per square ft. of flat surface.

„ 100 „ = weight of brickwork in mortar.

Fig. 8.

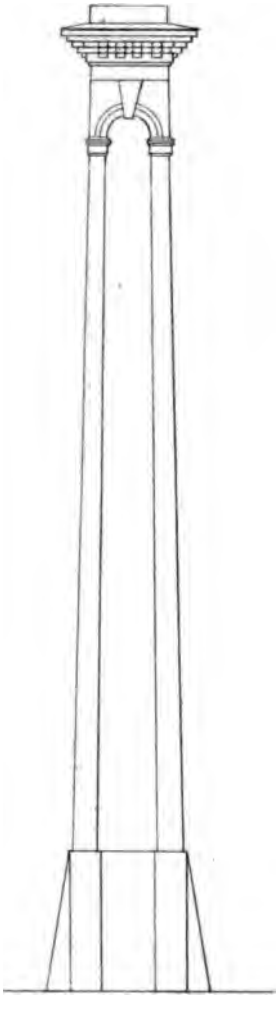
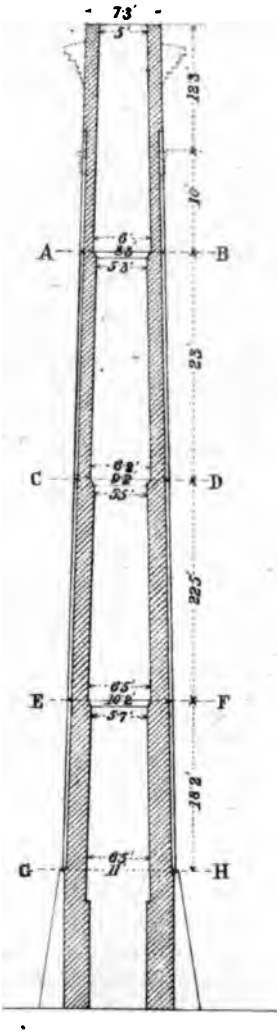


Fig. 9.



1st. As regards failure by overturning.

At the bed joint AB.

Mean external side above AB = 7·8′
„ internal „ „ „ = 5·5′
Mean area of hollow square = 30·6 square feet.
Weight of chimney above AB, deducting recess,
= (30·6 × 22·3 − 4 × 4′ × 11′ × $\frac{3}{8}$′) 100 lbs = 61,638.

Moment of stability at AB = 61,638 × $\frac{8·3′}{3}$ = 170,532 ft. lbs. (1)

Pressure of wind above AB = x × 7·8′ × 22·3′ = 174 x.
The position of the centre of pressure is at 10·9 ft. above AB.
Moment of pressure = 174 x × 10·9′ = 1,897 ft. lbs. (2)

Equating (1) and (2), $x = \frac{170,532}{1,897}$ = 89·8 lbs.

At the joint CD.

Mean external side between AB and CD = 8·75′
„ internal „ „ „ „ = 5·75′
„ area of hollow square „ „ = 43·5 square feet.
Weight of chimney above CD, deducting recess = 61,638 + (43·5′ × 23′ − 4 × 4·5 × 23′ × $\frac{3}{8}$′) 100 = 61,638 + 84,525 = 146,163 lbs.

Moment of stability at CD = 146,163 × $\frac{9·2′}{3}$ = 448,233 ft. lbs. (3)

Pressure of wind above CD = x × 8·25′ × 45·3′ = 373·7 x.
The position of the centre of pressure is at 21·1 ft. above CD.
Moment of Pressure = 373·7 x × 21·8′ = 8,147 ft. lbs. (4)

Equating (1) and (4), $x = \frac{448,233}{8,147}$ = 55 lbs.

At the joint EF.

Mean external side between EF and CD = 9·7′
„ internal „ „ „ „ = 6·0′
„ area of hollow square „ „ = 58·1 square feet.
Weight of chimney above EF, deducting recess = 146,163 + (58·1 × 22·5′ − 4 × 5′ × 22·5′ × $\frac{3}{8}$′) 100 = 146,163 + 113,850 = 260,013 lbs.

Moment of Stability = 260,013 × $\frac{10·2′}{3}$ = 884,044 ft. lbs. (5)

Pressure of wind above EF = x × 8·75′ × 67·8′ = 593·3 x.
The position of the centre of pressure is 32 feet above EF.
Moment of Pressure = 18,986 x ft. lbs. (6)

Equating (5) and (6),

$$x = \frac{884,044}{18,986} = 46·6 \text{ lbs.}$$

At the joint GH.

Mean external side between GH and EF = 10·6′
„ internal „ „ „ „ = 6·1′
„ area of hollow square „ „ = 75·2 square feet.

Weight of chimney above GH, deducting recess = 260,013 + (75·2 × 18·2′ − 4 × 5·5′ × 18·2′ × $\frac{3}{8}$′) 100 = 260,113 + 121,849 = 381,862 lbs.

Moment of Stability = 381,862 × $\frac{11}{3}$ = 1,400,161 ft lbs. (7)

Pressure of wind above GH = x × 9·15′ × 86′ = 786·9 x.
The position of the centre of pressure is 40·1 feet above GH.
Moment of pressure = 31,555 x ft. lbs.

Equating (7) and (8),

$$x = \frac{1,400,161}{31,555} = 44\cdot3 \text{ lbs.}$$

2nd. As regards the greatest intensity of the pressure.

At the bed joint AB.

Normal pressure = 61,638 lbs.
Area of bearing surface at AB = 32·9 square feet.

Mean intensity of pressure = $\frac{61,638}{32\cdot9′ \times 144}$ = 13 lbs.

Maximum intensity at leeward side = 26 lbs., which gives a factor of safety for strong brick of $\frac{1,100}{26}$ = 42.

At the bed joint CD.

Normal pressure = 146,163 lbs.
Area of bearing surface at CD = 46·2 square feet.

Mean intensity of pressure = $\frac{146,163}{46\cdot2′ \times 144}$ = 22 lbs.

Maximum intensity = 44 lbs.
Factor of safety = 25.

At the bed joint EF.

Normal pressure = 260,013 lbs.
Area of bearing surface at EF = 61·8 square feet.

Mean intensity of pressure = $\frac{260,113}{61\cdot8′ \times 144}$ = 29·2 lbs.

Maximum intensity = 58·4 lbs.
Factor of safety = 19 nearly.

At the bed joint GH.

Normal pressure = 381,862 lbs.
Area of bearing surface = 78·8 square feet.

Mean intensity of pressure = $\frac{381,862}{78\cdot8′ \times 144}$ = 33·6 lbs.

Maximum intensity = 67·2 lbs.
Factor of safety = 16·4.

So that the chimney is safe against crushing with any wind pressure, that it is capable of withstanding within Professor Rankine's limit, *i.e.*, 44 lbs. per square foot.

3rd. As regards failure by sliding.

It is only necessary to enquire into the stability of the joint A B.
The maximum wind pressure above A B = 174' × 55 = 9,570 lbs.
The weight of the chimney above A B = 61,638 lbs.
Taking the co-efficient of friction at ·5, the force to resist sliding = 61,638 × ·5, = 30,819 lbs., or three times as much as the force tending to produce it.

Summary.

It appears that this chimney, supposing no reliance to be placed on the tenacity and adhesion of the mortar is safe against a wind pressure of 44 lbs. per square foot of flat surface directly opposed to it. If the wind pressure were to exceed 44 lbs. per square foot, the centre of pressure would approach somewhat nearer to the leeward side of the chimney, and the maximum intensity of the pressure would be increased. There is, however, even at the weakest joint, a factor of safety of over 16, so that there would be little danger of the bricks crushing even under the increased intensity of the pressure.

To fulfil the conditions laid down by Professor Rankine, the dimensions of the chimney near the base should be somewhat increased, and for 80 lbs. wind pressure, they should be increased a good deal.

In regard to sliding the chimney is perfectly safe.

Chimneys should be built at a distance from the boiler buildings, and connected with them after the chimney has settled to its full extent. Without this precaution, unequal settlement and disruption of the work are, owing to the great weight of the chimney and consequent greater intensity of pressure on the soil below, sure to occur.

Enclosure Walls.

The following enclosure walls have been actually constructed. A long length, 550 feet, of one of them (No. 4) was blown over, as if it had been hinged at the footings.

They are given as typical examples of enclosure walls, and are intended to show, that if the wind acts as a horizontal uniformly distributed force normal to the wall, with a force of 55 lbs. on the square foot, these walls can only remain standing by virtue of the tenacity and adhesion of the mortar.

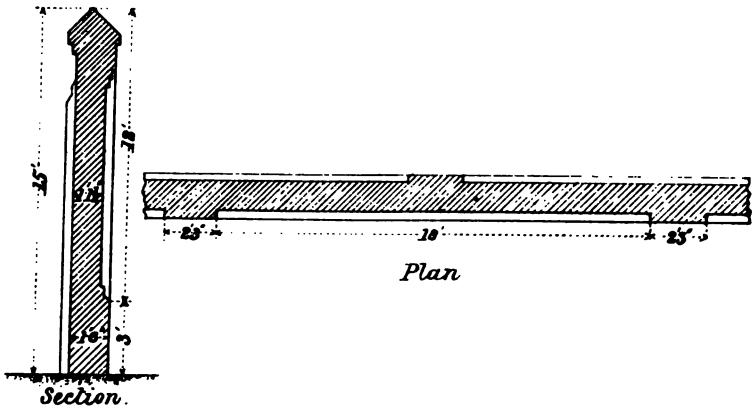
Wall No. 1.

Weight of material = 112 lbs. per cubic foot. Pressure per square foot required to overturn the wall = 27·3 lbs. This wall was built with brick in grey chalk lime mortar, 2 sands (see *Figs.* 13 and 14, Example V.).

Wall No. 2.

Weight of material = 112 lbs. per cubic foot. Pressure per square foot required to overturn the wall = 21 lbs. This wall was built in grey chalk lime mortar, 2 sands.

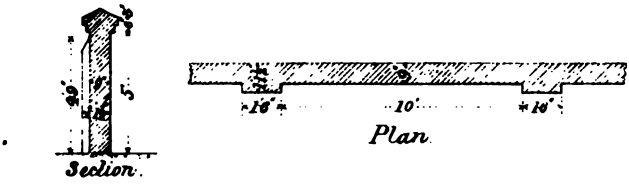
No. 2.



Wall No. 3.

Weight of material = 112 lbs. per cubic foot. Pressure per square foot required to overturn the wall = 19·6 lbs. This wall was built in grey chalk lime mortar, 2 sands. In this case the wall being very low, the friction of the wind against the ground would act more than in the other cases.

No. 3.

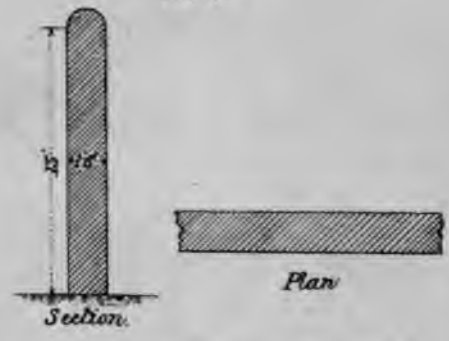


Wall No. 4.

Weight of material = 105 lbs. per cubic foot. Pressure per square foot required to overturn the wall = 18 lbs. This wall was built in pure lime mortar, 2 sands, and was blown down about 18 months after it was built.

It, therefore, appears advisable in constructing an enclosure wall of an ordinary section to provide a mortar of considerable tenacity and adhesion, and these walls should be treated as structures of cemented blocks.

No. 4.



STRUCTURES OF CEMENTED BLOCKS.

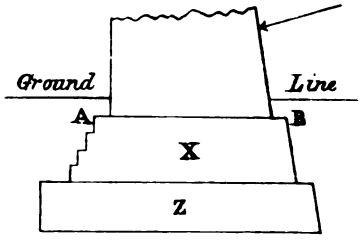
The following remarks apply only to structures whose weight and form alone are insufficient to ensure their stability as regards overturn, and in which consequently the tensile resistance of the material is called into action.

The danger of failure by sliding is done away with in structures of this kind, the force tending to produce sliding being resisted both by the friction due to the weight of the structure, and by the shearing strength of the material, which latter forces in every case far exceed the former.

It is therefore only necessary to enquire into the stability as regards overturn and as regards crushing.

In considering the stability of such structures, it may be assumed that the Engineer has so designed his footings X (*Fig.* 10), and his foundations R, that they cannot be torn out of the ground.

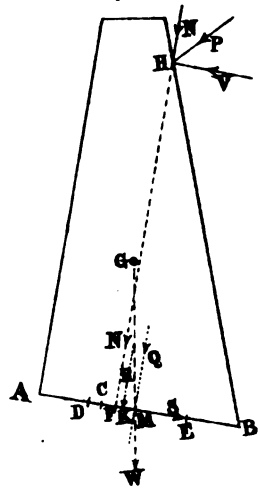
Fig. 10.



In this case the bed joint AB will, in every case occurring in practice, be the joint where fracture will first tend to take place, the moment of any force or forces applied to the structure being greater at AB, than at any joint above it.

Let *Fig.* 11 represent the transverse section of a structure of cemented blocks.

Fig. 11.



Let G be the centre of gravity of the section, and let W be the weight of a portion of the structure, one unit in length.

Let fall the vertical GM through G, intersecting AB in M.

Then M is the centre of the pressure due to the weight W of the structure, and this pressure (p. 233) is distributed as a uniformly varying pressure over a portion or the whole of AB, being greatest at that end of AB to which M is nearer, and decreasing uniformly towards the other side.

Let now the pressure P be applied gradually. The effect is to make the centre of pressure move gradually from M towards A, the intensity of the pressure at B decreasing, while that at A increases.

When the pressure P is sufficient to move the centre of pressure as far as C, so that $AC = \frac{1}{3}$ AB, the pressure at B is relieved entirely (p. 233).

If the pressure P be still further increased, the centre of pressure moves still further towards A, and a tensile resistance is simultaneously excited at B, and for a certain distance inside it, which retards the further movement of the centre of pressure.

If the pressure P be supposed to be sufficient to have moved the centre of pressure as far as D, notwithstanding the tensile resistance, the whole pressure is distributed, decreasing uniformly from A, over a part of the bed joint AE $=$ 3 AD, and a tensile resistance increasing uniformly from nothing at E to B is excited over the remaining part EB of the joint.

Now the position of the centre of pressure, and the amount of the tensile resistance excited are dependent on one another, and neither

can be assumed, and it is necessary to determine the relation between them.

Referring to *Fig.* 11; resolve the pressure P into two components, normal to and parallel to AB, and let them respectively be N and V. Draw HF perpendicular to AB, then F is the centre of pressure on AB of the force N.

Resolve W similarly into two components and let them be respectively Q and S.

Find the resultant pressure R of N and Q, and its point of application K on AB.

Then the normal pressures N and Q are distributed over the whole or a portion of AB as a uniformly varying stress, the intensity of which is greatest at the end of AB to which K is nearer, and diminishes towards the opposite end.

Let n and n_1 be the intensity of the pressure at A and B respectively, due to the normal pressure N + Q.

These values can be ascertained from the formulæ given at p. 240.

The force V produces a bending about a horizontal axis passing through the centre of gravity of AB, just as in a beam fixed at one end, and when equilibrium obtains, the moment of this force is equal to the moment of resistance of the section of the bed joint AB; *i.e.*,

$$\text{Moment of Flexure} = \text{Moment of Resistance} = \frac{f_o\, I}{y_o}$$

where f_o = intensity of the compression at A or of the tension at B.

y_o = distance of the neutral axis from A or B.

I = Moment of Inertia of the bed joint AB, one unit wide, about an axis passing through its centre of gravity.

$$\text{whence } f_o = \frac{\text{Moment of Flexure} \times y_o}{I}$$

and the total intensity of the compression at A $= n + f_o$.

" " " tension at B $= n_1 - f_o$.

In order that the structure may not tend to overturn, $n_1 - f_o$ must not exceed the working tenacity of the blocks or mortar, or their adhesion to one another.

In order that the structure may not tend to fail by crushing, $n + f_o$ must not exceed the working resistance to crushing of the blocks or mortar.

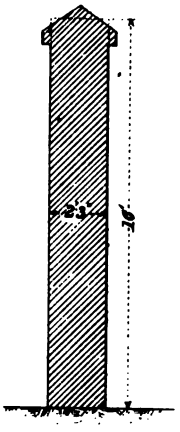
EXAMPLE IV.

A long enclosure wall 16 ft. high (*Fig.* 12) and three bricks thick, of brick in hydraulic mortar weighing 100 lbs. per cubic foot, is built in an exposed position in a country where the force of the wind has been registered at 55 lbs. on the square foot.

Determine the compression per square inch on the lee side of the bed-joint at the footings, and the tension per square inch on the windward side of the same joint, when the wind is blowing perpendicularly to the face of the wall, with its maximum force, and uniformly all over it.

The tenacity and resistance to crushing of the materials to be taken

Fig. 12.



from the tables (*U.R.T.*, p. 203, 204). With regard to their adhesion to one another, see note at the end of the paper.

Remarks.

The height of this wall is an ordinary height for the enclosure walls of dockyards, prison yards, &c.

Such walls are, however, seldom made so thick as three bricks, or of uniform thickness, being more generally of the plan and section given in Example V., but in this example an uniform width has been given for the sake of illustrating the principles as simply as possible.

There can be no doubt that in any but very open and exposed positions, the force of the wind never acts with its full registered force on a long length of a low wall, and even in such positions there must be some reduction in its force near the lower part of the wall, owing to the friction of the wind along the ground.

But it is certainly judicious in exposed positions to neglect this source of reduction of the force, and to provide for the maximum registered pressure.

As the wind seldom blows with its maximum registered force, and still less often with its maximum force normal to the face of the wall, it is justifiable to calculate upon the mortar having time to acquire a considerable proportion of its ultimate tenacity and adhesion to the blocks before the wall is subjected to the greatest pressure it can ever have to bear.

Referring to *Useful Rules and Tables*, p. 204, it would be justifiable to calculate in this case upon the tenacity of good hydraulic mortar one year after mixture, given at 140 lbs. per square inch, allowing a proper factor of safety (say 3, as the force is only occasional), and upon the adhesive resistance of the mortar being 36 lbs. per square inch (see note at end of paper) with the same factor of safety.

The ultimate crushing resistance of good brick is 1,100 lbs. per square inch (*U.R.T.*, p. 197) and the ultimate crushing resistance of good hydraulic mortar is certainly not less than 500 lbs. per square inch.

Preliminaries.

Let W lbs. = weight of a portion of the wall one foot long = $15' \times 1' \times 2\frac{1}{4}' \times \overset{\text{lbs.}}{100} = \overset{\text{lbs.}}{3,600}$.

In this case N = 0, and the total normal force = W = 3,600.

Also $n = n_1 = \dfrac{3,600}{2 \cdot 25' \times 144} = 11 \cdot 11$ lbs. per square inch.

Also Moment of Flexure = Wind pressure × half height of wall.
$= 16' \times 1 \times 55$ lbs. $\times 8' \times 12 = 84,480$ inch lbs.

Moment of Resistance $= \dfrac{f_o \times \frac{1}{12} \times 12 \times 27^3}{\frac{27}{2}}$

whence $f_o = 58$ lbs. per square inch (nearly).

and $n + f_o = 11 \cdot 11 + 58 = 69$ lbs. per square inch nearly (compression).
$n - f_o = 11 \cdot 11 - 58 = 47$ lbs. „ „ (tension.)

There is therefore a factor of safety of $\dfrac{500}{69} = 7 \cdot 2$ against crushing,

but the wall must fail by the opening of the joints on the windward side if the wind ever blows with its maximum force, normal to its face, the adhesive resistance which the materials are capable of exerting being far below what is necessary.

This source of danger might be entirely avoided by using cement mortar in the lower part of the wall.

EXAMPLE V.

A long enclosure wall, 16 feet high, and of the plan and section given in *Figs.* 13 and 14, of brick in hydraulic mortar, weighing 100 lbs. per cubic foot, is built in an exposed situation in a country where the force of the wind has been registered at 55 lbs. on the square foot.

Determine the compression per square inch on the lee side of the bed joint at the footings, and the tension per square inch on the windward side of the same joint, when the wind is blowing perpendicularly to the face of the wall with its maximum force, and uniformly all over it.

The tenacity of the materials, and their resistance to crushing, to be taken from the tables. With regard to their adhesion to one another, see note at the end of the paper.

The same remarks apply as in last example.

Fig. 13.

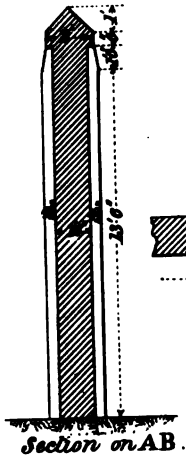
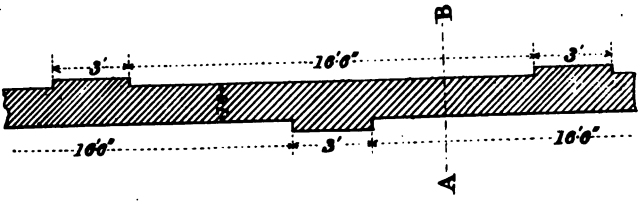


Fig 14.



Preliminaries.

As the wall is not uniform in section throughout, it is best to consider a length of the wall between the centres of two alternate piers, which is in fact a unit of the wall.

Then $W = (19{\cdot}5' \times 16' \times 1{\cdot}5' + 6' \times 16' \times \frac{3}{8}')\ 100 = 50{,}400$ lbs.

In this $N = 0$, and the total normal force $= W$.

Also $n = n_1 = 11{\cdot}11$ lbs. per square inch.

The height of 16′ is taken, because the coping being inclined does not offer the same resistance to the wind as a vertical surface.

$$\text{Moment of Flexure} = 19{\cdot}5' \times 16' \times \overset{\text{lbs.}}{55} \times 8' \times 12 = 1{,}647{,}360 \text{ inch lbs.}$$

$$\text{Moment of Resistance} = \frac{f_o}{y_o}\left(\tfrac{1}{12} \times 3' \times 12 \times 27^3 + \tfrac{1}{12} \times 16{\cdot}5' \times 12 \times 18^3\right).$$

$$= f_o \frac{155{,}277}{\frac{27}{2}} = f_o \times 11{,}502$$

$$\text{whence } f_o = \frac{1{,}647{,}360}{11{,}502} = 143 \text{ lbs. per square inch nearly.}$$

$n + f_o = 11{\cdot}1 + 143 = 154$ lbs. per sq. inch nearly (compression)

$n_1 - f_o = 11{\cdot}1 - 143 = 132$ lbs. „ „ (tension)

There is therefore a factor of safety of more than 3 in compression, but this wall must fail by the opening of the joints on the windward side, if the wind ever blows with its maximum force normal to its face, the adhesive resistance of the materials being far below what is necessary.

This source of danger may be entirely avoided by using cement mortar in the lower parts of the wall.

There would be a considerable gain in the strength of enclosure walls if they were built hollow.

NOTE ON THE TENACITY AND ADHESION OF MORTAR AND BRICKS.

It is evident that when a stress is applied to a structure of cemented blocks sufficient to call the tensile and adhesive resistance of the materials into existence, the Engineer can only count upon whichever of the two resistances is the smaller.

If the mortar used by the Engineer be divided into four classes, viz.: Cement mortars (mixed with such cements as Portland, not exceeding two sands, or Roman, not exceeding 1 sand); strong hydraulic mortars (mixed with such lime as Blue lias, not exceeding 2 sands); weak hydraulic mortars (mixed with such lime as Halling or Dorking grey chalk lime, commonly called stone lime, not exceeding 2 sands); pure lime mortars (mixed with such lime as white chalk lime, or Portland stone lime, not exceeding 3 sands); all the mortar being supposed to be properly mixed without excess of water, and the bricks or stone being supposed to be well wetted just before being put into the work, experiments and experience seem to show;

1st. That cement mortars are capable after a comparatively short time of exerting a tensile and adhesive resistance nearly equal to the tensile resistance of good brick, or about 250 lbs. on the square inch (*U.R.T.*, p. 195).

2nd. That the tensile resistance, which strong hydraulic mortar is capable of exerting is far in excess of the adhesive resistance of the same mortar, and that it is not safe to calculate upon the best hydraulic mortar adhering to the hardest grey stock bricks after a period of six months with a greater ultimate force than 36 lbs. per square inch. The adhesion to soft "place" bricks is only 18 lbs. per square inch (*M.P.I.C.E.*, Vol. xvii. p. 420).

3rd. That it is not judicious in designing any structure to take into consideration the tensile or adhesive resistance of the weaker hydraulic or pure lime mortars.

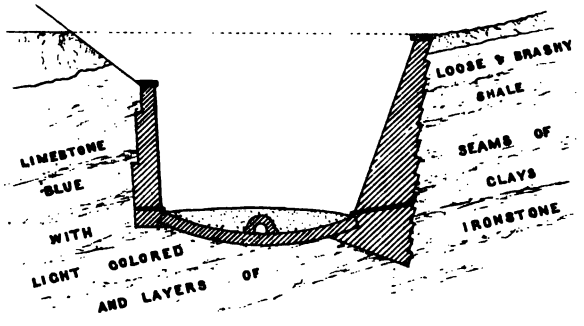
RETAINING WALLS.

A RETAINING wall, properly speaking, is a wall built to retain an artificial bank of earth, and is to be distinguished from a Breast wall, the office of which is often merely to protect a newly exposed surface of earth from the weather, and by so doing to prevent its getting into a condition to require greater support.

The dimensions of this latter class of wall must depend upon a variety of considerations, such as the length of time the newly opened ground is exposed to the action of the weather, the nature of the strata cut through, and their inclination.

Thus in *Fig.* 1, taken from *Dobson's Art of Building*, p. 22, the wall towards which the strata dip, should manifestly be made stronger than the wall on the other side of the cutting, where it need be little more than a thin facing to protect the ground from disintegration.

Fig. 1.



Again, if the soil cut through consists of clay which has been subjected to great pressure, it swells and exercises a force on the back of the wall, which it is very difficult to estimate, though this might perhaps be done by sinking trial pits behind the site of the wall, and inserting some means of measuring the force of expansion.

The dimensions of such walls must therefore be determined by experience.

In calculations for retaining walls proper, the Engineer requires to know or to assume the angle of repose of the earth to be supported, the angle of repose of masonry on masonry or brickwork on brickwork and on earth of different kinds, and the weights of earth and masonry.

The angle of repose of earth is the angle at which a bank of that particular earth will stand permanently, when the adhesion between the particles has been destroyed by the influence of the weather, and the earth depends for its stability upon friction alone, and when the state of the earth as regards moisture is the same as that in which the earth behind the wall can, by proper drainage, be ensured to remain.

The angle of repose of masonry on masonry, brickwork on brickwork, &c., is the angle of which the co-efficient of friction of the particular materials is the tangent (p. 235). This does not take into consideration the resistance of the material to shearing.

The average values of the weight of earth and masonry, and the angle of repose, and co-efficient of friction of various kinds of earth and masonry, are given in *Tables* I., II., and III.; but it is always better to ascertain these values by observation or experiment, where practicable.

TABLE I.

From *Rankine's Useful Rules and Tables*, and *Unwin's Lectures on Railway Construction*.

	Angle of repose.	Weight per cubic ft.
Sand, fine and dry	37° to 31°	89 lbs.
„ wet	26°	to
„ very wet	32°	118 lbs.
Vegetable earth, dry	29°	100 lbs.
„ „ moist	45° to 49°	to
„ „ very wet	17°	120 lbs.
„ „ punned	66° to 74°	
Clay, dry	29°	120 lbs.
„ damp ...	45°	to
„ well drained		135 lbs.
„ wet	16°	
Gravel, clean	48°	90 lbs.
„ with sand	26°	to
Loose shingle	39°	110 lbs.
Peat	14° to 45°	

TABLE II.

From the same source as *Table* I.

Brickwork in mortar	100 lbs. per cubic ft.
„ in cement	112 „ „
Masonry, rubble	140 „ „
„ ashlar, limestone	150 „ „
„ granite	160 „ „

TABLE III.

From *Rankine's Applied Mechanics*, p. 211.

	f	ϕ
Dry masonry and brickwork ...	0·6 to 0·7	31° to 35°
Masonry and brickwork, with damp mortar	0·74	36°½
Timber on stone	about 0·4	22°
Iron on stone	0·7 to 0·3	35° to 16°⅔
Timber on timber	0·5 to 0·2	26°½ to 11°⅓
Timber on metals	0·6 to 0·2	31° to 11°⅓
Metals on metals	0·25 to 0·15	14° to 8°½
Masonry on dry clay	0·51	27°
Masonry on moist clay	0·33	18°¼
Earth on earth	0·25 to 1·0	14° to 45°
Earth on earth, dry sand, clay, and mixed earth	0·38 to 0·75	21° to 37°
Earth on earth, damp clay	1·0	45°
Earth on earth, wet clay	0·31	17°
Earth on earth, shingle and gravel .	0·81 to 1·11	39° to 48°

The first step towards calculating the dimensions of a wall necessary to retain a bank of earth of known weight, and known angle of repose, is to determine:

1st. The amount of the resultant pressure of the earth upon the wall.

2nd. The direction of that resultant.

3rd. Its point of application.

The theory of the pressure of earth stated in this paper is that of Professor Rankine (*R.A.M.*, p. 212,; *R.C.E.* p. 318).

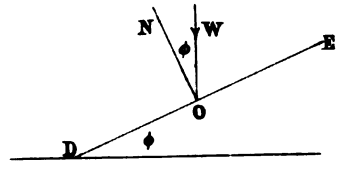
It is to be understood that the effect of the adhesion of the particles of earth to one another, is left out of consideration in this theory, and the stability of the earth is taken as depending on friction alone.

On the Stability of Earth.

In order that a body of any material may rest on a plane of any (the same or other) material by friction alone, the single condition is that the angle between the pressure on the plane and the normal to the plane shall not exceed the angle of repose of those materials.

In the case of a mass of loose earth, each particle, W, on the surface

Fig. 2.



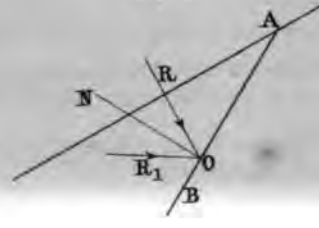
(*Fig.* 2) must fulfil this condition, and consequently the surface of the bank, when the inclination of each point in the surface is at the maximum consistent with stability is a plane D E with the uniform inclination ϕ, where ϕ is the angle of repose of the earth.

This condition, which determines the stability of a free surface of earth with respect to friction, is also the condition of stability with respect to any plane in the mass.

A particle is stable on a free surface, if the direction of its weight makes an angle less than the angle of repose with the normal to the surface; the mass of earth will be stable, if the direction of the pressure between the portions into which it can be divided by any plane in the mass, makes an angle less than the angle of repose with the normal to the plane.

Let *Fig.* 3 represent a vertical section through a mass of earth in the direction of the greatest slope, and let A B be a plane in any direction at right angles to the first plane.

Fig. 3.



The section is taken in the direction of the greatest slope, because it is in that direction that there is the greatest obliquity between the vertical and the normal to the plane, and it is therefore the direction in which motion would first ensue.

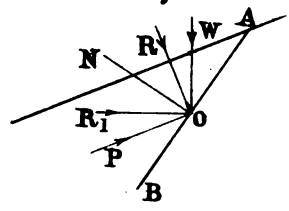
Then the mass will be stable in regard to sliding down this plane, if the direction of the resultant pressure R (from whatever cause arising) upon the plane makes an angle N O R with and above the normal O N less than the angle of repose, and it will be stable against sliding up the plane, if the resultant R_1 makes with and below the normal O N an angle R_1 O N less than the angle of repose.

And in order that the mass may be stable, this condition must be fulfilled for any position of the plane A B.

In a mass of earth loaded only with its own weight, this condition can only be fulfilled for any plane inclined at an angle greater than the angle of repose, by the existence of some other force which, combined with the weight resting on the plane, gives a resultant in the direction necessary for stability.

Let A B (*Fig.* 4) be any plane inclined at a greater angle than the angle of repose. Then the direction of the weight makes with the normal O N, an angle W O N, which by the conditions is greater than the angle of repose, and if this were the only force acting on the plane, motion must ensue along A B, in the direction A B. If motion does

Fig. 4.



not take place, *i.e.*, if the earth is stable, there must be some force P existing, which, combined with W, gives a resultant R, making an angle R O N not greater than the angle of repose.

If it be supposed that P is so large, that the resultant R_1 makes an angle R_1 O N greater than the angle of repose, motion will ensue along the plane in the direction B A, so that for stability on this plane, P must be large enough to give a resultant R, which shall make the angle R O N not greater than ϕ, and yet it must not be so large as to give a resultant R_1 which shall make the angle R_1 O N greater than ϕ.

In a mass of earth loaded with its own weight only, this force P is supplied by the tendency of the earth to spread laterally, which is produced by the vertical pressure due to the weight of the earth itself, and if its direction be given, it must, on the late Canon Moseley's Principle of least Resistance, have the least value necessary to the stability of the mass.

Professor Rankine shews (*R.A.M.*, p. 127);

Theorem. I. That in an indefinite homogeneous solid bounded above by a sloping plane, the pressure on any plane parallel to that sloping plane is vertical and of an uniform intensity equal to the weight of the vertical prism which stands on unity of area of the given plane.

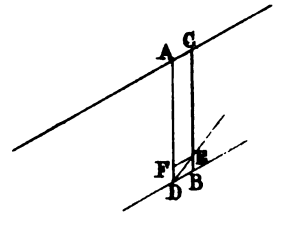
Theorem. II. The stress, if any, on any vertical plane is parallel to the sloping surface.

Theorem III. The state of stress at a given uniform depth below the sloping surface is uniform.

From Theorem II. the direction of P is known, and the least value of it necessary to maintain equilibrium obtained.

Let *Fig.* 5 represent a vertical section of a mass of earth in the direction of the greatest slope, and let A B, C D be two vertical planes intersecting the mass; let B E be another plane perpendicular to the

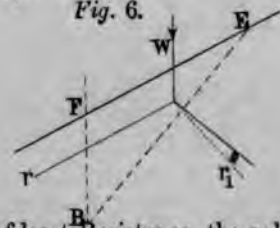
Fig. 5.



first plane, down which the prism of earth A B E C tends to slide. Draw E F parallel to A C.

Then from Theorems II. and III. the pressure on A F and C E balance each other, and the equilibrium of the prism A B E C is maintained by these pressures, by the reaction r of F B, which is parallel to A C, and the reaction r_1 of B E, which is inclined to the normal to B E at the angle ϕ (*Fig.* 6).

Fig. 6.



Then on the Principle of least Resistance, the value of r must be the least necessary to maintain equilibrium in every possible position of the plane B E.

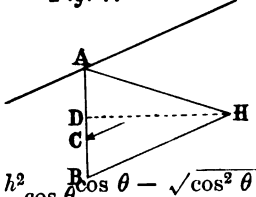
Professor Rankine has shewn (*R.A.M.*, p. 214, 216) that the intensity of the pressure on a vertical plane, such as A B, is

$$w\,h\cos\theta\frac{\cos\theta-\sqrt{\cos^2\theta-\cos^2\phi}}{\cos\theta+\sqrt{\cos^2\theta-\cos^2\phi}} \qquad (1)$$

where h is the depth of the plane B D below the surface A C (*Fig.* 5), w is the weight of earth per unit of volume, ϕ is the angle of repose of the earth, and θ is the inclination of the surface to the horizon, which cannot exceed ϕ.

The intensity of the pressure increases with h from the surface downwards, and the whole pressure on A B may be represented graphically by a triangle (*Fig.* 7) in which D H = intensity of pressure at the depth B, and A B = h; whence the centre of pressure is at C, B C being $\frac{A\,B}{3}$.*

Fig. 7.



The total pressure $P=\frac{w\,h^2}{2}\cos\theta\frac{\cos\theta-\sqrt{\cos^2\theta-\cos^2\phi}}{\cos\theta+\sqrt{\cos^2\theta-\cos^2\phi}}$ (2)

* There are two other pressures, equal to each other and in opposite directions, and both horizontal, acting on the prism of earth at right angles to the plane of greatest slope, the intensity of which is shewn (*R.A.M.*, p. 217) to be

$$\frac{w\,h\cos\theta\,(1-\sin\phi)}{\cos\theta+\sqrt{\cos^2\theta-\cos^2\phi}}$$

but which, balancing one another, do not affect the stability of retaining walls, and are therefore left out of consideration.

For a bank with a horizontal top $\theta = o$, and

$$p = w\,h\,\frac{1 - \sin\phi}{1 + \sin\phi} \tag{3}$$

$$\mathrm{P} = \frac{w\,h^2}{2}\,\frac{1 - \sin\phi}{1 + \sin\phi} \tag{4}$$

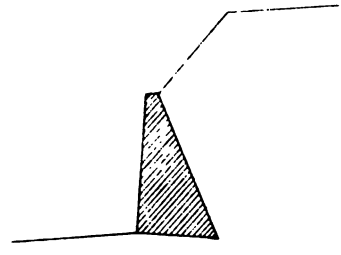
For a bank with an upper surface sloping at the angle of repose, which is the greatest slope possible.

$$p = w\,h\cos\phi \tag{5}$$

$$\mathrm{P} = \frac{w\,h^2}{2}\cos\phi \tag{6}$$

In most cases in practice, there is either no surcharge, or the surcharge rises a certain height, and then finishes in a horizontal top as in *Fig.* 8.

Fig. 8.



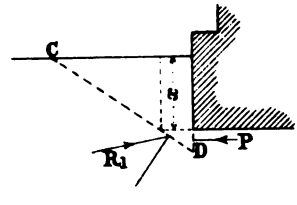
The latter case does not admit of an exact solution (*Philosophical Transactions*, 1856-57, *Stability of Loose Earth*, by Professor Rankine), and the approximate solution is far too difficult for ordinary use.

The direction and value of P in this case evidently lie between the two extreme cases of a bank without surcharge, and a bank with a surcharge indefinitely long, and the Engineer must in the absence of any better method use his judgment in interpolating.

A case is given in Example II.

When a horizontal bank is loaded with an external load, such as a building, the effect is to increase the value of P on the vertical plane at the end of the building, beyond what is due to the weight of the earth itself, and the effect is that in the adjoining prism of earth the weight of the prism combined with this greater value of P, may produce a resultant R_1 (*Fig.* 9), inclined to the plane C D at any angle

Fig. 9.



between ϕ on one side of the normal to the plane, and ϕ on the other side of the normal.

If on any plane it is inclined more than ϕ above the normal, sliding up the plane takes place, the earth under the building spreads, and settlement ensues.

As this sliding will take place on that plane where the force required to produce it is least, the weight of the building must not exceed that which will incline the pressure on C D beyond the angle necessary to stability.

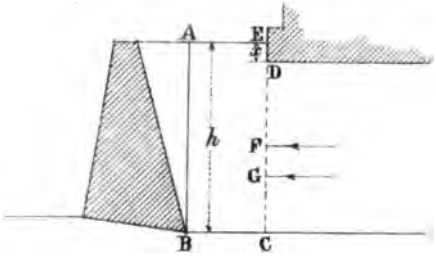
Professor Rankine shews that the weight of a building, the pressure of which is uniformly distributed over its base, must not exceed the weight of the earth which it displaces in a greater proportion than $(1 + \sin \phi)^2$ to $(1 - \sin \phi)^2$.

He further shews that if the pressure of a building is not uniformly distributed over its base, as may sometimes be the case with retaining walls, the intensity of the pressure on the side where the pressure is greatest must not exceed $w\, x \left(\frac{1 + \sin \phi}{1 - \sin \phi}\right)^2$, and its least intensity must not fall short of $w\, x$, x being the depth of the foundation below the surface of the earth (*Fig.* 9).

In retaining walls, the consideration of this case may be rendered unnecessary by widening the foundations of the wall on the outside.

The effect of buildings on the pressure of the earth below them is to increase the value of P (*Fig.* 10).

Fig. 10.



The intensity of the vertical pressure at D (*Fig.* 10) is the weight of the building per unit of surface, and calling this w_1, the intensity of the horizontal pressure due to it $= w_1 \frac{1 - \sin \phi}{1 + \sin \phi}$ (7)

This same intensity of horizontal pressure acts at every depth below the building on the vertical plane D C, its total amount being

$$w_1 (h - x) \frac{1 - \sin \phi}{1 + \sin \phi} \qquad (8)$$

x being the depth E D from the surface to the bottom of the foundation.

In addition to this, there is the horizontal pressure due to the weight of the earth itself, which on the plane D C$=\frac{w\,(h-x)^2}{2}\,\frac{1-\sin\phi}{1+\sin\phi}$, so that on the plane E C, the total pressure is made up of two components; the first, that due to the weight of the building and equal to

$$w_1\,(h-x)\,\frac{1-\sin\phi}{1+\sin\phi}$$

which has its centre of pressure at F, where F C $=\frac{1}{2}$ D C; the second that due to the earth itself$=\frac{w\,(h-x)^2}{2}\,\frac{1-\sin\phi}{1+\sin\phi}$, acting at G, where G C $=\frac{1}{3}$ D C.

In the case where the front of the building is at A, this gives the pressure on the plane A B, from which the stability of the wall may be determined; but where the front of the building is at a distance behind A, the pressure on the plane A B has not been determined, although it is certain that the further the building is from the wall, the less the pressure caused by the building must be.

An investigation of this case on Moseley's theory of the pressure of earth is given in Weisbach's *Principles of Machinery and Engineering*, Vol. II., p. 6.

Stability of Retaining Walls.

The sections of retaining walls used in practice are of various kinds, the forms of which are given in *Plate* I.

No. 1 is a simple rectangular mass of brickwork or masonry, and is called a rectangular retaining wall.

No. 2 is a sloping retaining wall.

No. 3 is a counter-sloping retaining wall.

No. 4 is a leaning retaining wall.

No. 5 is a modification of Nos. 2 and 3.

Nos. 6 and 7 are retaining walls having a curved batter on the face.

Nos. 8, 9 and 10 are counter-forted retaining walls.

No. 11 is a retaining wall with relieving arches, commonly called a "Revétement en Décharge."

No. 12 is a retaining wall consisting of buttressed horizontal arches.

In inquiring into the stability of any retaining wall, it is necessary to see that the conditions of stability required in an ordinary wall are fulfilled. These have been already stated (p. 236), and are as follows:

1st. The line of resistance must intersect every bed joint.

2nd. The angle which the resultant pressure at any bed joint makes with a normal to that joint must not be greater than the angle of which $\frac{1}{3}$ths* of the co-efficient of friction of the material is the tangent.

3rd. The point of intersection must fall sufficiently far within the outer edge to prevent any risk of crushing at that edge.

And it is advisable in every case to inquire both into the stability of the wall itself, as regards the most dangerous bed joint in the masonry,

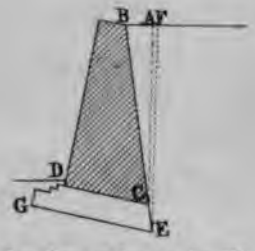
* This is of course an arbitrary quantity, which should be smaller if possible.

and into the stability of the wall as regards the earth on which it stands.

In designing a wall for any particular case, the simplest method is, 1st, to determine what form of section the wall is to have, and 2nd, assuming certain dimensions, to inquire whether they fulfil the conditions of stability.

In doing this for any bed joint, any earth resting on the wall above that bed joint is considered as forming part of the wall (*R.A.M.*, p. 250; *R.C.E.*, p. 402).

Fig. 11.



Thus in a wall of the form shown in *Fig.* 11, the prism of earth B A C is regarded as forming a part of the wall, and its weight as adding to the stability of the wall above the bed joint D C, while the prism of earth B F E is regarded as forming a part of the wall above the base G E, and its weight as adding to the stability of the whole wall.

This weight, acting through its centre of gravity, is then compounded with the pressure on the vertical plane A C, if the bed joint C D is under examination, and with the pressure on the vertical plane E F, if the whole wall is under consideration, the resultant determining whether the wall is sufficiently stable, as shown in the Examples.

In the case where the back of the wall rests on the earth as in No. 4 or No. 7, *Plate* I. the part of the masonry cut off by the vertical plane is to be regarded as adding to the stability of the wall only to the extent of the difference between its weight and that of an equal prism of earth. (*R.A.M.*, p. 250; *R.C.E.*, p. 402).

It follows in this case that if the earth and the material of the wall be of the same specific gravity, the part of the wall behind the vertical plane is useless, and this form of wall is not a good one.

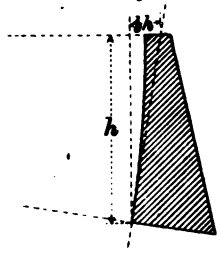
It also follows that the wider the base of a wall can be made with a given quantity of material, the greater will be the quantity of earth adding to the stability of wall, and the greater that stability.

This points to the advisability of reducing the thickness of these walls at the top to the minimum consistent with sound construction.

As regards giving the face a batter, straight or curved, it is generally agreed among Engineers, that the batter should not exceed $\frac{1}{6}$ of the height, a greater inclination being found to afford too great a facility for the entrance of water into the joints; and as the lowest part of the

wall is that most liable to injury from this cause, it is evident that to be consistent, the face of a curved batter wall should be a tangent at the foot to a line having an inclination not greater than $\frac{1}{6}$ of the height (*Fig.* 12).

Fig. 12.



As regards the advantage, in point of stability, of either straight or curved batter in comparison with a vertical face, it may be said, that in some cases the wall with a batter is somewhat more stable as regards overturning, and in others it is not quite so stable, but in every case it is more stable against sliding forward either on any of the bed joints or on its base, *i.e.*, if the joints are at right angles to the face of the wall, which is the usual construction. (*Plates* II. and III.)

A batter has the advantage in point of apparent stability, for if the wall is originally vertical, and rotates ever so little, it offends the eye, and gives the idea of instability, whereas if the wall has a batter originally, the alteration in inclination is not perceived.

Curved batter walls are more difficult, and expensive to construct, and unless a good deal of material is cut to waste (involving expenditure in labour), the joints must be thicker at the back than at the front of the wall, leading to unequal settlement.

They are, however, a good deal used, and generally the chord of the curve has an inclination of $\frac{1}{6}$ the height.

It is therefore recommended to give all walls some slight batter where the circumstances admit of it, preferring the straight to the curved, and in cases where the fining down of the quantity of material is a matter of great importance, it is better to prepare alternative designs.

Wing walls of bridges carrying railways over roads which have to sustain the pressure of the railway embankment, are frequently made curved on plan, by which some extra resistance is obtained against the pressure of the earth.

As regards walls with counterforts, it is shown by Professor Rankine that in the cases of a vertical rectangular wall with and without rectangular counterforts, a small saving of masonry is effected by the use of counterforts.

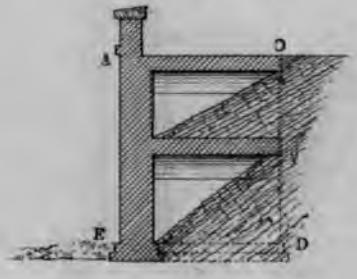
The form of counterfort which appears likely to be strongest is that shewn in *No.* 10, *Plate* I., because the resistance to fracture at the counterfort either at A B or B C, increases with the pressure from the top of the wall to the bottom, because the base of the counterfort is

made as large as possible with a given quantity of material, and therefore has (as compared with the wall without counterforts) an increased quantity of earth tending to the stability of the wall,

Relieving Arches, or Revêtements en Décharge.

The rule given by Professor Rankine in *Civil Engineering*, p. 413, enables the Engineer to calculate the stability of this class of retaining walls, the method of proceeding being the same as in the case of ordinary walls, *i.e.*, determine first the pressure, and then assuming a form of wall, ascertain whether it fulfils the condition of stability.

Fig. 13.



This is stated as follows :

A row of arches having their axes and the faces of their piers at right angles to the face of a bank of earth are called "relieving arches." There may be either one or several tiers of them, and their front ends may be closed by a vertical wall, which thus presents the appearance of a retaining wall, although the length of the archways is such as to prevent the earth from abutting against it. *Fig.* 13 represents a vertical transverse section of such a wall, with two tiers of relieving arches behind it. To compute the length of a relieving arch from its clear height, or its clear height from its length, the following approximate formulæ may be used, in which

x denotes the depth of the crown of an arch below the surface,
h, its clear height,
l, its length, and
ϕ, the angle of repose of the earth.

$$l = \operatorname{cotan} \phi \left(h + \frac{x}{(1 + \sin \phi)^2} \right); \qquad (1)$$

$$h = l \cdot \tan \phi - \frac{x}{(1 + \sin \phi)^2} \qquad (2)$$

To determine the conditions of stability of such a structure as a whole, the pressure against the vertical plane O D (*Fig.* 13) may be determined, and compounded with the weight of the combined mass of masonry and earth O A E D in front of that plane, to find the resultant pressure on the base.

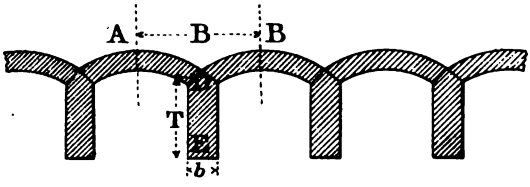
In soft ground the bases of the piers of the lowest tier of relieving arches should be connected by means of inverted arches, so as to distribute the pressure over the whole area covered by the structure.

Buttressed Horizontal Arches.

Professor Rankine gives the following rules for these arches,

Fig. 14 represents a plan, or horizontal section, of part of a row of buttresses, connected by horizontally arched walls.

Fig. 14.



To find the thickness, T = D E, required for such buttresses,

let B denote AB, the breadth of the mass of earth which one buttress has to sustain;

„ b, the breadth of the buttress;

„ t, the thickness which would be required for an uniform wall, to sustain the same bank of earth, computed as in p. 268;

Then

$$T = t \cdot \sqrt{\frac{B}{b}}; \; b = B \cdot \frac{t^2}{T^2}$$

In soft ground the bases of the buttresses may be connected by means of inverted arches, to distribute the pressure; and their tops may if necessary, be connected by means of arches, in order to support a platform, or a surcharged bank of earth.

Forms of Walls for particular Cases.

Where a wall has only the pressure of a bank of earth, or the pressure of a bank with a building on it, to sustain, there is no reason why it should be made any thicker at the top, than is necessary for an efficient coping.

Where it is liable to shocks of any kind, as where a road runs behind the wall on the bank, or as in the walls of basins, or locks, the thickness at the top should be considerable, and the wall itself strengthened.

The latter may be effected by multiplying the value of the pressure by some empirical factor, say 1·5 to 2, and designing the wall for this increased pressure.

In walls for works of defence, counterforts were formerly used to increase the difficulty of breaching, but with the artillery of the present day, it does not seem possible to delay the attack for any time worth considering by making unarmoured escarp walls thicker than is necessary for their purpose as simple retaining walls.

If counterforts are used, their efficiency will entirely depend on the care with which they are bonded into the face portion, or mask of the wall.

Construction of Retaining Walls.

The angle of repose of the earth should be ascertained by experiment if it cannot be arrived at by observation, the earth being in that state in which, by proper drainage, it can be ensured to remain.

The weight of the earth should be ascertained in a similar manner.

In most cases there should be numerous weepholes left, or pipes inserted in the wall, to prevent any accumulation of water in the earth behind. Professor Rankine (*R.C.E.*, p. 410) gives the proportion of 1 weephole to every 4 square yards of face of wall. These weepholes should be pointed and lined with cement. If pipes are used they should be pointed in cement. Along the front of the retaining wall there should be a surface drain to carry off the water from the weepholes, which otherwise might sink into the ground below the wall, and render it less stable.

It is well also to pack up behind the wall a layer of loose stone or gravel, to facilitate the passage of the water to the weepholes.

The back of retaining walls should be left rough, and if built in steps the steps should be as numerous as practicable. The number of steps or offsets will in brickwork be governed by the slope of the back of the wall, each offset being half a brick, or $4\frac{1}{2}$ inches in width.

The earth behind the wall should be put in thin layers, inclined slightly upwards from the wall, and rammed, not too heavily. Some Engineers prefer to ram the layers well, stopping the ramming within a short distance of the back of the wall. The earth backing should in no case be tipped in. If the backing be of clay, care should be taken that it is not too dry, as it will swell on becoming moist, and produce unnecessary pressure on the wall.

If the earth retained be of such a nature, that with excess of water it becomes mud, and if it is impossible by drainage to prevent excess of water, it is best to calculate the wall for a fluid having the weight of the mud.

Hoop iron is worse than useless in retaining or any walls, unless it is protected from oxydation by the use of very strong mortar or cement. Wet causes the iron to rust and expand, with a force sufficient to open the joints of the work. Where properly protected, hoop iron is most valuable as a bond.

The pressure on foundations in firm earth should not (*R.C.E.*, p. 381) exceed from 2,500 to 3,500 lbs. per square foot, and it is a common rule to limit it to 1 ton on the square foot, which however appears a low limit.

The maintenance of a thoroughly good bond is of the greatest importance in retaining walls.

EXAMPLE I.

It is required to design a retaining wall of brickwork in mortar, the bricks having an average resistance to crushing, to sustain a bank of earth with a horizontal top.

The data are as follows:

Weight of brickwork, 100 lbs. per cubic foot (Table II., p. 260).

Ultimate resistance of brickwork to crushing, 600 lbs. per square inch (*U.R.T.*, p. 197).

Factor of safety not less than 8 (*U.R.T.*, p. 205).

Co-efficient of friction of brickwork (the mortar being moderately fresh)..... ·7, (Table III., p. 261).

Weight of earth, 110 lbs. per cubic foot (Table I., p. 260).

Angle of repose of the earth behind the wall = 37° (Table I. p. 260).

Angle of repose of the earth on which the wall stands=25° (Table I., p. 260).

Pressure on the earth below the wall, not to exceed $1\frac{1}{4}$ tons per square foot (*R.C.E.*, p. 381), the soil being sandy gravel.

Factor of safety, against sliding, not less than 1·2. both as regards the joints of the masonry, and the wall on its base.

Height of wall, 18 feet above footings.

There is supposed to be no reason why the wall should not be as thin at the top as good construction allows, *i.e.*, having sufficient width for a solid coping; and the top may be taken as 2 bricks thick.

The face batter may be taken at $\frac{1}{12}$ = 18 inches.

Assume for trial, a thickness at the top of the footings of 6 feet, which will allow of the back of the wall being built with off-sets of $4\frac{1}{2}$ inches at every 18 inches in height, *i.e.*, at every six courses of brickwork.

Then as the moment of pressure on the vertical plane A B (*Plate* II., *Fig.* 1) increases as the cube of the height, while the moment of stability increases very little faster than the square of the height, the wall must be more likely to overturn at the joint at the top of the footings than at any other, and it is sufficient to enquire into the stability in this respect at this joint.

Moreover, as the pressure tending to produce sliding increases as the square of the height, while the weight resisting it increases very little faster than the height, this same joint must be that on which the tendency to slide is greatest.

Then considering this bed joint A C for a unit of the wall 1 foot in length.

The weight of the wall above the bed joint ... = 6,750 lbs.

" " earth resting on the wall, *i.e.*, in front of the plane A B ... = 3,025 lbs.

Total weight on A C ... ... = 9,775 lbs = W

The centre of gravity of the mass is found to be at G.

The pressure on the vertical plane A B $= \frac{w\,h^2}{2} \cdot \frac{1 - \sin\,\phi}{1 + \sin\,\phi} = 4{,}594$ lbs. which acts at D, A D being $\frac{\text{A B}}{3}$.

Through G draw a vertical and through D a horizontal, intersecting in O. On any scale of parts, make O a = W = 9,775 lbs., and O b = P = 4,594 lbs. Complete the parallelogram; then O c represents the resultant pressure on A C, and E is the centre of pressure.

Resolve O c normal to and parallel to A C, then $c\,d$ = 10,100 lbs. is the normal pressure on A C. On the principles explained at p. 233, this pressure is spread over a surface equal to three times C E multiplied by the length of the section of the wall under consideration, which is one foot, and the maximum pressure at C is equal to twice the mean pressure.

Hence the pressure per square inch at C $= \dfrac{2 \times 10{,}100}{3 \times 9\frac{1}{2}'' \times 12''} = 59$ lbs.

This gives a factor of safety of 10, and somewhat more than fulfils the required condition of stability; if it had not fulfilled the condition it would have been necessary to assume a greater thickness at A C, and recalculate.

Next, measure the angle O $c\,d$; which is found to be 20°, the tangent of which is ·36, so that as the co-efficient of friction of this joint is given at ·7, there is a factor of safety of nearly 2.

The wall is therefore stable as regards this joint.

To design the footings and foundations (*Plate* II., *Fig.* 2).

Assume the general outline of the footings and foundations to be as shewn, the underside of the foundation being at its outer end about three feet underground, so as to get below the effects of frost (*R.C.E.*, p. 380).

Draw a vertical F H through the back of the foundation. The weight of the wall and earth in front of this plane is found to be 13,401 lbs.; and the centre of gravity of the mass is at K.

The pressure on the plane F H=6,320 lbs., acting at L, at a distance from F $= \frac{1}{3}$ F H.

Compounding the pressure, and the weight as before, M e represents the resultant pressure on the base of the wall F N, and R is the centre of pressure.

As N R $= \frac{1}{3}$ N F nearly, the intensity of the pressure at F is very small, while that at N is considerable. This would lead to a greater compression of the soil at N than at F, and the wall would lean forward somewhat from its original position.

This would be avoided by lengthening the toe of the foundation. If R S be made equal to R F, the pressure on the soil below will be uniform. The normal pressure, represented by $e\,f$ = 14,000 lbs. = $6\frac{1}{4}$ tons, and it would be spread over $12\frac{1}{2}$ square feet; so that the pressure would be only $\frac{1}{2}$ ton per square foot.

The toe might, therefore, be made somewhat shorter than this.

Finally, the angle M $e\,f$ is 20°, the tangent of which is ·36; and the co-efficient of friction of the soil on which the wall stands is ·47; so that there is a factor of safety as regards sliding forward of 1·3.

If there were any doubt as to the condition of the earth below the wall in regard to moisture, it would be better to incline the concrete backwards.

The wall may now be designed as shewn. (*Plate* II. *Fig.* 3.)

EXAMPLE II.

It is required to design a retaining wall, with the same data as in Example I., except that the bank of earth rises at the natural slope above the wall to the height of 50 feet.

It is necessary in this case to calculate first the dimensions of the wall, supposing the slope to be indefinitely continued, and then to interpolate between that thickness and the thickness necessary for a wall supporting a horizontal topped bank of the same earth.

Assume for trial a thickness at the top of footings of 9′ $4\frac{1}{2}$″. (*Plate* III., *Fig.* 1.)

Then considering, as in last Example, the bed joint A C, for a unit of the wall one foot in length:

The weight of the wall above this bed joint = 9,788 lbs.
„ „ earth in front of A B = 8,186 „
17,974 „ = W

The centre of gravity of the mass is found to be at G.

The pressure on the vertical plane A B $= \frac{w\ h^2}{2} \cos \phi = 24{,}464$ lbs. acts at D, A D being $\frac{A\ B}{3}$.

Compounding these two forces, the following results are obtained.

Normal pressure represented by $c\ d = 34{,}700$ lbs.

Surface over which it is spread = 1,260 inches.

Maximum pressure per square inch = 55 lbs.

Angle O $c\ d = 25°$, of which the tangent is ·47, or there is a factor of safety as regards sliding of 1·5.

This wall would therefore be stable against the pressure o an indefinitely long surcharge, and therefore is too strong for the actual surcharge.

As there is, however, no way of accurately determining the actual thrust of a surcharged bank having a horizontal top, it becomes necessary to interpolate approximately.

As the wall 6 feet thick at the bottom is stable against a bank without a surcharge, and has a pressure of 59 lbs. per square inch on the most compressed edge, and as the wall 9′ $4\frac{1}{2}$″ thick at the bottom is stable against an indefinitely long bank sloping at the angle of repose, and has a pre sure of 55 lbs. on the most compressed edge, the proper thickness must be between the two, and the object of the Engineer being safety first and economy afterwards, it would probably not be wise with so high a bank as 50 feet to reduce the thickness very much.

Let the wall be reduced to 8′ 3″ at the bottom, and be drawn in a similar manner to *Plate* II., *Fig.* 3.

Supposing that the surcharge had been only ten feet high, the wall would be more nearly of the dimensions of the wall without a surcharge, and might be made at 6′ 9″ at the bottom.

The footings and foundations should also be designed for both walls, and a width between the two assumed.

WALLS FOR RETAINING WATER.

The Pressure of Water at Rest.

The pressure of water, regarded as a perfect fluid and incompressible, which for the purposes of the Engineer, it may be taken without appreciable error to be, is:

1st. Proportional to the surface on which it presses.

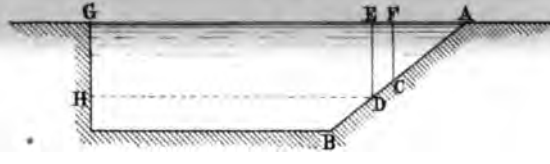
2nd. Proportional to the depth of that surface below the surface of the water.

3rd. Normal to the surface against which it presses.

4th. Of equal intensity in all directions.

Thus, let AB (*Fig.* 1) be a plane surface, such as the side of a

Fig. 1.



reservoir, pressed upon by water which it retains; then the pressure on the surface CD, supposed to be one unit in width, is equal to the weight of the column of water CDEF, one unit in width, the centre of pressure is at the intersection of a vertical through the centre of gravity of the prism CDEF with its base; and the direction of the pressure is normal to CD.

For the vertical wall GH (*Fig.* 1), the vertical pressure per unit of surface at the depth GH is equal to the weight of a column of water of the height GH, having a base of one unit of surface; and as water presses equally in all directions, the intensity of the pressure normal to GH at the depth GH is given by the weight of the same column.

Expressing this symbolically, the intensity of the pressure, on either the sloping or the vertical wall, at a depth x below the surface $= w\,x$, where w is the weight of water per solid unit and x the depth in linear units.

In dealing with water pressure, it is usual in this country to take the units of measure in feet, and the units of weight in lbs., the weight of water per cubic foot being taken at 62·5 lbs.

The graphic representation of the pressure in the two cases is as shewn in *Figs.* 2 and 3, the base BC of the right angled triangle

Fig. 2.

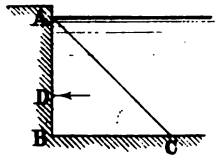
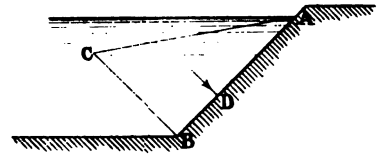


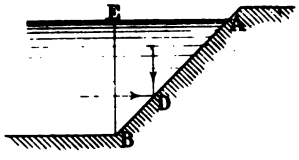
Fig. 3.



ABC representing in each case, on any scale of parts, the depth of the water in feet, multiplied by the weight of water per cubic foot; the area of the triangle representing the whole pressure on the face AB; and the centre of pressure being at D; where BD = $\frac{1}{3}$ BA.

It is sometimes convenient to treat the vertical and horizontal components of the pressure separately, as in *Fig.* 4, the vertical

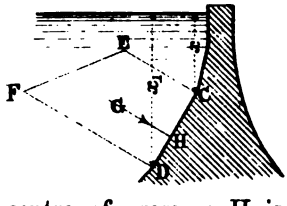
Fig. 4.



component being the weight of the prism of water AEB, acting through its centre of gravity at D; the horizontal component being the pressure on the vertical plane EB, acting at D, where BD = $\frac{1}{3}$ BA.

Where the water presses on a polygonal face, as is sometimes the case in large reservoir dams, the pressure on any part such as CD (*Fig.* 5), may be represented by a trapezium, CEFD, where CE = wx, FD = wx_1.

Fig. 5.



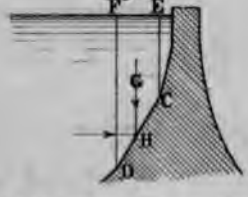
In this case the centre of pressure H is found by drawing HG normal to CD through the centre of gravity G of the trapezium.

In designing reservoir dams, it is sometimes convenient to treat the vertical and horizontal components of the pressure, on such faces as CD, separately. (*Fig.* 6.)

In this case, the vertical component is the weight of the prism of

water CEFD (*Fig.* 6), lying vertically above CD, the centre of pressure H being the intersection of the vertical through G the centre of gravity of the prism with its base CD; the horizontal component

Fig. 6.



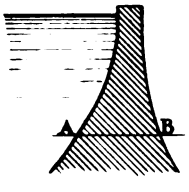
being the difference of the pressures on the vertical plane CE, and DF, acting at H.

Where the surface on which the water presses is curved, the pressure may be found by replacing the curve by a polygon having a large number of sides, dealing with each side of the polygon, as in the last case.

Conditions of Stability of a Reservoir Wall or Dam.

The conditions of stability which guided the French Engineers in the design of the great reservoir walls on some of the head waters of the Loire (as described in the Memoires by Messieurs Graeff and Delocre, Numbers 134 and 135, Annales des Ponts et Chaussées, Tome XII., 1866) were; that the intensity of the vertical pressure on the inner (or water) face of the section of the wall, the reservoir being empty, and on the outer face of the section, the reservoir being full, should not exceed a certain limit, *i.e.*, the pressure per unit of surface at A (*Fig.* 7), reservoir empty, and the pressure per unit of surface at

Fig. 7.



B, reservoir full, should not exceed a certain assumed limit, AB being any horizontal section.

In the Barrage du Furens, which was commenced in 1861, and completed in 1866, and which had a height of 50 mètres, this limit was taken at six kilogrammes per centimètre quarrè, equivalent to 12,289 lbs. on the square foot, while in the later design for the Barrage du Ban, which had a height of 42 mètres, the limit was, owing to the complete success of the Barrage du Furens, raised to 8 kilogrammes per centimètre quarrè, equivalent to 16,385 lbs. on the square foot.

Monsieur Graeff, in his Memoire (No. 134), quoted above, gives the section of several stone dams, and in one of these, that of Almanza in Spain, the intensity of the pressure, when the reservoir is full, reaches 14 kilogrammes per centimètre quarrè, equivalent to 28,674 lbs. on the square foot, the height of the dam being 20·7 mètres.

Monsieur Graeff regards 6 kilogrammes and 14 kilogrammes as the extremes of pressures, the latter being, in his opinion, extremely bold.

Considering the very disastrous consequences of a reservoir wall giving way, an Engineer is not justified in adopting a high limit of pressure, even though a large saving in the quantity of masonry is certain to be effected by so doing.

The conditions just given were the sole conditions which guided the French Engineers.

In a report on the design and construction of Masonry Dams, published in the *Engineer*, Jan. 5, 1872, Professor Rankine proposes another condition, viz., "that there ought to be no practically appreciable tension at any point of the masonry, whether at the outer face when the reservoir is empty, or at the inner face when the reservoir is full. Experience has shown that in structures of brickwork and masonry that are exposed to the overturning action of forces, which fluctuate in amount and direction (as when a factory chimney is exposed to the pressure of the wind), the tendency to give way first shews itself at that point at which the tension is greatest. In order that this principle may be fulfilled, the line of resistance should not deviate from the middle of the thickness of the wall to an extent materially exceeding one-sixth of the thickness. In other words, the lines of resistance, when the reservoir is empty and full respectively, should both be within, or but a small distance beyond, the middle third of the thickness of the wall."

This condition must be of primary importance on the inner face of reservoir dams, where any penetration of water would decrease by its upward pressure the stability of the dam.

It determined the extreme positions of the centre of pressure, given at p. 229, *Rankine's Applied Mechanics*, and at p. 234.

It may be here said that this condition is more than fulfilled both in the Barrage du Furens and the Barrage du Ban, but it is not even approximately fulfilled in that of Almanza, when the reservoir is full; nor is it nearly fulfilled in a section designed for the Furens, with a limit of pressure of 14 kilogrammes per centimètre quarrè.

Professor Rankine also considers that the limit of the vertical pressure should decrease in some proportion as the latter increases.

The inside or water face in reservoir walls of moderate height is vertical, an internal batter being given only when the weight of the dam becomes such as to make an increased width of base necessary, in order to reduce the intensity of the pressure on the inner face; which is the case only in dams of great height.

But the batter of the outer face in dams of moderate height, and of the lower part of the inner face in dams of great height, is

considerable, and gradually increases; and the excess of the intensity of the pressure, tangential to the face, over the intensity of the vertical pressure ought, in Professor Rankine's opinion, to be met by reducing the limit of pressure in some proportion as the batter increases, though in what proportion the present state of knowledge as to the distribution of stress in a solid body is not sufficiently precise to determine.

Bearing in mind however, that the first object of the Engineer is safety, it is easy to fix such a diminution, as will sufficiently satisfy this additional condition.

The intensity of the pressure tangential to the face, must certainly be less than the intensity of the vertical pressure multiplied by the secant of the angle of the batter, and inversely the limit of pressure cannot be required to be reduced so much as by multiplying it by the cosine of the angle of the batter. In the examples which follow, the limit of pressure has been taken at a mean between the vertical pressure and that pressure multiplied by the cosine of the angle of the batter, *i.e.*, limit of pressure = 20,000 lbs. $\times \frac{1 + \cos. \theta}{2}$, where θ is the angle of the batter with the vertical.

It is to be observed that it is the safe resistance of the mortar to crushing, which in every case determines the limit of pressure.

An Engineer is not justified in using weak stone or brick for a reservoir dam, and such stone or brick as is fit for the purpose always affords greater resistance to crushing than the mortar with which the work is put together.

The limit proposed by Professor Rankine, viz. 20,000 lbs. per square foot, vertical pressure, is therefore suitable for all cases where strong stone and good hydraulic mortar is used. Without good hydraulic mortar, the construction of a masonry dam should not be undertaken, neither should it be undertaken without such superintendence as will ensure the careful execution of the work or without rock foundations.

Summary of Conditions.

The summary of conditions is as follows:

Condition A. The centre of pressure, when the reservoir is full, must fall within the centre third of the thickness of the wall, at every level.

Condition B. The centre of pressure, when the reservoir is empty, must fall within the centre third of the thickness of the wall at every level. In the upper parts of dams with a road at the top it falls well within the centre third, for some distance from the top.

Condition C. The intensity of the vertical pressure at the outer face, reservoir full, must not exceed the limit laid down, which may be taken at 20,000 lbs. $\times \frac{1 + \cos. \theta}{2}$ per square foot.

Condition D. The intensity of the vertical pressure, reservoir empty, must not exceed the same limit.

Forms of Reservoir Walls.

The actual section adopted in any particular case depends to some extent upon the thickness which it may be necessary to give to the top.

Where the circumstances are such as to allow of a feather edge at the top, and the water is on a level with the top, a triangular section may be adopted for any height not requiring increased width of base to distribute the pressure, and this section "may be said (*R.A.M.*, p. 247) to be of *uniform stability*."

Further, it can be shewn that in such a dam, where the specific gravity of the material is twice, or more than twice the specific gravity of water, a vertical inner face is best, a batter on the inner face being advantageous, only when the specific gravity of the material is less than twice that of water.

The following are the batters proper for materials of different specific gravities, that of water being taken as 1.

Water.			Masonry.			Batter.
1	...	...	1·5	...	...	$\frac{h}{5}$
1	...	...	1·8	...	...	$\frac{h}{13}$
1	...	...	2·0	...	...	0

Where the weight of the masonry is more than twice that of water, the inner face would, considering economy only, have a overhanging batter, but a overhanging batter with a triangular section renders the fulfillment of condition B (p. 280), impossible, even if it were otherwise advantageous.

The case where it is possible to have a feather edge at top seldom occurs.

It more generally happens that for some reason, such, for instance, as to resist the shocks of waves in a large reservoir, or to afford a passage across the valley either for carts or foot passengers, or for the transport of materials for repairs, it is necessary to have some considerable thickness at the top.

The Barrage du Furens has a thickness of 5·7 mètres, and the Barrage du Ban of 5 mètres.

In such cases the section of the wall may be divided into four parts, each of which is considered in a somewhat different way.

1st. Working downwards from the top, there is a certain depth at which condition A is just fulfilled, and condition B more than fulfilled, by maintaining the same thickness of wall as at the top.

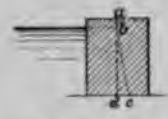
At any other section between the top of the dam and that depth, conditions A and B are more than fulfilled.

Conditions C and D are also more than met in this part of the wall, the height not being in any actual case sufficient to produce the intensity of pressure laid down as the limit.

The lines of resistance are as shewn in *Fig.* 8.

Fig. 8.

a b d line of resistance, reservoir empty.
a b c „ „ „ full.

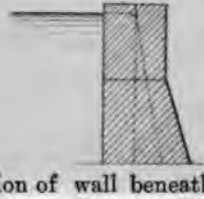


2nd. There is a section of wall below the part just dealt with, at the bottom of which conditions A and B are just fulfilled, by battering the outer face, leaving the inner face vertical, condition B being more than fulfilled at any less depth.

Conditions C and D are also fulfilled, or more than fulfilled in this part of the wall.

Fig. 9 shews this part of the wall, and the lines of resistance.

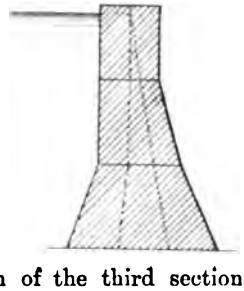
Fig. 9.



3rd. There is a section of wall beneath these two, where, in order just to fulfil condition B as well as condition A, it is necessary to increase the thickness of the wall by battering both the inner and outer faces, conditions C and D being also fulfilled or more than fulfilled in this section.

Fig. 10 shews this part of the wall, and the lines of resistance.

Fig. 10.

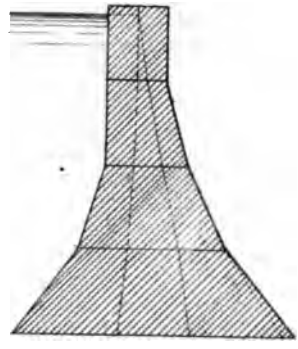


4th. At the bottom of the third section, condition C is only just fulfilled, and it becomes necessary in this fourth section to increase the batter of one or both faces, so as to spread the pressure over a larger surface, and so keep its intensity within the assigned limit.

In this section conditions A and B are more than complied with.

Fig. 11 shews this part of the wall, and the lines of resistance.

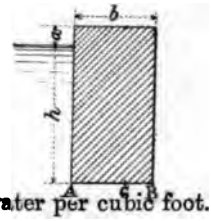
Fig. 11.



DESIGN OF A RESERVOIR WALL OR DAM.

To design the first part (*Fig.* 12), all measurements being taken in feet, all weights in lbs.

Fig. 12.



Let w = weight of water per cubic foot.
„ w_1 = „ „ masonry „ „
„ a = depth from the top of the wall to top water level.
„ b = breadth of wall at top.
„ h = unknown height at which condition A is just fulfilled.

Then the centre of pressure C on AB is at a distance from B = $\frac{1}{3}$ A B, and dealing with a layer of the wall 1 foot in length,

Moment of Pressure, $\frac{wh^3}{6} = w_1 (h+a) b \times \frac{b}{6}$ Moment of stability (1)

from which the value of h is obtained.

Let p = intensity of pressure at A, reservoir empty,
„ p_1 = intensity of pressure at B, reservoir full,

then $p = w_1 (h + a)$ (2)

$p_1 = 2w_1 (h + a)$ (3)

In no actual case is the height of this part sufficient to give p_1 a value so great as the limit of pressure for good hydraulic mortar.

In a rectangular wall, reservoir empty, the pressure at every section is uniformly distributed, and condition B is thus more than fulfilled.

To design the second part (*Fig.* 13).

Fig. 13.

Let it be determined to design this part in layers, m feet high.

Let W = weight of the portion already designed.
" c = distance of the vertical passing through the centre of gravity of this part from the inner edge.
" e = width of base of part already designed.
" x = width at the depth, $a + h + m$, below the top.

Then,

$$\frac{w\,(h+m)^3}{6} = \mathrm{W}\left(\tfrac{2}{3}x - c\right) + w_1\left\{em\left(\tfrac{2}{3}x - \tfrac{e}{2}\right) + (x-e)\,\frac{m}{2}\left(\tfrac{2}{3}x - e - \tfrac{1}{3}(x-e)\right)\right\}$$

$$= \mathrm{W}\left(\tfrac{2}{3}x - c\right) + \frac{w_1\,m\,(x^2 + e\,x - e^2)}{6} \qquad (4)$$

from which the value of x at the depth, $a + h + m$, is known.

To determine the pressure, p_1, at the outer face, reservoir full, find the weight of the masonry above the depth, $a + h + m$, *i.e.*, above DE (*Fig.* 13). Call this W_1.

Then $p_1 = \dfrac{2\,\mathrm{W}_1}{x}$, which must not exceed the limit of pressure proper for the angle of the batter, *i.e.*, 20,000 lbs. $\times \dfrac{1 + \cos.\,\theta}{2}$ per square foot.

To determine the pressure p at the inner face, when the reservoir is empty, find the centre of gravity of the section of the wall above DE, and draw a vertical through it.

If this vertical falls within the centre third of DE, the stability is assured, because the pressure cannot be greater than it is at the outer face, while the limit of pressure owing to there being no batter is greater. If it does not, the height of the layer must be reduced and the calculation repeated with this new value of m.

The intensity of the pressure is given by the formula at p. 240, *i.e.*,

$$n = \frac{2l - 3d}{l}\,\frac{2\,\mathrm{N}}{l},$$

where n is the intensity of the pressure on that edge of the base to which the centre of pressure is nearer,
l is the width of the base,

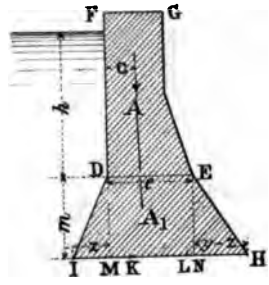
d is the distance from the centre of pressure to the nearer edge of the base,

N is the total pressure on the base.

This calculation is to be repeated with other layers, m feet deep, until the vertical through the centre of gravity of the section ceases to fall within the centre third of the base of that section.

To design the third part (*Fig.* 14).

Fig. 14.



In this part, the problem is to find the inner and outer batters, which will just fulfil conditions A and B.

Let FDEG (*Fig.* 14) represent the part of the dam already designed.

„ DEHI represent the new layer, the inner and outer batters of which it is required to determine.

„ L be the centre of pressure, reservoir full, and K the centre of pressure, reservoir empty.

Then by the conditions, IK, KL, LH, each $= \frac{1}{3}$ IH.

Let A = area of FDEG.

„ A_1 = „ „ DEHI.

„ e = DE.

„ y = addition to breadth of dam at IH, *i.e.*, IH $= e + y$.

Neglecting the small vertical component of the pressure of the water on the inner batter, DI, and taking moments about L,

$$\frac{w(h+m)^3}{6} = w_1 (A + A_1) KL = \frac{w_1}{3}(e+y)\left(A + \frac{m(2e+y)}{2}\right) \quad (5)$$

from which y is obtained.

Next, to find how this addition to the width is to be made, so as to fulfil condition B.

Let c = the horizontal distance from I to the vertical passing through the centre of gravity of A.

Let z = IM, the base of the inner batter.

Then NH, the base of the outer batter $= y - z$.

Taking moments about I, knowing that the vertical through the centre of gravity of the whole is at a distance from I $= \frac{1}{3}(e+y)$.

$$\frac{e+y}{3} = \frac{A(z+c) + \frac{mz^2}{3} + me\left(z + \frac{e}{2}\right) + \frac{m}{2}(y-z)\left(z + e + \frac{1}{3}(y-z)\right)}{A + A_1}.$$

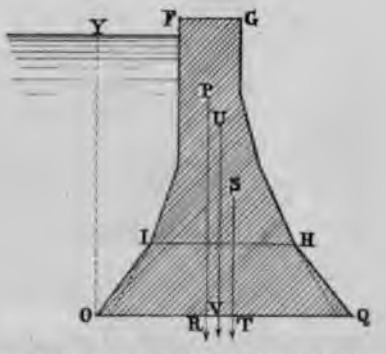
$$= \frac{A(z+c) + \frac{me}{2}(e+y+z) + \frac{my}{6}(y+z)}{A + A_1} \qquad (6)$$

from which z is obtained.

This calculation is to be repeated with other layers until the limit of pressure is reached on the outer face of the dam.

To design the fourth part (*Fig.* 15).

Fig. 15.



To this depth the batter of the inner face has been so small that the vertical component of the water, *i.e.*, the weight of the water lying vertically above the batter has been left out of consideration, the effect being to underestimate a little the stability of the dam, reservoir full; the stability, reservoir empty, not being affected by it.

From this point downwards, the internal batter becomes considerable; the effect of the water lying vertically above it is to increase materially the stability of the dam, and it becomes a question for the Engineer whether he will take the weight of the water into consideration or not.

Supposing the work to be so done that the dam is sound, and the leaks through it inconsiderable, and unless it is, the wall cannot be safe, there appears to be no valid reason why the advantage to be derived from the weight of the water should not be considered, and the section reduced accordingly.

In the Examples the design has, for illustration, been made both ways.

Fourth part, omitting the weight of the water resting on the internal batter.

The simplest way of designing this part is in layers by trial and error.

Let FGIH (*Fig.* 13), represent the part already designed.

Assume for trial a layer IOQH, the batter both of the inner and outer face being increased, the outer batter at a greater rate than the inner.

Let PR be the vertical passing through the centre of gravity of the part FGIH, and ST the vertical passing through the centre of gravity of the assumed new part IOQH.

Find the vertical UV passing through the centre of gravity of the whole section.

It must fall within the centre third of OQ. If it does not the inner batter must be increased, the calculations being repeated with the new dimensions.

Determine the intensity of the pressure at O by the formula given at p. 284.

Find the centre of pressure, reservoir full, by combining the pressure of the water, and the weight of the dam.

The resultant must fall within the centre third of the base, and if it does not the outer batter must be increased, and the calculations repeated.

Determine the intensity of the pressure at Q, reservoir full, by the formula given at p. 284.

If the intensity of the pressure at O is too small, while that at Q is too large, it points to a reduction of the inner batter, and an increase of the outer batter; if the intensities of the pressure at O and Q are both too great, it points to an increase of both batters, while if the intensity at O is too great, while that at Q is too small, it points to an increase of the inner, and a decrease of the outer batter.

By a few trials, the proper dimension, which just fulfils the conditions, will be arrived at.

This method is troublesome, but the calculations are very simple, and it would be more troublesome to arrive at the proper dimensions by means of equations.

The remaining layers of the dam would be designed in the same manner.

Fourth part, taking the weight of the water into account.

In this case the method of proceeding as far as the determination of the pressure at the inner edge, reservoir empty, is the same as before.

For the determination of the pressure at the outer edge, find the vertical passing through the centre of gravity of the dam, and of the water (FIOY) (*Fig.* 15), resting on it.

Combine the horizontal pressure of the water with the weight of the dam, and of the water resting on it, to find the centre of pressure, reservoir full.

It must fall within the centre third of the base, and the intensity of the pressure at the outer edge as determined by the rule given at p. 284, must not exceed the limit laid down.

The alterations in the assumed addition would be made in the manner already described for the case, where the weight of the water is left out of consideration.

EXAMPLE I.

A wall or dam is to be constructed across a somewhat broad gorge, so that no diminution of section can be effected by building the wall in a curve on plan, although it may be well so to build it. There is no opening through the wall.

The material of which the wall is to be built is trap in strong hydraulic mortar, the latter being capable of resisting with safety a vertical pressure of 20,000 lbs. per square foot; the vertical pressure to be gradually diminished as the batter increases, the limit of pressure being 20,000 lbs. $\times \frac{1 + \cos.\theta}{2}$ per square foot, where θ is the angle which the outer face makes with the vertical.

The weight of the masonry, which is to be well built rubble, not in courses, is to be taken at twice that of water, which may be taken at 62·5 lbs. per cubic foot.

The co-efficient of friction of the masonry may be taken at ·7.

The top of the dam is to be 19 feet in width to allow of a road, and the top water level is on a level with this.

The total depth of water is to be 200 feet. The foundations and sides of the valley are of trap, and able to bear this weight of water.

The design is to be made in accordance with the conditions laid down at p. 280.

Remarks.

The French Engineers gave the Barrage du Furens a curvature towards the reservoir of 5 mètres, the chord of the arc being 100 mètres, and in the Memoire by Monsieur Delocre, already referred to, a method is given for calculating the reduction, which may be made on account of the transmission of the pressure to the sides of the valley.

On this subject, Professor Rankine says (*Engineer*, 5th Jan. 1872):

"As regards the effect of giving the wall a curvature in plan, convex towards the reservoir, I look upon this as a desirable, and in many cases an essential precaution, in order to prevent the wall from being bent by the pressure of the water into a curved shape concave towards the water, and thus having its outer face brought into a state of tension horizontally, which would probably cause the formation of vertical fissures, and perhaps lead to the destruction of the dam. I consider, however, that calculations of stability which treat the dam as a horizontal arch are so uncertain as to be of very doubtful utility; and I would not rely upon them in designing the profile. In fixing the radius of horizontal curvature, I consider that the Engineer should be guided by the form of the gorge in which the dam is to be built, making that radius as short as may be consistent with convenience in execution, and with making the ends of the dam abut normally against the sound rock at the sides of the gorge."

As regards there being no opening through the dam; it is better to avoid such openings, as likely to induce unequal settlements, and there

is seldom any serious difficulty in running a tunnel round the end of the dam in the natural rock to receive the outlet pipe with the necessary arrangements.

The limit of vertical pressure 20,000 lbs. per square foot is that given by Professor Rankine, while the reduction of the limit as the batter increases, is proposed as safe, and approximately meeting the increase of stress due to the increase of the batter.

The co-efficient of friction is given, but if the work is properly built, and not in layers, no sliding can take place, unless accompanied by the shearing of a great number of blocks.

The width of the dam at the top is enough for a road and guard walls.

The depth of water 200 feet is considered to be as much as or more than is likely to be required.

It is to be observed, that in order to shorten the calculations, the layers have been taken of greater depth, than it would be desirable to take them in preparing a design for actual execution.

It is obvious, that the greater the number of the layers, the more accurately will the section fulfil the required conditions at every level.

First part. ABCD. (*Plate* I., *Fig.* 1.)

Here $w = 62{\cdot}5$ lbs.
$w_1 = 125$,,
$b = 19$ feet
$a = 0$,,
h is unknown.

Then by eq. (1) p. 283.

$$\frac{wh^3}{6} = w_1 (h + a) \frac{b^2}{6}$$

and substituting the values

$$h = 26{\cdot}87 \text{ feet}$$

By eq. (2) p. 283, $p = w_1 (h + a) = 3{,}358$ lbs.
(3) ,, ,, $p_1 = 2w_1 (h + a) = 6{,}716$,,

Drawing this part to a large scale, and tracing the resultant of the pressure, and of the weight, a check is afforded on the accuracy of the calculations.

Second part. DCEF. (*Plate* I., *Fig.* 1.)

Assume a layer $23{\cdot}13$ feet deep.

Here $W = 63{,}816{\cdot}25$ lbs.
$c = 9{\cdot}5$ feet
$e = 19$,,
$m = 23{\cdot}13$,,
$h + m = 50$,,
x = width of base at a depth of $(h + m)$ feet

Then by eq. (4) p. 284,

$$\frac{w(h + m)^3}{6} = W (\tfrac{1}{2} x - c) + \frac{w_1 m (x^2 + ex - e^2)}{6}$$

and substituting the values

$$e = 31{\cdot}2 \text{ feet.}$$

The weight of the dam above this level=136,386·62 lbs., whence (p. 283), $p_1 = \frac{2 \times 136{,}386{\cdot}62}{31{\cdot}2} = 8{,}743$ lbs.

To find the value of p.

The vertical passing through the centre of gravity of the dam above this level is at a distance of 11·25 feet from the inner face, and it falls within the centre third of the base.

The intensity of the pressure at the inner face must, therefore, be less than at the outer face, and substituting in the formula at p. 284,

$$p = \frac{2 \times 31{\cdot}2 - 3 \times 11{\cdot}25}{31{\cdot}2} \times \frac{2 \times 136{,}386{\cdot}62}{31{\cdot}2} = 7{,}845 \text{ lbs.}$$

Adding this part to that already drawn, and tracing the resultant of the pressure and of the weights, the accuracy of the calculation is checked.

Second part continued. FEGH. (*Plate* I., *Fig.* 1.),

As the centre of pressure, reservoir empty, at the last level was not very far (only ·85 foot) within the centre third of the width, it is probable that a layer of small depth, say 10 feet, will give the level at which conditions A and B are just fulfilled.

Assume for trial a layer 10 feet deep.

Then W = 1,091·09 feet × 125 lbs. = 136,386·62 lbs.
$c = 11{\cdot}25$,,
$e = 31{\cdot}2$,,
$m = 10$,,
$h+m = 60$,,

and substituting these values in equation (4), p. 284.

$$x = 37{\cdot}86 \text{ feet.}$$

Also $p_1 = 9{,}485$ lbs.

The vertical passing through the centre of gravity of the dam above this level, is 12·71 feet (*i.e.*, ·09 foot) within the centre third. This depth may therefore be taken as fulfilling with sufficient accuracy, conditions A and B.

Then $p = p_1 = 9{,}485$ lbs.

Adding this layer to those already designed, and tracing the resultant of the pressure, and of the weight, the accuracy of the calculation is checked.

Third part. HGIK. (*Plate* I., *Fig.* 1.)

Assume a layer of 30 feet deep.

Here A = 1,436·39 square feet.
$c = 12{\cdot}71$ feet.
$e = 37{\cdot}86$,,
$m = 30$,,
$h+m = 90$,,
y = addition to width at the depth $(h + m)$.

Then by eq. (5), p. 285.

$$\frac{w(h+m)^3}{6} = \frac{w_1}{3}(e+y)\left(A + \frac{m(2e+y)}{2}\right)$$

and substituting the values

$$y = 24{\cdot}22 \text{ feet.}$$

$$\text{and } e + y = 62{\cdot}08 \text{ ,,}$$

Now let z = base of the inner batter.

A_1 = area of additional section.

Then by eq. (6) p. 286.

$$\frac{e+y}{3} = \frac{A(z+c) + \frac{me}{2}(e+y+z) + \frac{my}{6}(y+z)}{A + A_1}$$

and substituting the values

$$z = 2{\cdot}05 \text{ feet.}$$

$$\text{Also } p_1 = p = 12{,}145 \text{ lbs.}$$

$$\text{The limit for } p_1 = 20{,}000\,\frac{1+\cos. 37^\circ}{2} = 17{,}986 \text{ lbs.}$$

$$\text{,, for } p = 20{,}000\,\frac{1+\cos. 4^\circ}{2} = 19{,}976 \text{ lbs.}$$

so that the limit of pressure is not reached either at the outer or the inner edge of the layer.

This layer may now be drawn, and the centres of pressures laid down as in previous layers.

Third part continued. KILM. (*Plate* I., *Fig.* 1.)

Assume a layer 30 feet deep.

Here $A = 2{,}936$ square feet.

$c = 20{\cdot}69$ feet.

$e = 62{\cdot}08$,,

$m = 30$,,

$h + m = 120$,,

y = addition to width at a depth of 120 ft.

Substituting these values in eq. (5), p. 285, as before,

$$y = 22{\cdot}19 \text{ feet.}$$

$$\text{and } e + y = 84{\cdot}27 \text{ ,,}$$

$$\text{Also } A_1 = 2{,}195{\cdot}25 \text{ square feet.}$$

Substituting the values in eq. (6), p. 286, as before,

$$z = {\cdot}61 \text{ feet.}$$

$$\text{Also } p = p_1 = 15{,}223 \text{ lbs.}$$

$$\text{The limit for } p_1 = 20{,}000\,\frac{1+\cos. 36^\circ}{2} = 18{,}092 \text{ lbs.}$$

$$p = 20{,}000\,\frac{1+\cos. 2^\circ}{2} = 19{,}994 \text{ lbs.}$$

so that the limit of pressure is not reached either at the outer or the inner edge of the layer.

This layer may be drawn as in the case of previous layers.

Third part continued. MLNO. (*Plate* I., *Fig.* 1.)

Assume a layer 30 feet deep.

Here A = 5,131·25 square feet.
c = 28·09 feet.
s = 84·27 „
m = 30 „
$h+m$ = 150 „
y = addition to width at a depth of 150 feet.

Substituting these values in equation (5) p. 285.

y = 21·72 feet.
$c + y$ = 105·99 „
Also A_1 = 2,850·9 square feet.

and substituting these values in eq. (6) p. 286.

z = ·37 feet

Also $p = p_1$ = 18,825 lbs.

The limit for $p_1 = 20{,}000 \frac{1 + \cos. 36^\circ}{2}$ = 18,092 lbs.

The limit for $p = 20{,}000 \frac{1 + \cos. 2^\circ}{2}$ = 19,994 lbs.

so that the limit of pressure is exceeded at the outer edge, and it would be desirable in practice to assume a smaller depth for this layer and recalculate.

For the purpose of this paper, it is near enough, and the layer may be drawn and the calculations checked as before.

Fourth part. ONPQ. (*Plate* I., *Fig.* 1), neglecting the weight of the water on the inner batter.

Assume a layer 50 feet deep and a base of 187·5 feet as shewn (this width having been arrived at after three trials).

Then area of new layer = 7,337·25 square feet
„ „ part already designed = 7,981·15 „ „
15,318·4 „ „

The vertical through the centre of gravity of the whole is at a distance of 67 feet from the inner lower edge of the whole section.

Then by formula, p. 284.

$$p = \frac{2 \times 187{\cdot}5 - 3 \times 67}{187{\cdot}5} \times \frac{2 \times 125 \times 15{,}318{\cdot}4}{187{\cdot}5} = 18{,}954 \text{ lbs.}$$

the limit for p being $20{,}000 \frac{1 + \cos. 19^\circ}{2}$ = 19,455 lbs.

Drawing by construction, the resultant of the pressure and of the weight, the centre of pressure, reservoir full, is found to be 76·7 feet from the outer lower edge of the whole section.

Then by formula, p. 284.

$$p_1 = \frac{2 \times 187{\cdot}5 - 3 \times 76{\cdot}7}{187{\cdot}5} \times \frac{2 \times 125 \times 15{,}318{\cdot}4}{187{\cdot}5} = 15{,}784 \text{ lbs.}$$

the limit for p_1 being $20{,}000 \frac{1 + \cos. 53^\circ}{2}$ = 16,018 lbs.

or this breadth is a little too great, and might be reduced a trifle.

It is to be observed that there is no check by construction upon the accuracy of the calculations in this last part of the dam.

The actual pressure at P, reservoir full, taking the weight of the water into consideration is 13,291 lbs. per foot super. so that there is an unnecessary quantity of material in this layer, due to the neglect of the weight of the water.

Fourth Part. ONRS. (*Plate* I., *Fig.* 1), taking the weight of the water into consideration.

Assume a layer 50 feet deep, and a base of 179·5 feet as shewn.

Then area of new layer = 7,137·25 square feet.
„ „ part already designed = 7,981·15 „ „
15,118·4 „ „

The vertical through the centre of gravity of the whole is at a distance of 68·5 feet from the inner lower edge of the whole section.

Then by formula, p. 284.

$$p = \frac{2 \times 179{\cdot}5 - 3 \times 68{\cdot}5}{179{\cdot}5} \times \frac{2 \times 15{,}118{\cdot}4 \times 125}{179{\cdot}5} = 18{,}018 \text{ lbs.}$$

the limit for p being $20{,}000 \frac{1 + \cos. 22^\circ}{2} = 19{,}272$ lbs.

or this inner batter is probably rather too much.

The vertical through the centre of gravity of the water lying on the inner face of the dam, and of the dam itself being drawn, and the pressure, and the whole weight of dam and water being compounded, the centre of pressure, reservoir full, is found to be 79 feet from the outer lower edge of the whole section.

Then by formula, p. 284.

$$p_1 = \frac{2 \times 179{\cdot}5 - 3 \times 79}{179{\cdot}5} \times \frac{2 \times 2{,}128{,}238}{179{\cdot}5} = 16{,}116 \text{ lbs.}$$

the limit for p_1 being $20{,}000 \frac{1 + \cos. 47^\circ}{2} = 16{,}820$ lbs.

From this it appears that the width is a little too much, and might be reduced somewhat on both faces.

EXAMPLE II.

The conditions in this case are precisely the same as in Example I., with the single exception that the weight of the masonry is taken at 162·5 lbs. per cubic foot, instead of at 125 lbs., the object being to show the difference of section resulting from the different weight of the material. It is not considered necessary to give the calculations as in Example I. the method being precisely the same in both; but the section calculated is given in *Plate* I., *Fig.* 2, and in *Plate* III. the two sections are imposed on one another.

COMPARISON OF THE TWO DAMS.

It will be seen that for a dam of any less height than about 180 feet, there is less finished material in the dam of the heavier

masonry, while above this height the lighter masonry gives a smaller section.

Supposing an Engineer to have a choice of materials for his masonry, it does not of course follow that the heavier masonry would be cheaper, even where less in quantity, the increase of expense in raising and laying the heavier stone being probably greater than the saving arising from the smaller quantity of it required.

Professor Rankine's Section.

Fig. 2, *Plate* II., is the section of the reservoir dam proposed by Professor Rankine (see the *Engineer*, 5th January, 1872, in which the width at the top is 18·74 feet, the weight of materials the same as in Example I., and the faces are given by two logarithmic curves.

This design fulfils all the conditions laid down by him, and provides for the reduction of the pressure at the outer face as the batter increases in a far greater degree than in Example I.

There is more masonry all the way down in this design than in that given in Example I.

Underground Tanks or Reservoirs.

Masonry or brickwork tanks are almost invariably constructed underground, the water being thereby kept cooler and purer.

Plate III. gives the plan and section of an ordinary service reservoir.

When the reservoir is empty, the side-walls are subjected to the thrust of the arch outwards, and the pressure of the earth inwards, and when the reservoir is full, the side walls are subjected, in addition, to the pressure of the water outwards.

They must therefore be designed to meet both pressures.

The end walls from which no arches spring, are frequently built as horizontal arches to enable them better to resist the pressure of the earth when the reservoir is empty.

They are subjected to the pressure of the earth or of the water only.

In doubtful ground, inverts are sprung between the feet of the piers to distribute the pressure caused by the weight of the arched covering and its loading.

STONE AND BRICK ARCHES.

EVERY theory of the arch is strictly applicable only to arch-rings, composed entirely of wedge shaped blocks (voussoirs) of incompressible materials, resting on or against immoveable abutments.

But experience shews that, when the centres are struck, the deformation of a well built arch-ring is so small, that the results obtained from the consideration of the stability of a stone arch-ring, supposed to be incompressible, may with perfect safety be applied, not only to actual arch-rings having slightly compressible stone voussoirs, but also to well built rubble and brick arch-rings.

For ordinary spans for ordinary purposes, such as small road or railway arches, or casemate arches, the thickness of the arch-ring, as well as the dimensions of the abutments necessary to resist the thrust of the arch have been sufficiently determined by actual practice, and there is generally no necessity for calculating them.

For larger spans, the weight of the arch-ring and its permanent load is so large in comparison with any moving load that can come upon it, that it has usually been the practice, unless there has been some special reason for thinning down the arch-ring, to treat the moving load as a uniformly distributed load, and to include it with the weight of the structure.

In very large arches, the difference between the permanent load, and the greatest possible moving load is still greater, but it is generally an object to keep down the depth of the voussoirs, and the intensity of the thrust is so great that it is necessary to enquire into the stability of the arch-ring.

In both of the last classes of arch, where the experience obtained from actual practice is not so great as in the case of smaller arches, it is necessary to determine the dimensions of the abutments by calculation, or diagram, or the two combined, and to do this, the thrust of the arch must be ascertained.

In this paper the following method is followed:

(1.) On the assumptions, that the arch-ring consists of incompressible voussoirs resting on, or against immoveable abutments; that each voussoir has its load, which is entirely vertical, and includes the weight of the voussoir, applied to it directly without the intervention of any backing; and that Moseley's Principle of least Resistance is applicable to the arch, it is shown how to trace the true line of resistance through a proposed arch-ring, and from it to determine whether the arch-ring is stable under the intended load; (*a*) when the arch-ring and load are both

symmetrical with regard to the crown; (*b*) when either the arch-ring or the load, or both are unsymmetrical with regard to the crown.

(2.) It is shewn how the true line of resistance through a slightly compressible arch-ring of the same original form may be deduced from that of the incompressible arch-ring with sufficient accuracy for the purposes of the Engineer, and the dimensions of the abutments determined from it.

The method is taken from a work by Dr. Hermann Scheffler, Baurath, of Brunswick, entitled "*Theorie der Gewölbe, Futtermauern, and Eisernen Brücken*," translated into French, under the title "*Traité de la Stabilité des Constructions*," by M. Victor Fourniè, Ingenieur des Ponts and Chaussèes, and it is the method which was explained by Mr. Cawthorne Unwin, B.Sc., in the course of Lectures on *Railway Construction*, delivered at the School of Military Engineering, in 1869-70.

Conditions of Stability of an Incompressible Arch-ring.

The conditions of stability in an incompressible arch-ring are the same as in a wall or buttress of incompressible materials (p. 236), and they are:

(1.) The line of resistance must fall everywhere within the arch-ring.

(2.) The direction of the pressure on any joint must not make with the normal to the joint an angle greater than the angle of friction, *i.e.*, the angle of which the co-efficient of friction of the material of the joints is the tangent.

First Condition of Stability.

The forces maintaining the equilibrium in any voussoir ABCD (*Plate* I. *Fig.* 1), are the weight W of the voussoir and its load, acting through their common centre of gravity, the pressure P of the voussoir above, and the reaction R of that next below; and it is necessary to the stability of this voussoir, that the centre of pressure K on AB should lie within AB.

If R the resultant of P and W were to pass below A, the voussoir would rotate about A, as in *Fig.* 2, *Plate* I., and if it were to pass above B, the voussoir would rotate about B, as in *Fig.* 3, *Plate* I.

The condition of stability necessary for one voussoir is necessary for each and all of the voussoirs, and hence it follows that, for stability against rotation, the line of resistance must lie everywhere within the arch ring.

Second Condition of Stability.

As regards the second condition of stability it may be at once stated that arches seldom, probably never, tend to fail by the sliding of the voussoirs on one another, and this condition will therefore be left out of consideration for the present, although it will be shewn ultimately how to determine whether an arch-ring, found to be stable as regards rotation is stable as regards sliding.

Application of the Principle of Least Resistance.

In general, where an arch-ring is stable, it is possible to draw within the arch-ring an infinite number of lines of resistance, each consistent with equilibrium, and it is necessary, in order to determine which of these is the actual line of resistance, to resort to Moseley's Principle of least Resistance.

This principle as stated by Professor Rankine (*R.A.M.*, p. 215) is as follows:

"If the forces, which balance each other in or upon a given body or structure be distinguished into two systems, called respectively *active* and *passive*, which stand to each other in the relation of cause and effect, then will the passive forces be the least which are capable of balancing the active forces, consistently with the physical condition of the body or structure."

The result of this is, that of all lines of resistance lying within the arch-ring, the real line of resistance is that which gives to the horizontal thrust in the arch-ring, which, with vertical forces, is the same everywhere, its minimum value, and the problem is to find that line.

Method of Drawing a Line of Resistance through an Arch-Ring.

Let *Fig.* 4, *Plate* I., represent an arch-ring of any form, loaded with vertical loads in any manner whatever, and let the value, direction, and point of application of the resultant, R, at any joint AB be supposed to be known.

Draw vertical lines, 1 to 8, through the centres of gravity of the voussoirs, and their loads.

Produce R to intersect 4 in a, make $a\ b$ to represent R (on any scale of parts) and $a\ c$ to represent W_4. Complete the parallelogram; then $a\ d$ is the resultant pressure on C D, and K is the centre of pressure, which falls outside of the arch-ring.

Produce $a\ d$ to intersect 3 in e; make $e\ f = a\ d$, and $e\ g = W_3$, complete the parallelogram; then $e\ h$ is the resultant pressure on E F, and L the centre of pressure.

In a similar manner, find the centres of pressure M and N, on the joints HG and OP, and on the right side of the arch-ring.

Joining NMLK, and so on to the right, the line of resistance is obtained.

It will be seen that this line lies above the extrados in the neighbourhood of the joint CD, and below the intrados on the right of the arch-ring.

It is not therefore the true line of resistance, but the object now is to show how to draw a line of resistance with any assumed pressure and any point of application.

It is obviously the same as in other blockwork structures (p. 232).

It is a polygon, but if the voussoirs were infinitely thin, it would evidently be a curve of some kind or other, and the lines of resistance shewn in *Plates* II., III., and IV. are drawn as curves on this supposition.

DETERMINATION OF THE LINE OF RESISTANCE, WHICH CORRESPONDS TO THE MINIMUM THRUST, IN A SYMMETRICAL ARCH-RING SYMMETRICALLY LOADED.

Let ABCD (*Plate* II., *Fig.* 1) be a half arch-ring of any form, and W its weight, including any external loads (all vertical), GW the vertical passing through the centre of gravity of W.

Let KI be any line of resistance, Q the thrust at the crown, which from the symmetry of the arch-ring and load must be horizontal, and which, applied at K, gives rise to the line of resistance K I.

Let p be the horizontal distance of the vertical GW from I, and h the vertical height of K above I.

Then taking moments about I

$$h\,Q = p\,W$$

$$Q = \frac{p\,W}{h}$$

For any other line of resistance, Q will be less, as p is less and h is greater; so that the least value of Q will be obtained by drawing a line of resistance through D and A, (*Plate* II., *Fig.* 1.)

From this proceeds the following method of enquiring into the stability of an arch-ring.

A line of resistance, which may or may not lie within the arch-ring, is drawn (as described at p. 297) through the highest point D of the crown, and the innermost point A of the springing. Five cases then present themselves:

FIRST CASE. The line of resistance DA lies entirely within the arch-ring (*Plate* II., *Fig.* 1). It complies with both conditions, of being a minimum, and lying within the arch-ring, and is therefore the true line of resistance.

SECOND CASE. The line of resistance DA cuts the intrados in one point (*Plate* II., *Fig.* 2), or more points (*Plate* II., *Fig.* 3), but does not cut the extrados.

Then the line of resistance corresponding to the minimum value of Q passes through the extrados at the crown D, and touches the intrados CA; and provided it lies everywhere within the arch-ring, it is the true line.

DEI (*Plate* II., *Fig.* 4) is such a line, E its point of contact with the intrados.

If the line of resistance cuts (*Plate* II., *Fig.* 5) the extrados between D and E, the equilibrium of the arch-ring is to be considered as belonging to the Fourth Case, further on.

If the line of resistance cuts the extrados below E and above the springing (*Plate* II., *Fig.* 6,) equilibrium is impossible with that arch-ring, and that load, and one or the other or both must be altered.

In this case the arch-ring would fail, on the removal of the centres, by the falling in of the crown, and the spreading of the haunches (*Plate* II., *Fig.* 7).

The limit of stability is attained, when the line of resistance D E I passes through the exterior point B of the springing (*Plate* II., *Fig.* 8),

and the further I is inside of B, the greater the stability of the arch-ring.

THIRD CASE. The line of resistance D A cuts the extrados at the crown, and again lower down, but does not cut the intrados above the point A (*Plate* III., *Fig.* 1). Then the line of resistance corresponding to the minimum value of Q starts from some point in the crown below D, touches the extrados between crown and springing as at F (*Plate* III, *Fig.* 2), and cuts the intrados at A; and provided it lies everywhere within the arch-ring, it is the true line.

A F K (*Plate* III., *Fig.* 2) is such a line, F its point of contact with the extrados.

If the line of resistance A F K (*Plate* III., *Fig.* 3) cuts the intrados below the point F, the equilibrium of the arch-ring is to be considered as belonging to the Fourth Case, further on.

If it cuts the intrados above the point F (*Plate* III., *Fig.* 4), equilibrium is impossible with that arch-ring and that load.

In this case, the arch-ring would fail on the removal of the centres, by the lower parts falling inwards, lifting the upper part of the arch (*Plate* III, *Fig.* 5).

The limit of stability is attained, when the line of resistance A F K passes through the lowest point of the crown (*Plate* III., *Fig.* 6), and the further K is from C the greater the stability of the arch.

FOURTH CASE. The line of resistance D A cuts the extrados and intrados without passing more than once through the arch-ring (*Plate* IV., *Fig.* 1).

The line of resistance corresponding to the minimum value of Q touches the extrados and the intrados, the point of contact with the intrados being below that with the extrados, and provided it lies everywhere within the arch-ring, it is the true line.

K F E I is such a line, F and E its points of contact with the extrados and intrados (*Plate* IV., *Fig.* 2).

If the line of resistance cuts the intrados above the point F (*Plate* III., *Fig.* 4), or the extrados below the point E (*Plate* II., *Fig.* 6), or both at once (*Plate* IV., *Fig.* 3), equilibrium is impossible with that arch-ring and that load.

In this case the arch-ring would fail, on the removal of the centres, in one of the ways shewn in *Figs.* 4 and 5, *Plate* IV.

The limit of stability is attained when the line of resistance passes through B or C, or through both these points (*Plate* IV., *Fig.* 6), and the arch-ring is more stable in proportion as the points I and K are respectively nearer to A and D.

FIFTH CASE. The line of resistance D A cuts the intrados and extrados in any number of points, and traverses the thickness of the arch-ring more than once.

Equilibrium with this arch-ring and this load is impossible, the arch failing as shewn in *Plate* IV., *Fig.* 7.

TO TRACE THE TRUE LINE OF RESISTANCE.

To do this, it is necessary either to trace a line of resistance which

passes through two given points (Case 1), or which, passing through one point, touches a given curve (Cases 2 and 3), or which touches two given curves (Case 4).

To do this analytically is very difficult, but it may be done by construction with comparative facility and with sufficient accuracy for the purposes of the Engineer, as described further on.

It may be said here, however, that the three sets of conditions just mentioned, under which the real line of resistance may have to be drawn, may in every case be reduced to one, viz., the first of the three, drawing a line of resistance through two given points.

For if the trial line of resistance D A cuts the intrados near the springing (*Plate* II., *Fig.* 2), or the extrados near the crown (*Plate* III., *Fig.* 1), or both (*Plate* IV., *Fig.* 1), the form of that line indicates with considerable accuracy the points of contact of the real line with the extrados and intrados, those points of contact being approximately the points where the line D A is farthest from the extrados or intrados.

If a second line of resistance traced through these approximate points of contact does not satisfy exactly the required conditions, it will nevertheless very nearly do so, and a third line drawn in a manner similar to the second will certainly be near enough for all purposes.

In illustration of this, let *Fig.* 1, *Plate* V, represent the half of a symmetrically loaded arch-ring, and D A the trial line of resistance drawn through the extrados at the crown and the intrados at the springing.

It cuts the extrados below the crown and the intrados above the springing.

Then, approximately, the points of contact of the real line of resistance are at F and E, where the trial line of resistance is furthest from the extrados and intrados respectively.

Let K be the point of application of the thrust Q in the crown; then taking moments about F,

$$Q \times K a = W_1 \times F b \qquad (1)$$

where W_1 is the weight of the arch-ring, and its load between the joint F G and the vertical D c, acting through their common centre of gravity.

Taking moments about E,

$$Q \times K c = W_2 \times E d \qquad (2)$$

where W_2 is the weight of the arch-ring, and its load between the joint E H and the vertical D c, acting through their common centre of gravity.

Let $K a = x$, $c a = a$, $F b = b$, $E d = c$

Then equations (1) and (2) become, respectively,

$$Q x = W_1 b \qquad (3)$$

$$\text{and } Q (x + a) = W_2 c \qquad (4)$$

from which Q and x are obtained, and the new line of resistance drawn.

This line may cut the extrados or intrados, or both; and if it does, it may be necessary to choose two other points, whose position will

approximately be, where the new line of resistance is farthest from the extrados and intrados respectively, and to take moments about them and construct a third line of resistance. But this accuracy will seldom be necessary for the purposes of the Engineer.

The case just stated is the general case (No. 4, p. 299), of which cases (2) and (3) are particular cases.

In case (2), as the true line passes through the extrados at the crown, it is only necessary to take moments about the point E (*Plate* II., *Fig.* 4).

In case (3) the true line passes through the intrados at the springing, and moments are taken about the point F (*Plate* III., *Fig.* 2), and the intrados at the springing.

In many cases it saves trouble to draw the trial line through points in the crown and springing other than those hitherto chosen, and a little practice will point out when this is the case.

THICKNESS OF THE ARCH-RING AT THE CROWN.

Before applying the principles above laid down to any particular case, it is necessary to assume the figure of the arch-ring; and it is therefore desirable, before giving special examples, to give some rule for determining the thickness of the arch-ring at the crown and springing, so that the pressure shall not exceed what the material can safely bear.

Professor Rankine's rule (*R.C.E.* p. 425, and *U.R.T.* p. 188) is as follows:

For the depth of the keystone, take a mean proportional between the radius of curvature of the intrados at the crown and a constant whose values are:

For a Single Arch ... ... ·12 foot
For an Arch forming one of a series... ·17 „

or in symbols,

Depth of Keystone, for a single arch, in ft. $= \sqrt{\cdot 12 \times \text{radius at crown.}}$

Depth of Keystone, for an arch of a series, in ft. $= \sqrt{\cdot 17 \times \text{radius at crown.}}$

The depth at the springing in stone arches is generally more than that at the crown, the pressure being far greater there; but it is generally determined in the first instance by considerations of appearance, and tested afterwards in the manner shewn in Example I.

THE LINE OF RESISTANCE, WHICH CORRESPONDS TO THE MINIMUM HORIZONTAL THRUST, IN AN UNSYMMETRICAL OR UNSYMMETRICALLY LOADED ARCH.

Let ABCD (*Plate* V., *Fig.* 2), be an arch-ring under any distribution of vertical load.

The pressure of the load gives rise to reactions, R_1, R_2, at the abutments, which in all actual cases are inclined.

Let W be the weight of the arch-ring and its load; then resolving R_1, R_2 vertically and horizontally, equilibrium requires that

$$Q_1 = Q_2$$
$$P_1 + P_2 = W$$

and by the Principle of least Resistance, the points of application of R_1 and R_2 must be such that $Q_1=Q_2$ shall have the minimum value, consistent with the construction of a line of resistance lying everywhere within the arch-ring.

For one joint, EF, in the arch-ring, the thrust is horizontal, and this joint divides the arch into two parts, such that P_1 is equal to the weight of the arch-ring and its load, lying on that side of EF, while P_2 is equal to the weight of the arch-ring and its load lying on the other side of EF.

Then, assuming the points A and C (*Plate* V., *Fig.* 3) as two points in the trial line of resistance, the vertical components of the reactions R_1 and R_2 are obtained in the usual manner of calculating reactions.

Assuming F as a third point in the trial line of resistance, that line is drawn through A, F, and C.

If that line lies everywhere within the arch-ring, it is the true line of resistance. If it passes outside the arch-ring (*Plate* V., *Fig.* 3), let N, M, L be the points in the intrados and extrados, at which the trial line of resistance is farthest from them.

A second line of resistance is drawn through the points N, M, L.

This line will approximate more closely than the trial line to the true line of resistance, and if it is not sufficiently accurate, a third line must be traced, unless it is considered better to alter the form of the arch.

Method of Drawing a Line of Resistance through three points.

Let I, L, K (*Plate* V., *Fig.* 4), be three points, through which it is required to draw a line of resistance, and ACB a horizontal line through the highest point of the arch;

Let Q be the horizontal thrust in the arch, which, with vertical loads, is the same everywhere;

„ H, the vertical component of the thrust S in the vertical passing through the crown;

„ q, the vertical distance from the point of application of S to the horizontal line ACB;

„ g_1, h_1; g_2, h_2; g_3, h_3, the horizontal and vertical co-ordinates of I, K, L, the origin of co-ordinates being at C.

„ P′, P″, P‴, the vertical components of the thrusts at I, K, L:

„ P_1, P_2, P_3, the weights of the segments CI, CK, CL with their loads.

„ p_1, p_2, p_3, the horizontal distances of the centres of gravity of these segments from the point C.

Also for abbreviation, let

$g_1-p_1=a_1$	$g_2-p_2=a_2$	$g_3-p_3=a_3$
$h_1-q=b_1$	$h_2-q=b_2$	$h_3-q=b_3$
$g_2+g_3=d_1$	$g_1-g_3=d_2$	$g_1+g_2=d_3$
$h_2-h_3=e_1$	$h_1-h_3=e_2$	$h_1-h_2=e_3$

Then
$$P'+P=P_1 \quad (1)$$
$$P''-P=P_2 \quad (2)$$
$$a_1P_1-g_1P=h_1Q \quad (3)$$
$$a_2P_2-g_2P=b_2Q \quad (4)$$
$$a_3P_3-g_3P=b_3Q \quad (5)$$

When the third point given is in the vertical passing through the crown, the value of q is known, and $g_3=o$, $h_3=q$, $P_3=o$.

Then from (3) and (4)

$$P=\frac{a_1 e_1 P_1-a_2 e_2 P_2}{e_2 d_3-e_3 d_2} \quad (6)$$

$$Q=\frac{a_1 d_1 P_1-a_2 d_2 P_2}{e_2 d_3-e_3 d_2} \quad (7)$$

When the third point L is not in the vertical passing through the crown,

$$P=\frac{a_1 e_1 P_1-a_2 e_2 P_2+a_3 e_3 P_3}{e_2 d_3-e_3 d_2} \quad (8)$$

$$Q=\frac{a_1 d_1 P_1+a_2 d_2 P_2-a_3 d_3 P_3}{e_2 d_3-e_3 d_2} \quad (9)$$

$$q=h_1-\frac{a_1 P_1-g_1 P}{Q} \quad (10)$$

In both cases the vertical components of the thrusts at I and K are already known:

$$P'=P_1-P \quad (11)$$

$$P''=P_2+P \quad (12)$$

Having thus P, Q, and their resultant S, which is the thrust in the crown, the line of resistance is constructed to right and left in the manner already described (p. 297).

The Slightly-Compressible Arch-Ring.

All materials of which arches are built being more or less compressible, it follows that in no case can the line of resistance actually touch either the intrados or the extrados; for, if it did, the thrust would be concentrated on the end of the voussoir at the point of contact, and its edge would crush or split off.

This effect may be seen in many arch-rings of insufficient stability, *i.e.*, where the centre of pressure has approached too near to the extrados or intrados.

The actual distance of the centre of pressure from the extrados or intrados at those joints, where on the supposition that the material is incompressible, the true line of resistance touches them, depends upon the compressibility of the materials.

As the centres are relieved the thrust comes gradually on to the voussoirs, and the materials compress until the pressure is spread over such a surface that further compression is prevented, the centre of pressure moving towards the centre line of the arch-ring at these joints, and stopping so soon as the surface is sufficient to bear the pressure.

The line of resistance in this case corresponds to a greater horizontal thrust than in the incompressible arch-ring of the same form.

The present state of knowledge in respect to the distribution of the thrust, and of the resistance of the materials to compression, does not admit of the actual position of the centre of pressure being accurately determined in each case; and, in fact, any accurate determination in an arch-ring would be very troublesome.

But it is not difficult to lay down certain limits outside of which the centres of pressure ought not to be, and if a line of resistance drawn through these points does not approach at any other point nearer to the extrados or intrados than at these points, the arch-ring is stable and the thrust on the abutment corresponding to this line is the thrust to be provided for in designing the abutments.

The limit proposed by Dr. Scheffler is, that the line of resistance shall lie everywhere within the centre half of the depth of the arch-ring, the form or the thickness of the ring being altered until this condition is complied with, it being supposed that the Engineer has taken care to select a strong material.

To apply this, after designing the arch-ring, two curves are drawn through points in each joint at a distance from the intrados and extrados of one-fourth of the depth of the arch-ring at that joint; and the rules already given for determining the true line of resistance in arch-rings of incompressible materials are applied to the part of the arch-ring lying between the curves.

Thus, let *Fig.* 5, *Plate* V., be the arch-ring; then at every joint DB=CE=$\frac{1}{4}$ DC, and the part of the arch-ring to be examined is the zone contained between the curves BBB and EEE.

Equilibrium of an Arch-Ring as regards Failure by Sliding.

In drawing the line of resistance to ascertain the stability of an arch-ring as regards rotation, the pressure on each joint is known in magnitude and direction, and it is only necessary to see that the direction of the pressure at each joint makes with the normal to the joint an angle less than that of which the co-efficient of friction of the material is the tangent.

This condition will be fulfilled in almost all cases, but it may not be in the lower joints of an arch such as the flat three-centred arch shewn in *Fig.* 5, *Plate* V., or in the four-centred arch in Example I.

In such a case the backing may be legitimately counted on to prevent the sliding, or the form of the arch and direction of the joints may be altered.

EXAMPLE I.

It is required to enquire into the stability of the arch-ring shewn in *Fig.* 1, *Plate* VI., the material of the arch and of its load being the same, and solid across the arch, its weight being 1 cwt. per cubic foot.* It is slightly compressible, so that the allowance on this account, recommended above, is to be made, viz., that the line of resistance shall be everywhere within the centre half of the arch-ring.

The arch-ring and its load being symmetrical, it is only necessary to consider one-half of it; and the load being uniform across the arch, it is sufficient to consider a section one foot in width.

* In finding the weights and centres of gravity, in cases where the load is not uniform across the arch (as where spandril walls are used instead of solid backing, or as in the case of the rolling load on a railway bridge, or of the parapet walls of a bridge), the load may be treated as if it were uniformly distributed, but consisting of a material, the weight of which, per cubic foot, is to the actual weight of this part of the load, per cubic foot, as the width of the parts actually covered by the load is to the total width of the arch.

i.e., as $a+b+c+d$ is to $a+e+b+f+c+g+d$. (*Plate* VI., *Figs* 2 and 3.)

A C is the intrados; B D is the extrados, and U V the extrados of the load.

The depth, D C, of the arch-ring at the crown is calculated by Professor Rankine's rule for an arch above ground, standing solitary between its abutments, *i.e.*,

$$D C = \sqrt{r \times \cdot 12} = \sqrt{90 \cdot 3 \times \cdot 12} = 3 \cdot 3 \text{ feet.}$$

The radial depth, B A, of the arch-ring at the springing is made a good deal greater than the depth at the crown, as, from the form of arch it is known, that the line of resistance will approach the extrados at the springing, and the pressure is also greater there.

Divide the arch-ring into seven parts by the joints 1 to 6, drawn each to its proper centre, and from the intersection of each with the extrados draw a vertical line.

Find the weights of each part of the arch-ring with the load resting directly on it, and find also the centre of gravity of each part.

These weights are as follows:

$$\begin{aligned} W_1 &= 3 \cdot 99 \text{ tons.} \\ W_2 &= 4 \cdot 47 \quad \text{,,} \\ W_3 &= 4 \cdot 92 \quad \text{,,} \\ W_4 &= 5 \cdot 61 \quad \text{,,} \\ W_5 &= 4 \cdot 61 \quad \text{,,} \\ W_6 &= 5 \cdot 30 \quad \text{,,} \\ W_7 &= 8 \cdot 54 \quad \text{,,} \end{aligned}$$

$$\text{Total} = 37 \cdot 44 \text{ tons} = W.$$

Draw a vertical through the centre of gravity of each of the loads, as well as through the centre of gravity of the whole.

Lay off on the arch-ring two curves, G G and H H, the radial distance of G from the extrados B D, and the radial distance of H from the intrados A C, at any point, being one-fourth of the radial depth of the ring at that point.

To draw a line of resistance passing through G at the crown and H at the springing, take moments about H (in the springing). Then the horizontal thrust at the crown,

$$Q = \frac{W \times Ha}{Gb}$$

By measurement, Ha = 11·4 feet, Gb = 27·3 feet,

whence Q = 15·63 tons.

Draw the trial line through G at the crown in the manner described at p. 297, by compounding Q with W_1, this resultant with W_2, and so on.

It cuts the curve G G at the crown, and again between joints 2 and 3; and is furthest from G G opposite joint 1.

It cuts the curve H H between the joints 3 and 4, and is furthest from it opposite joint 6.

The true line will touch the curve G G somewhere near F, and the curve H H near E.

It happens in this case that the points E and F coincide with two joints. Supposing either had fallen between two joints, it would have been necessary to draw another joint through it, and to find the weight

of the part of the arch-ring lying between this new joint and the joint next above it, and also to find the centre of gravity of that part, before calculating the new value of Q. Call this new value Q_1.

Find the centre of gravity of the part of the arch-ring and its load lying above E, and draw a vertical through it. It is shewn on *Fig.* 1, *Plate* VI., marked $W_{1\cdot6}$

Then from equations (1) and (2) p. 300.

$$Q_1 x = W_1 \times 2{\cdot}8' = 11{\cdot}172 \text{ foot tons}$$

$$Q_1 (x + 15{\cdot}3) = W_{1\cdot6} \times 11{\cdot}5' = 28{\cdot}9 \times 11{\cdot}5 = 332{\cdot}35 \text{ foot tons.}$$

$$\text{whence } Q_1 = 21 \text{ tons}, x = {\cdot}5 \text{ foot.}$$

Draw the line of resistance, with this new value of Q, through a point ·5 foot below G at the crown.

It touches the curves G G and H H very nearly at the points F and E, and is sufficiently accurate for the purposes of the Engineer.

It, however, cuts the curve G G about 2·3 feet above the springing, and the stability of the arch-ring is therefore not sufficiently assured below this point, as the intensity of the pressure might be enough to cause crushing of the material.

It will be better to increase the thickness as shewn by the curve K 6; B K being made $\frac{1}{3}$ B A.

It is clear from this Example that this form of arch requires a great thickness of arch-ring.

Now although it is usually agreed that the exact maximum pressure in an arch-ring cannot be ascertained, yet, on the supposition that the line of resistance is really as drawn, the pressure can be calculated, at any rate approximately on the principles stated at p. 233.

Thus at the joint F, the normal pressure = 21·5 tons, and it is spread over a surface = 3 × 1 × 1 = 3 square feet or there is a mean pressure of 14·34 tons per square foot, or ·1 ton = 224 lbs. per square inch, which would give a factor of safety of only 5 with strong brick. Such an arch-ring should therefore be built of some stronger material.

At the springing, the normal pressure = 37·44 tons, which is spread over 6·6 square feet, the maximum pressure being 11·4 tons per square foot, or ·08 tons per square inch.

As regards failure by sliding.

It is evident that as far as joint 6, the pressure is so nearly normal to the joints that sliding cannot occur; and even at the springing the angle between the normal and the direction of the pressure B*f* is only 30°, while the angle of friction of masonry, in the condition which would allow of the centres being struck, is not less than 37°, so that there is a good margin of security even at this joint.

Sliding may always be prevented by slate dowels; but in fact the masonry of the abutment prevents any sliding on this, or other joints. The abutment is dealt with as an ordinary buttress, subject to an inclined thrust, and it is not considered necessary to deal with it in detail.

EXAMPLE II.

It is required to enquire into the stability of the arch shewn in *Plate* VII., the material of the arch and its load being the same, and solid across the arch, its weight being one cwt. per cubic foot. It is slightly compressible, so that the allowance on this account recommended at p. 304 is to be made, viz., that the line of resistance shall be everywhere within the centre half of the arch-ring.

There is an isolated load at X equal to 2·5 tons per foot run of the width of the arch.

As the loading of the arch-ring is not symmetrical, it is necessary to consider the whole arch-ring, and the loading being uniform across the arch, it is sufficient to consider a section one foot in width.

ACA_1 is the intrados; BDB_1 is the extrados; UVU_1 the extrados of the load.

The depth of the arch-ring at the crown is calculated by Professor Rankine's rule for an arch standing solitary between its abutments, *i.e.*,

$$DC = \sqrt{r \times \cdot 12} = \sqrt{25 \times \cdot 12} = 1{\cdot}75 \text{ feet.}$$

The depth at the springing is the same as at the crown, as usual in brick arches.

Divide each half of the arch-ring into six parts by the joints 1 to 5, and from the intersection of each with the extrados, draw a vertical line.

Find the weight of each part of the arch-ring with the load resting directly on it, and find also the centre of gravity of each part.

These weights are as follows:

$W_1 = \cdot 8$ tons.
$W_2 = 1{\cdot}0$,,
$W_3 = 1{\cdot}2$,,
$W_4 = 1{\cdot}7$,,
$W_5 = 1{\cdot}5$,,
$W_6 = 2{\cdot}0$,,

Total, 8·2 tons.

The isolated load $W_7 = 2{\cdot}5$ tons.

Draw a vertical through the centre of gravity of each of the loads, as well as through the centre of gravity of the load on each half of the arch-ring, including the isolated load.

Lay off on the arch-ring two curves, GGG_1 and HHH_1, at a distance of one-fourth of the depth of the arch-ring from the extrados and intrados.

Assume the trial line of resistance to pass through H, and H_1 both at the springings.

Then the vertical component of the thrust

$$\text{at H} = 8{\cdot}2 \text{ tons} + \frac{2{\cdot}5 \text{ tons} \times 30{\cdot}75}{41} = 10 \text{ tons},$$

$$\text{and at } H_1 = 8{\cdot}2 \text{ tons} + \frac{2{\cdot}5 \text{ tons} \times 10{\cdot}25}{41} = 8{\cdot}9 \text{ tons}.$$

Then commencing at AB, and adding the successive loads together, it is seen that joint 1 divides the total load into two parts, which are very nearly respectively equal to the reactions on each side of it; *i.e.*, the load to the left of joint 1 = 9·9 tons, while the load to right of joint 1 = 9 tons.

Joint 1 is therefore very nearly the joint where the thrust is horizontal.

Assume the point K in this joint to be a third point in the trial line of resistance; and it is now required to draw a line of resistance through H, K, and H_1.

The value of the symbols in equations (8), (9), (10), p. 303, are as follows:

$g_1 = 20{\cdot}4$ ft.; $P_1 = 10{\cdot}7$ tons; $p_1 = 13{\cdot}2$ ft.
$h_1 = 11{\cdot}2$ ft.; $P_2 = 8{\cdot}2$ tons; $p_2 = 13{\cdot}6$ ft.
$g_2 = 20{\cdot}4$ ft.; $P_3 = {\cdot}8$ tons; $p_3 = 2{\cdot}0$ ft.
$h_2 = 11{\cdot}2$ ft.;
$g_3 = 4{\cdot}0$ ft.;
$h_3 = {\cdot}3$ ft.;

which applied in those equations give P=·52 tons, Q=6·1 tons, q=·3 ft. with which the trial line is drawn through HKH_1.

On the left it cuts the curve G G at K, and again lower down, and is farthest from G G opposite joint 2.

On the right, it nearly touches the curve H H_1 at joint 3, and coincides with it very nearly from that point to H_1.

Then the true line of resistance will pass through H, will touch the curve G G (on the left) approximately in F and the curve H H_1 (on the right) approximately in E; and if it remains between G G_1 and H H_1 at every point below E, the arch-ring is stable.

Draw a new line of resistance through H, F, and E by help of equations (8), (9), (10), which give P=·4 tons, Q=6·6 tons, q=·77 ft.

This line is a sufficient approximation for the purposes of the Engineer, and it lies everywhere between G G G_1 and H H H_1.

The greatest pressure in this arch-ring is at the springing on the left, where it amounts to 12 tons and it is spread over a surface = 4·5 square feet.

The maximum pressure is therefore $\frac{2 \times 12}{4{\cdot}5} = 5\frac{1}{2}$ tons and the pressure per square inch = 81 lbs., which gives a factor of safety for strong brick of 1·4 nearly.

There is manifestly no danger of sliding in this arch-ring.

N.B.—For an arch-ring under a moving load or loads, it would be necessary to repeat this operation for several positions of the load, and in designing the abutment to provide for the maximum thrust produced.

THE ABUTMENT.

Let the proposed abutment be of the form and dimensions shewn in *Fig.* 1, *Plate* VIII., viz., A B C D E F, and of one uniform material.

Find the centre of gravity of a section, one foot in width, having the

weight W. Let P be the thrust of the arch-ring. Then compounding P and W the resultant thrust R on E F is obtained in the usual manner.

If this resultant falls within the centre third of E F, the abutment is at least stable enough, and may be decreased. If it falls outside the centre third, it is not stable enough, and must be made larger, the last operation being repeated, until the proper dimensions are arrived at.

It is considered desirable that there should be no tension on any part of the abutment of an arch, or in other words that the pressure should be spread over the whole surface of each joint. If the point where the resultant pressure cuts E F be nearer to the outside edge than $\frac{1}{3}$ E F, there would be no pressure on the part of the joint near F, and the pressure would not be distributed in the most advantageous manner. It does not however follow that crushing would ensue at E, and in ordinary materials it would not ensue.

The intensity of the pressure can be ascertained as described at p. 233.

NOTE.—If the abutment is built, as is frequently the case, with counterforts, the whole thrust of the arch at the springing, which of course is equal to the thrust due to 1 foot in width multiplied by the total width in feet, is to be taken, the centre of gravity and total weight of the proposed abutment are to be found, and the resultant of the thrust and weight should pass just within the centre third of the width of the abutment with its counterforts at the footings, *i.e.*, CD = $\frac{1}{3}$ CE. (*Plate* VIII., *Figs.* 2 and 3.)

To ensure security against sliding on E F, the angle N K R must not exceed that of which the co-efficient of friction of the masonry is the tangent; and at p. 236 it was proposed to make the tangent of this angle not greater than $\frac{4}{5}$th of the co-efficient of friction.

If this condition is not fulfilled, the joints should be inclined upwards to the back of the abutment, as shewn in *Fig.* 4, *Plate* VIII.

FOUNDATIONS.

Having determined the proper dimensions of the abutment, it only remains to distribute the pressure sufficiently uniformly over the surface below to ensure that no very unequal settlement shall take place, and to that sliding along the soil shall not occur.

To ensure the first the pressure must not exceed what the soil below can safely bear. (*U.R.T.*, p. 181).

Draw in the footings and foundations (*Plate* VIII., *Fig.* 1), making the bottom line H G of the foundations at a depth necessary to get below the effects of frost, *i.e.*, in England, about 3 feet for sand and 4 feet for clay (*R.C.E.*, p. 380).

Compound R with the weight of the footings and foundations, and resolve the resultant normal to and parallel to the line of the base.

Divide the normal pressure thus obtained, for one foot in width of the arch and abutments by the safe pressure per square foot, which the soil can bear, and the quotient will be very nearly the least safe width of the foundations.

One half of the foundation should be set off on each side of the point L, L G being made equal to L H, even if G H be greater than the minimum safe width found above.

If the direction of the resultant makes with the normal to the base

of the foundation an angle greater than the action of friction of the abutment and the soil below, it will be necessary to incline the base until the angle contained between the direction of the pressure and the normal to the base is considerably less than the angle of friction. As already stated, it was proposed at p. 236 to limit the tangent of this angle to $\frac{1}{3}$th of the co-efficient of friction of the wall and the earth below.

Groined Arches.

The Military Engineer has frequently to construct ranges of groined arches, such as are shewn in *Fig.* 5, *Plate* VIII.

In this case any pier A B D C, has to support the thrust of the half arch C Q R D, as well as the thrust of the two quarter groins S E Q C, T F R D in the directions E C, F D, the two together producing an additional thrust in the direction of the axis of the pier, while it is strengthened against overturning by the weight of the two half arches S U A C and T V B D, the horizontal thrusts of which neutralize one another. First, then, find the horizontal thrust per foot run at the crown of the arch C D O N, according to previous examples. Then the total thrust on C D will be this thrust multiplied by E F in feet. Find the weight of the part of the arch S E Q R F T, with its load, and its centre of gravity. Find also the weights of the two half arches S U A C, T V B D, with their loads. Then the forces acting on the pier are as shewn in *Fig.* 6, *Plate* VIII., *i.e.*, the horizontal thrust, H, along E F, acting at the crown, W the weight of the part of the arch, C S E Q R F T D acting through its centre of gravity, which will be somewhere in the continuation of the centre line of the pier, W_1 the weight of the pier and the two half arches S U A C, T V B D resting upon it, acting through their common centre of gravity.

The resultant of these forces should pass through a point Y (*Fig.* 6, *Plate* VIII.), at a distance Y B from B = $\frac{1}{3}$ B D, and the pressure at B should not be more than the material can safely bear.

As the centre of gravity of W is difficult to find, it may be easier to trace the line of resistance for the half arch C Q R D to the springing joint, and transfer both H and W to that point, *i.e.*, to A, *Fig.* 6, the result being the same.

Skew Arches.

In inquiring into the stability, as regards rotation of a proposed skew arch, supposing it to be properly constructed, it is only necessary to ascertain whether a square arch of the same dimensions and figure, similarly loaded, fulfils the condition necessary to stability.

If it does, the skew arch which is merely a parallel projection of the square arch will be stable also. (*R.A.M.*, p. 232, and *R.C.E.*, p. 220).

Thus, let *Figs.* 7 and 8, *Plate* VIII., represent the plan and elevation of a square arch which has been found to be stable.

Then will the arch shewn in *Figs*, 9 and 10, *Plate* VIII., being a parallel projection of the square arch, be stable also, as regards rotation.

As regards sliding, it is necessary to enquire, whether the increased

obliquity of the thrust caused by the skewing of the arch is sufficient to cause sliding at any joint.

Thus if *ab* (*Fig.* 8) is the line of resistance through the arch and abutment of the square arch, *cd* (*Fig.* 10) will be the line of resistance through the skew arch, and this line must fulfil at every joint the second condition of stability.

For the method of laying off the joints, &c. of skew arches, see *Buck's Oblique Arches.*

Construction of Arches.

On this subject, see *Rankine's Civil Engineering* p. 413, *et seq.*

Centres of Arches.

On this subject, see *Rankine's Civil Engineering*, pp. 415 and 485, *et seq.*; also *Useful Rules and Tables*, p. 190.

Table of Thickness of Arches.

On the following page is a table of thicknesses for arches, in ordinary use in the War Department.

TABLE OF THICKNESS REQUIRED FOR ARCHES. SEMICIRCULAR ARCHES AND ARCHES OF 120°.

	ORDINARY VAULTS, BRIDGES, &c.						COVERING FOR BOMBPROOF CASEMATES, &c.						REMARKS.
	Semicircular.			Arches of 120°.			Semicircular.			Arches of 120°.			
	Block Stone.	Brick.	Rubble Stone.	Block Stone.	Brick.	Rubble Stone.	Block Stone.	Brick.	Rubble Stone.	Block Stone.	Brick.	Rubble Stone.	
	′ ″		′ ″	′ ″		′ ″	′ ″		′ ″	′ ″		′ ″	
Span of 5 feet and less.	0 8	1	0 10	0 9	1	0 $10\frac{1}{2}$	0 9	1	0 11	0 10	1	1 0	The arches are assumed to be built in well made hydraulic lime mortar.
,, 6 ,,	0 9	1	0 11	0 10	1	1 0	0 10	1	1 0	0 11	$1\frac{1}{2}$	1 2	
,, 8 ,,	0 10	1	1 0	0 11	$1\frac{1}{2}$	1 $1\frac{1}{2}$	1 0	$1\frac{1}{2}$	1 $1\frac{1}{2}$	1 1	$1\frac{1}{2}$	1 4	
,, 10 ,,	0 11	$1\frac{1}{2}$	1 $1\frac{1}{2}$	1 0	$1\frac{1}{2}$	1 3	1 $1\frac{1}{2}$	$1\frac{1}{2}$	1 3	1 2	$1\frac{1}{2}$	1 6	The bombproof arches are considered sufficiently strong to support any quantity of covering; but if the covering be not sufficient to protect them from actual contact with shells the thickness of the arches may require to be increased.
,, 12 ,,	1 0	$1\frac{1}{2}$	1 3	1 1	$1\frac{1}{2}$	1 $4\frac{1}{2}$	1 3	$1\frac{1}{2}$	1 $4\frac{1}{2}$	1 4	2	1 8	
,, 14 ,,	1 1	$1\frac{1}{2}$	1 4	1 2	$1\frac{1}{2}$	1 6	1 4	$1\frac{1}{2}$	1 6	1 5	2	1 10	
,, 16 ,,	1 2	$1\frac{1}{2}$	1 5	1 3	2	1 7	1 5	2	1 8	1 6	$2\frac{1}{2}$	2 0	
,, 18 ,,	1 3	2	1 6	1 4	2	1 8	1 6	2	1 10	1 $7\frac{1}{2}$	$2\frac{1}{2}$	2 $1\frac{1}{2}$	
,, 20 ,,	1 4	2	1 7	1 5	2	1 9	1 7	$2\frac{1}{2}$	2 0	1 9	3	2 3	
,, 22 ,,	1 5	2	1 8	1 6	2	1 $10\frac{1}{2}$	1 8	$2\frac{1}{2}$	2 $1\frac{1}{2}$	1 $10\frac{1}{2}$	3	2 $4\frac{1}{2}$	
,, 24 ,,	1 6	2	1 9	1 7	$2\frac{1}{2}$	2 0	1 9	3	2 3	2 0	3	2 6	

APPENDIX.

TESTS FOR WROUGHT IRON.

For detailed information on the subject of wrought iron, see Kirkaldy's *Experiments on Wrought Iron and Steel* (especially p. 91 *et seq.*) Pole's *Iron as a Material of Construction*, p. 110; Rankine's *Civil Engineering*, p. 503; Anderson's *Strength of Materials*, p. 46; Unwin's *Wrought Iron Bridges and Roofs*, p. 96.

The last writer says, "The one irremediable quality in iron intended for a rivetted structure is brittleness or want of ductility. The best iron is that which is tough as well as strong. Brittleness is accompanied by a small elongation under strain, and generally by a bright silvery crystalline or granular fracture."

ORDINARY SPECIFICATION.

The ordinary specification in this country for wrought iron for use in bridges is that the iron shall be equal in quality to Best Staffordshire, the plate iron being specified to resist 20 or 21 tons, lengthways of the grain, and the bars to resist 22 or 23 tons, which is equal, as regards tenacity, to G and F classes in Mr. Kirkaldy's scale (p. 314); and very lately, a certain minimum contraction of area at the section of fracture has been introduced by some Engineers into their specifications.

The test of tenacity, without any condition as to the elongation of the iron under tension, or as to its reduction of area at the section of fracture is insufficient, as hard brittle irons often have both a very high tenacity and elastic limit, but break under comparatively small blows, and are therefore unsuitable for such structures as railway bridges, the iron in which is daily working under small shocks, and may be called upon, in case of accident, to resist very heavy blows.

As there is a difference of opinion as to whether the reduction of area under a gradually increasing stress, free from shock, is a certain guide to the behaviour of the iron under a blow, it might be well, in testing iron for railway purposes, to apply both tests, where it can conveniently be done.

No class of iron is so frequently defective as that of ready-made

rivets, and yet there is scarcely any on which so much depends. It is better, therefore, to have the rivets made at the works, and to test each bar intended for rivets by breaking it across at the ends, any bars showing very large facets of crystals being rejected.

The following tests applied by three Government Departments to iron required for their respective works are all directed to ensuring that the iron shall be both strong and ductile.

INDIAN STORE DEPARTMENT.

The following is the "Scale of tensile tests for the supply of iron of various qualities" prepared by Mr. Kirkaldy, and adopted by the Secretary of State for India.

SCALE OF TENSILE TESTS FOR IRON OF VARIOUS QUALITIES.

DESCRIPTION.	CLASS C.		CLASS D.		CLASS E.		CLASS F.		CLASS G.	
	Ultimate stress per square inch.	Contraction of area at fracture.	Ultimate stress per square inch.	Contraction of area at fracture.	Ultimate stress per square inch.	Contraction of area at fracture.	Ultimate stress per square inch.	Contraction of area at fracture.	Ultimate stress per square inch.	Contraction of area at fracture.
	Tons.	per cnt	Tons.	per cnt	Tons.	per cnt	Tons.	per cnt	Tons.	per cnt
Bars, round or sq.	27	45	26	35	25	30	24	25	23	20
Bars, flat	26	40	25	30	24	25	23	20	22	18
Angle and Tee or T	25	30	24	22	23	18	22	15	21	12
Plates, lengthway	24 } 23	20 } 16	23 } 21½	15 } 12	22 } 20½	12 } 9½	21 } 19½	10 } 7½	20 } 18½	8 } 5½
Plates, crossway ...	22 }	12 }	20 }	9 }	19 }	7 }	18 }	5 }	17 }	3 }

N.B.—Classes A and B are reserved for any special qualities of Iron which might be required at any future time.

SWEDISH BARS.

Ultimate Stress per square inch } 23 tons. Contraction of area at fracture. } 60 per cent.

The conditions of Contract specify that "materials representing 4 per cent. of the value shall be selected, from which will be cut pieces 20 inches in length, and of plates and sheets 20 inches by 18 inches," to be tested. "The iron will be accepted, although under the specified strain, provided the contraction of area at fracture is the same per centage higher, or in other words, softer iron than that specified will be accepted."

The tension in these tests is applied gradually and without shock.

WAR DEPARTMENT.

For shield frames, for the girder work of iron forts, and for armour bolts, all of which are intended to resist heavy blows, it is necessary to have a specially ductile iron, and the present (October, 1872) specification is as follows:

The method of conducting the test by blows for the War Department is described at page 315.

	Tensile Strain per inch of original Section to be borne without breaking.	Reduction of Area at the Point of Fracture.
Plate iron, lengthways	20 tons	12 per cent.
,, crossways	16 ,,	5 ,,
Angle iron 6″ × 6″ × ¾″ and upwards, lengthways,	22 ,,	15 ,,
Angle iron, less sections, lengthways	22 ,,	20 ,,
Rivet iron, lengthways	23 ,,	40 ,,

Bar and T iron to be equal to angle iron of equivalent section.

For armour bolts, the specified reduction of area is 40 per cent., (whether broken in a testing machine or by a falling weight,) with an uniformly fibrous fracture.

The iron is tested both in the testing machine by gradually increased tension and by the blow of a falling weight, or more lately by blows from a steam hammer, the apparatus being arranged to produce tension on the bolt, (*Professional Papers*, vol. xviii., 1870, p. 122), it being found that when the iron is of good quality a ton weight falling 30 feet, or an equivalent blow from the hammer, will pull a bolt 2 feet long and nearly 3 inches diameter in two in six or seven blows. The fracture should be silky fibrous, not crystalline in any degree.

This apparatus affords a comparative measure both of the tenacity and the ductility of the iron, but the actual tensile stress is not given by it.

It has, however, been found by many experiments, that with the quality of iron necessary to give a fibrous fracture, and good reduction of area under the blow of a falling weight, there is always associated a very fair resistance to tensile stress.

In applying this test, and indeed any other test, care must be taken that the blow or stress acts along the axis or mean fibre of the test bar.

ADMIRALTY TESTS.

The following are the tests applied to iron in Her Majesty's Dockyard: 1872.

BEST BEST, OR 1ST CLASS PLATE IRON, ¼-INCH THICK AND ABOVE.

Tensile strain per sq. in. { Lengthways 22 tons. 21 tons for *Boiler Plates*.
Crossways. . 18 ,,

Forge Test (Hot).

All plates of the 1st Class of 1 inch in thickness and under, should be of such ductility as to admit of bending hot, without fracture, to the following angles:

Lengthways of the grain.....	125 degrees.			
Across ,, ...	90 ,,	for Ship Plates.		
,, ,, ...	100 ,,	,, Boiler Plates.		

Forge Test (Cold).

All plates of the 1st class should admit of bending cold, without fracture, as follows:

With the Grain.

1 inch and $\frac{15}{16}$	of an inch in thickness	to an angle of	15	degrees.
$\frac{7}{8}$ "	$\frac{13}{16}$	" "	20	"
$\frac{3}{4}$ "	$\frac{11}{16}$	" "	25	"
$\frac{5}{8}$, $\frac{9}{16}$ "	$\frac{1}{2}$	" "	35	"
$\frac{7}{16}$ "	$\frac{3}{8}$	" "	50	"
$\frac{5}{16}$ "	$\frac{1}{4}$	" "	70	"

Across the Grain.

1 in., $\frac{15}{16}$, $\frac{7}{8}$, and $\frac{13}{16}$	of an inch in thickness	to an angle of	5	degrees.
$\frac{3}{4}$ and $\frac{11}{16}$		" "	10	"
$\frac{5}{8}$, $\frac{9}{16}$, and $\frac{1}{2}$		" "	15	"
$\frac{7}{16}$ " $\frac{3}{8}$		" "	20	"
$\frac{5}{16}$ " $\frac{1}{4}$		" "	30	"

BEST, OR 2ND CLASS PLATE IRON, $\frac{1}{4}$-INCH THICK AND ABOVE.

Tensile strain per square inch. { Lengthways ..... 20 tons.
Crossways ..... 17 "

Forge Test (Hot).

All plates of the 2nd Class of 1 inch in thickness and under, should be of such ductility as to admit of bending hot, without fracture, to the following angles:

Lengthways of the grain	90	degrees.
Across "	60	"

Forge Test (Cold).

All plates of the 2nd Class should admit of bending cold, without fracture, as follows:

With the Grain.

1 inch and $\frac{15}{16}$	of an inch in thickness	to an angle of	10	degrees.
$\frac{7}{8}$ "	$\frac{13}{16}$	" "	15	"
$\frac{3}{4}$ "	$\frac{11}{16}$	" "	20	"
$\frac{5}{8}$, $\frac{9}{16}$, "	$\frac{1}{2}$	" "	30	"
$\frac{7}{16}$ "	$\frac{3}{8}$	" "	45	"
$\frac{5}{16}$ "	$\frac{1}{4}$	" "	55	"

Across the Grain.

$\frac{3}{4}$ inch and $\frac{11}{16}$	of an inch in thickness	to an angle of	5	degrees.
$\frac{5}{8}$, $\frac{9}{16}$, "	$\frac{1}{2}$	" "	10	"
$\frac{7}{16}$ "	$\frac{3}{8}$	" "	15	"
$\frac{5}{16}$ "	$\frac{1}{4}$	" "	20	"

The following conditions apply to both Classes:

Plates both hot and cold should be tested on a cast iron slab, having a fair surface, with an edge at right angles, the corner being rounded off with a radius of $\frac{1}{2}$ an inch.

The portion of plate tested, for both hot and cold tests, is to be 4 ft. in length, across the grain, and the full width of the plate, with the grain.

The plate should be bent at a distance of from 3 to 6 inches from the edge.

All plates to be free from lamination and injurious surface defects.

The Inspecting Officer may select for testing one plate in fifty of each thickness or of each parcel of plates rolled. Should the parcel be accepted the plate will be paid for, but not otherwise.

BEST BEST OR 1ST CLASS THIN PLATE OR SHEET IRON.

Tensile strain per square inch { Lengthways ..... 22 tons. Crossways ..... 18 „

Forge Test (Hot).

All plates of the 1st class should be of such ductility as to admit of bending hot, without fracture, to the following angles:

Lengthways of the grain ..... 125 degrees.
Across „ ..... 90 „

Forge Test (Cold).

All plates of the 1st Class should admit of bending cold, without fracture, as follows:

With the grain, to an Angle of 90 degrees.
Across the grain, to an Angle of 40 degrees.

BEST OR 2ND CLASS THIN PLATE OR SHEET IRON.

Tensile strain per square inch { Lengthways ..... 20 tons. Crossways ..... 17 „

Forge Test (Hot).

All plates of the 2nd Class should be of such ductility as to admit of bending hot, without fracture, to the following angles:—

Lengthways of the grain ..... 90 degrees.
Across „ ..... 60 „

Forge Test (Cold).

All plates of the 2nd Class should admit of bending cold, without fracture, as follows:

With the grain, to an Angle of 75 degrees.
Across the grain, to an Angle of 30 degrees.

The following conditions apply to both classes of sheet iron:

Plates both hot and cold should be tested on a cast iron slab, having a fair surface, with an edge at right angles, the corner being rounded off with a radius of half an inch.

The portion of plate tested, for both hot and cold tests, is to be 4 ft. in length, across the grain, and the full width of the plate, with the grain.

The plate should be bent at a distance of from 3 to 6 inches from the edge.

All plates to be free from lamination and injurious surface defects.

One plate to be taken indiscriminately for testing from every thickness of plate sent in per invoice, provided they do not exceed fifty in number. If above that number, one for every additional fifty, or portion of fifty.

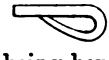
The thickness for testing plates is to be ascertained in all cases by weighing the plate, or the piece to be tested; assuming that a square foot of $\frac{1}{3}$ inch plate weighs five pounds, and that other thicknesses have the same proportionate weight.

ANGLE, BULB, TEE, ANGLE BULB, TEE BULB, CHANNEL, OR OTHER IRON OF ORDINARY FORM.

1. Samples will be taken indiscriminately for testing from every description of angle, bulb and other iron included in any one invoice, provided the number of irons so included does not exceed fifty, and, if above that number, one for every fifty or portion of fifty of each description. The samples may be tested to show the strength, ductility, and other qualities of the iron, and if not found satisfactory, the lot from which they are taken may be rejected.

2. The whole of the samples of every description of iron may be tested with the grain to a tensile strain of 22 tons to the square inch.

3. Angle iron may be tested hot by being bent thus,

and also by being flattened, thus and the end bent over, thus One sample may be notched and broken across cold, to show the quality of the iron, and one flange of the angle iron may be cut off and bent cold, thus 

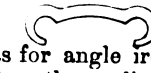
4. Tee iron may be tested hot by being bent thus

The cold test will be the same as for angle irons.

5. Tee bulb may be tested hot by cutting off the bulb and testing the remainder in the same way as tee iron, the bulb being notched on one side and broken cold to show the quality of the iron.

6. Angle bulb may be tested in the same way as angle iron after the bulb has been cut off; and the bulb itself tested as in the case of tee bulb.

7. Bulb iron may be tested hot by cutting off the bulb and bending the web across the grain, thus and the bulb may be notched on one side and broken cold to show the quality of the iron.

8. Channel iron may be tested hot, thus  and one of the flanges may be cut off and bent cold as for angle iron, and one sample may be notched and broken cold to show the quality of the iron.

9. All other descriptions of iron may be tested in a similar manner according to their forms.

BEST MERCHANT OR MALLEABLE ONCE-WORKED IRON, FIRST BRAND; BEST BEST BAR IRON; MOULDING, SASH BAR, HALF-ROUND AND SEGMENTAL IRON; FIRE BAR IRON; BEST BEST NAIL ROD; BEST BEST HOOP IRON.

The iron marked "Best Best" in this list to stand a tensile strain of 22 tons per square inch, and the whole of the iron to stand such forge tests, both hot and cold, as the receiving Officers may deem expedient to satisfy themselves that it is fit for her Majesty's Service.

NOTE ON THE STRENGTH OF LONG TIMBER PILLARS AND STRUTS.

It has been objected lately by very competent judges that Professor Rankine's formula (p. 86) for long pillars gives results so different from those obtained from later experiments, as to render necessary some modification of it.

It certainly appears that, even if the formula is correct, the method of applying it at present in use can hardly be so.

Thus, a pillar 19 times as long as its least dimension is assumed, irrespective of the way in which it is secured, to fail by crushing only, while another pillar 21 times as long as its least dimension is assumed to fail by cross breaking.

Now there cannot be any sudden change of this kind, and in fact the change from failure by crushing to failure by cross breaking must be very gradual, passing through a succession of stages, in which failure takes place by the stress being unequally distributed over the section sufficiently to cause crushing on one side, but not enough to set up tension on the other.

If the formula be accepted as correct, the intermediate stages would be roughly met by using the formula for cross breaking for all ratios of l to h, and not only for those of 20 and upwards.

As to the correctness of the formula, it is objected that it gives too high results for comparatively short pillars, and too low results for very long pillars, and no fixed alteration of the constants would meet this objection. The error, if there is one, must therefore be in the theory of the formula in which, however, no flaw has yet been pointed out.

On the whole it does not seem advisable, in default of a better formula, to cast aside the present one, although, as it is known to be, to some extent, empirical, it is wise to use it with a liberal factor of safety.

The objection has at present been confined to long pillars or struts of timber, and the above remarks apply only to them.

The following rule for long pillars of wood is taken from the *Engineer*, 1st December, 1871, and has been devised by Mr. Stoney, C.E., (author of the *Theory of Strains*).

RULE FOR SQUARE PILLARS OF SOFT FOREIGN TIMBER, WITH THE ENDS ADJUSTED AS IN ORDINARY PRACTICE.

Ratio of length to side of pillar...	10,	15,	20,	25,	30,	35,	40,	45,	50
Breaking weight in tons, per sq. ft. of section	120,	118,	115,	100,	90,	84,	80,	77,	75

The breaking weight of the strut by this rule is found as follows: If ratio of length to side=24; breaking weight by table=103 tons per square foot,

$$\therefore \text{ for 13 inches square, breaking weight} = \frac{13 \times 13 \times 103}{12 \times 12} = 120{\cdot}8 \text{ tons.}$$

Divide by factor of safety of 4 for such work as a cofferdam.

The following rule for cast iron columns is taken from *The Engineer and Contractor's Pocket Book*, for 1866, and is given on the authority of Mr. Sheilds, C.E.; the columns being supposed of good construction with flat ends, and with base plates at their bearings.

From 20 to 24 diameters in length,			Columns may be loaded with,		
$\frac{3}{4}$	inch thick or upwards.....	2	tons per square inch of iron.		
$\frac{5}{8}$	" "	$1\frac{3}{4}$	"	"	
$\frac{1}{2}$	" "	$1\frac{1}{2}$	"	"	
$\frac{3}{8}$	" "	$1\frac{1}{4}$	"	"	

NOTE ON PAGE 145.

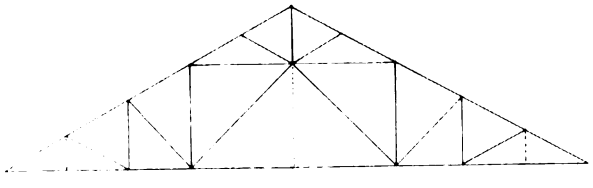
The load on the stiffener is overstated here, as it would only consist of the weight of the cross beam and the load on it, the web of the girder between the stiffeners receiving directly the weight of the booms or flanges, in the same way that the stiffener receives the weight of the cross beam.

This reduces the load on the stiffener to 15·4 tons, and makes $t = \frac{5}{16}$ inch, instead of $\frac{7}{16}$ inch, and the T iron might be somewhat reduced in dimension.

NOTE ON EXAMPLE III.—ROOFS.

In the figure for this Example, the principals are shown carried up to the ridge, which is required for the strict application of Professor Maxwell's method.

In practice this is never done, unless the length of common rafter between the upper purlin E and the ridge G (*Fig.* 1, *Pl. III. Roofs*) is so long as to require support, in which case the upper part of the truss becomes a king-post truss, and the roof is of the form shewn below.



Professor Maxwell's method will give the stresses with sufficient accuracy, by assuming the lines of stress to be as shewn in the diagrams, (*Plate III. Roofs*), even supposing the upper parts of the principals to be omitted.

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It is suggested that the Errata be transferred to their proper pages, before studying the Volume.

ERRATA AND ADDENDA.

Page 2, line 1, *for* 1861, *read* 1868.
„ 15, line 13, *after* one, *put* a comma.
„ 16, line 8, *after* axis, *add* passing through the centre of gravity of the section.
„ 17, line 1, *for* total amount, *read* moment.
„ 17, lines 5 to 7, *for* $\int_{-y^0}^{+y^0}$ *read* $\int_{-y_0}^{+y_1}$
„ 17, *after* line 5, *add* y_1 being the distance from the neutral axis to the upper side of the beam.
„ 22, *Fig.* 43, the arrows in the rectangle under *b should be*
„ 23, line 22, *after* demonstrated, *add* for a rectangular section.
„ 24, *Fig.* 51, *add* the letters UV at the neutral axis (see p. 25, line 11).
„ 28, line 24, *for* 324, *read* 342.
„ 42, line 25, *for* bs. *read* lbs.
„ 43, line 13, *for* 13, *read* 12.
„ 43, line 5 from bottom, *for* 1·8960, *read* 1·703.
„ 43, last line, *for* p. 38, *read* p. 36.
„ 44, line 3, *for* 1·8960, *read* 1·703; and *for* ·474, *read* ·424.
„ 44, line 13, *for* $\frac{1}{3}$ W c^3, *read* $\frac{1}{8}$ W c^3.
„ 45, line 29, *omit* x, and *after* C, *add* distant x from A.
„ 45, line 31, for ×, *read* +.
„ 46, line 17, *for* m, *read* n.
„ 47, line 7, *for* $E \times 144 \times d_3$, *read* $E \times 144 \times d^3$.
„ 47, *Fig.* 5, *after* $\frac{2{,}300{,}000}{E \times 144 \times d^3}$, *add* to one inch.
„ 52, line 3, *omit* vertical.
„ 54, line 13, *for* c_3, *read* c^3.
„ 56, line 14, *for* x_4, *read* x^4.
„ 56, line 16, *for* Wl_3, *read* Wl^3.
„ 59, line 8 from bottom, *for* $\frac{\frac{f_0}{5}}{y_0}$ *read* $\frac{\frac{f^0}{5}I}{y_0}$.
„ 60, line 3, *for* $\frac{w\,c^2}{12}$, *read* $\frac{w\,c^2}{2}$.
„ 62, line 1, *after* best, *add* in beams.
„ 63, line 14, *for* extremes, *read* extreme.
„ 78, line 8, *after* usual, *omit* the comma.

Page 83, line 21, *after* 20, *add* Appendix, p. 319.
„ 89, line 33, *for* 3·53, *read* 3·7.
„ 89, line 36, *for* near together, *read* far apart.
„ 91, line 7, *after* safe, *add* Appendix, p. 319.
„ 92, line 21, *after* possible, *add* uniformly.
„ 92, line 2 from bottom, *for* 16, *read* 18.
„ 93, line 4. Transverse pressure here means at right angles to the fibres of the wood, as distinct from pressure in the direction of the fibres.
„ 93, line 6, *for* transvere, *read* transverse.
„ 94, line 28, *for* 14·400, *read* 14,400.
„ 95, line 13. Transverse pressure here means at right angles to the fibres of the wood, as distinct from pressure in the direction of the fibres.
„ 95, footnote, *for* 1,5, *read* 1·5.
„ 96, line 18, *after* A, *add* (*Fig.* 5).
„ 98, *Fig.* 6. *Write* T on tongue, close to E E.
„ 100, line 4, *for* p, *read* b.
„ 101, line 23, *for* π, *read* $\frac{\pi}{4}$.
„ 101, line 34, *for* π, *read* $\frac{\pi}{4}$.
„ 104, line 5 from bottom, *for* main, *read* mean.
„ 105, line 17, *for* 5·42, *read* 5·115.
„ 105, line 19, *for* ·42, *read* ·115.
„ 111, line 18, *for* $\frac{5}{15}$, *read* $\frac{5}{16}$.
„ 111, line 19, *for* $x=5$, *read* $n=4$.
„ 113, line 11, *for* $1\frac{1}{16}$, *read* $1\frac{3}{16}$.
„ 114, line 21, *after* lattice, *add* comma, *omit* and; and, *after* warren, *add* N or Whipple-Murphy.
„ 117, line 35, *after* being, *add* so great.
„ 120, last line, *for* $\frac{2}{w\,c^3}$, *read* $\frac{w\,c^3}{2}$.
„ 121, line 20, *add* a bracket (at commencement of line.
„ 124, line 15, *after* web, *add* a comma.
„ 125, line 5, *for* $\frac{S}{P}$, *read* $\frac{P}{S}$.
„ 127, line 17, *add*, If as near together as 4′ 6″, iron longitudinals are sometimes dispensed with.
„ 128, *Fig.* 20, the arrows should not be at the extreme edges of the angle irons, but about $\frac{3}{4}$ inch inside both at top and bottom.
„ 130, line 19, *for* $\frac{\cdot 50}{5}$, *read* $\frac{\cdot 05}{5}$.
„ 131, line 4, *for* $\frac{16}{19}$, *read* $\frac{19}{16}$.
„ 134, line 10. In dealing with the shear, it would be more accurate and not more troublesome to consider the actual loads,

instead of the equivalent uniformly distributed load. This would reduce the shear at the ends, but it would, in this case, hardly affect the shear at 3 feet from the centre, when only one train is on.

Page 134, line 13, *for* 33 ft. tons, *read* 32·5 ft. tons.
„ 134, line 26, *for* 66, *read* 69⅓.
„ 134, line 29, *for* ·46, *read* ·44.
„ 135, *Fig.* 30, *for* w, *read* w_1, and *for* w_1, *read* w.
„ 140, line 1, *for* 2, *read* 2·5.
„ 140, *Fig.* 36, the arrows should not be at the extreme edges of the flanges, but about 1⅛ inch inside, both at the top and bottom.
„ 144, line 12 from bottom. It would be more correct in calculating the resistance of the web to buckling to take the shear at the end of each division of the web nearest the pier instead of that furthest from it, *i.e.*, for division 1, take the shear at the stiffener between 1 and 2; for division 2, take the shear at the stiffener between 2 and 3.
„ 144, line 9 from bottom, *for* ·87, *read* ·88.
„ 145, line 19, *for* 17 tons, *read* 15·4 tons, and see Appendix, p. 320.
„ 149, line 16, *omit* They are shewn on *Plate* II.
„ 150, line 16, *for* $2 \times 150 \times 2$, *read* $4 \times 150 \times 2$.
„ 150, *Plate* III., *Built-up Wrought Iron Beams*, the scale of the rocker is about ½ inch to 1 foot.
Table II., *Lattice Girder*, column 6, *for* $\frac{1}{14}$, *read* $\frac{1}{15}$.
„ V., *Lattice Girder*, column 4, *for* $+ \frac{1}{6} Wn$, *read* $- \frac{1}{6} Wn$.
Page 160, line 7, *after* =, *add* $\frac{1}{6}$.
„ 160, line 6 from bottom, at end of line, *add* bars.
„ 162, line 15, *for* +(1), *read* (+1).
„ 164, line 20, *before* 2·1204, *add* −.
„ 164, line 21, *before* 41·3626, *add* −.
„ 165, line 25, *for* $w_1 \times \frac{5}{8}$, *read* $w_1 \times \frac{5}{8}$.
„ 166, line 13, *for* 60 08, *read* 60·80.
„ 167, line 4 from bottom, *before* =, *add* Stress in *b* A.
„ 169, line 3, *for* *b*, *read* *d*.
„ 169, line 16. The effect of the difference of these horizontal components is to produce two horizontal reactions in the booms, one of which relieves the stress all through one boom, while the other increases it in the other boom; and these stresses must be added to or deducted from the stresses obtained in the manner given at p. 167, in order to make the total stresses correct. If this is not done, the results obtained by Rankine's Method of Sections, as applied in p. 167, will not agree with those obtained in the other way.

Thus, suppose a girder with four sets of triangles, in which case two bars meet half-way up the end pillar. Let the difference of the horizontal components be 1 ton tension, then a compression will be produced on each boom $= \frac{1}{2}$ ton, and this compression is additional to that calculated for the upper

boom, and is to be deducted from the tension calculated for the lower boom, all through in both cases.

Page 178, line 7, *after* points, *add* each part is in the condition of a beam fixed at both ends, and

„ 178, line 10, *for* $\frac{3wc}{4}$, *read* $2wc$.

Page 178, *Fig.* 9, *for* c, c, c, *read* $2c, 2c, 2c$.

„ 186, line 6, *for* $\frac{3}{6}$ W and $\frac{1}{6}$ W, *read* $\frac{1}{6}$ W and $\frac{3}{6}$ W.

Plate I., *Fig.* 2, *Bowstring Beams.* This diagram is drawn to a wrong scale; F_1 and F_6 *being drawn as* 18 cwt., *instead of* 1 ton.

„ I., *Fig.* 3, *Bowstring Beams, for* B_5 and B_6, *read* B_7 and B_8.

Page 187, line 13, *omit* latter.

„ 188, line 22, *for* B_{27}, *read* B_1.

„ 194, line 25, *for* P, *read* P_h.

„ 195, line 1, *for* 50°, 60°, and 70°, *read* 60°, 70°, and 80°.

„ 199. Column of remarks in Table IV., *add* with reference to the spans of 14, 16, and 18 feet "If there is no ceiling, the collars may be reduced in depth but they should be made thicker to prevent side buckling, as they will no longer be stiffened by the lathing.

„ 201, before line 10 from bottom, insert the following lines:

Weight of tie, ceiling joists, and ceiling=$12\frac{3}{4}$ lbs. per square foot of horizontal surface covered.

Weight of snow=5 lbs. per square foot of horizontal surface covered=4·5 lbs. per square foot of roofing surface.

„ 205, line 22, *for* F_3, *read* F_8.

„ 206, column 4 in table, *for* 2700, *read* 4700.

„ 210, line 27, *for* 16, *read* 15·6.

„ 210, line 28, *for* 30, *read* 25, and *for* 15, *read* 13·9.

„ 210, line 29, *for* 30, *read* 25, and *for* 45 lbs., *read* 39 lbs. nearly.

„ 210, line 3 from bottom, *for* 45×45^2, *read* 39×45^2, and *for* 45^3, *read* 39×45^2.

„ 211, line 3, *for* 45^3, *read* 39×45^2.

„ 211, line 5, *for* 7·8, *read* 5·8.

„ 211, line 14, *for* $7\frac{3}{4}$ or 8, *read* $5\frac{3}{4}$ or 6.

„ 211, line 33, *for* last example, *read* Example I.

„ 211, *after* line 37, *insert* the following lines:

Weight of tie, ceiling joists, and ceiling=13 lbs. per square foot of horizontal surface covered.

„ 212, *Fig.* 22, *see* Note in Appendix, p. 320.

Plate II., *Roofs*, *Fig.* 3, *for* R_2 (the lower of the two reactions), *read* R_1.

„ „ *Fig.* 2, *for* R_2, *read* R_3.

Page 216, line 8, *for* $\frac{1}{10}$, *read* $\frac{1}{20}$.

„ 216, line 10, *for* 7·6, *read* 7·13.

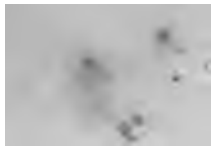
Plate IV., *Fig.* 7, *Roofs.* The tie should be horizontal.

Page 226, line 9, *for* 222, *read* 220.

„ 233, line 3, *for* $C_1 D_1$, *read* $O_1 D_1$.

„ 234, line 23, *for* eclipse, *read* ellipse.

Page 237, lines 20 and 21, *omit from* It is also *to* consequently, *and insert*, It can be shewn that.
„ 237, line 22, *omit* must, and *for* approach, *read* approaches.
„ 237, line 27, *omit* by similar reasoning to that just given, it is, *and insert* it can be shewn to be.
„ 239, line 2, *for ad, read ab.*
„ 240, line 9, *for* n_1, *read n.*
„ 240, line 16, *for* satety, *read* safety.
Page 245, line 5, *for* 458,530, *read* 485,530.
„ 246, line 3, *for* 107,984, *read* 107,084.
„ 246, line 14, *after* increased, *add* and tension set up on the windward side.
„ 246, line 6 from bottom, *after* Example, *add* II.
„ 249, lines 12, 18, 24, and 30. The recess should be deducted in all these bearing surfaces. The intensity of the pressure is thereby increased.
„ 250, line 15, *after* increased, *add* and tension set up on the windward side.
„ 250, line 7 from bottom, *for* 27·3, *read* 25·4.
„ 252, line 15, *for* R, *read* Z.
„ 253, line 5 from bottom, *for* =, *read* >.
„ 256, line 7, *for* 15, *read* 16.
„ 263, *Fig.* 5, *transpose* the letters B and D.
„ 265, last line, *after* resultant, *add* equal and opposite to the reaction.
„ 266, line 3, *for* it, *read* R_1.
„ 273, line 5, *after* bricks, *add* and mortar.
„ 273, line 32, *omit* the square of.
„ 279, line 41, *for* latter, *read* batter.
„ 279, line 43, *before* vertical, *add* nearly, and *after* batter, *add* of any importance.
„ 281, lines 22 and 23, before overhanging, *for* a, *read* an.
„ 284, *Fig.* 13, *insert* letters D and E at either end of *x*.
„ 285, line 7 from bottom, *for* I, *read* D.
„ 286, line 4 from bottom, *for Fig.* 13, *read Fig.* 15.
„ 288, line 11, *for* the outer, *read* either.
„ 290, line 30, *after* feet, *add* from H, *and transfer bracket from after* foot, *to after* third.
„ 293, line 4, *for* 13,291, *read* 13,921.
„ 293, line 5 from bottom, *for Plate* III., *read Plate* II.
„ 302, line 29, *for* H, *read* P.
„ 302, line 2 from bottom, *for* −, *read* +.
„ 306, line 27, *after* pressure, *add* measured on the diagram.
„ 306, line 28. $3 \times 1 \times 1$ signifies three times the distance, F 1, of the centre of pressure from the extrados multiplied by 1 foot in width of the arch (see p. 233).
„ 307, lines 35 and 36, the two last verticals are required to give p_1 and p_2, p. 308. They are not shewn in *Plate* VII.
„ 308, line 34, *after* tons, *add* measured on the diagram.



Page 309, line 22, strictly, C D ought, in order to prevent tension at E, to be rather more than $\frac{1}{3}$rd C E; but how much more, depends upon the relative dimensions of the wall, and counterforts forming the abutment. In an abutment of ordinary proportions C D would equal about $\frac{4}{10}$ C E.

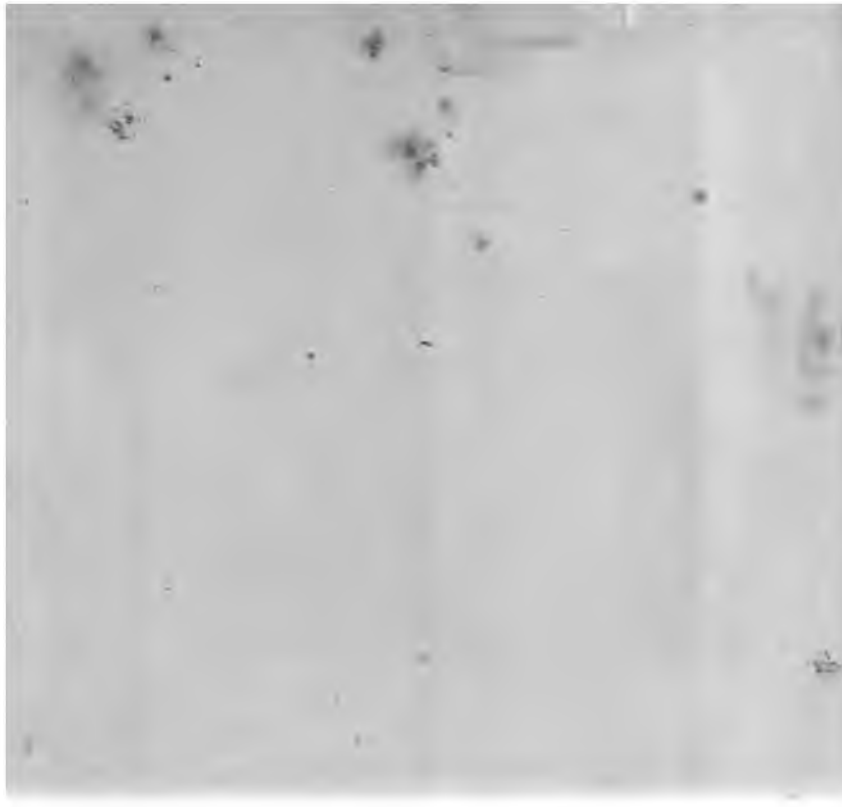
„ 309, line 32, at end of line, *omit* to.

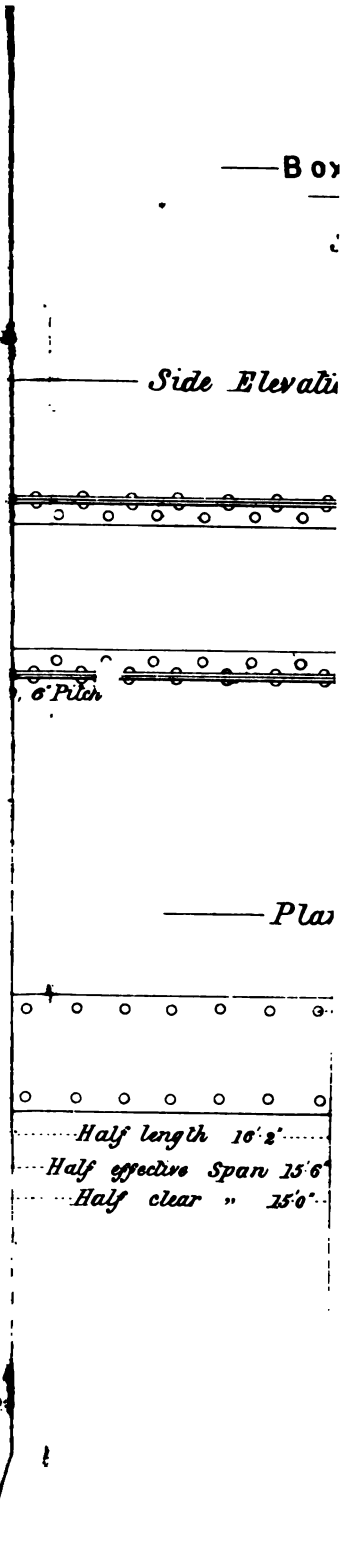
Plate IV., *Arches.* *Insert Fig.* 3 over figure in upper right hand corner.

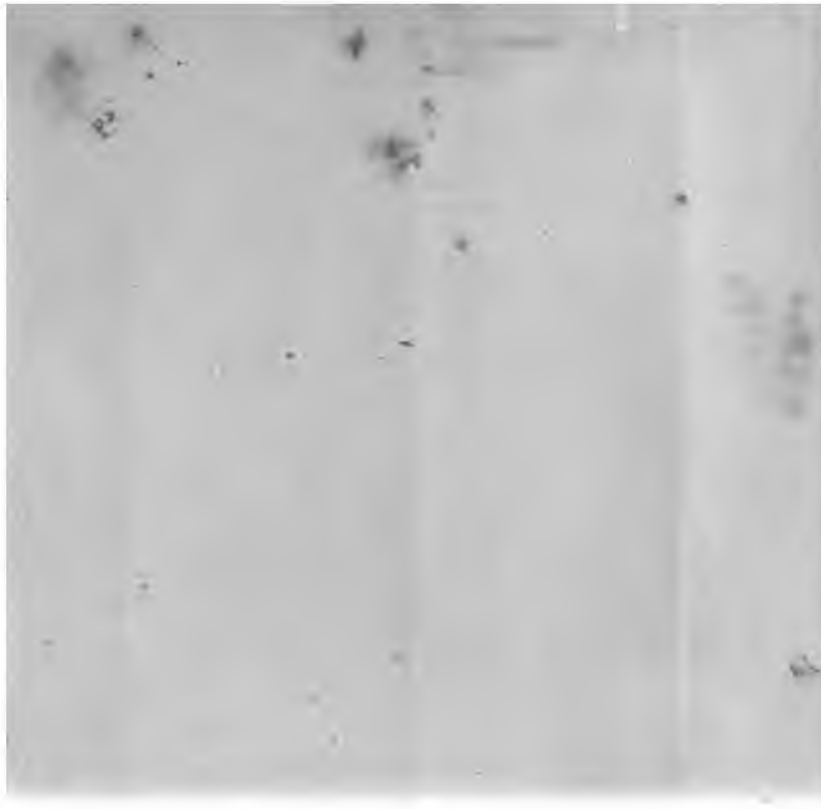
„ VI., „ the scale of *Fig.* 1 is $\frac{7}{8}$ inch to 10 feet.

„ VII., „ the scale is 5 feet to 1 inch.

„ VIII., „ *Fig.* 4. *Strike out* weakest joint.




Box
Side Elevation
6" Pitch
Plan
Half length 10'2"
Half effective Span 15'6"
Half clear " 15'0"



—— Box

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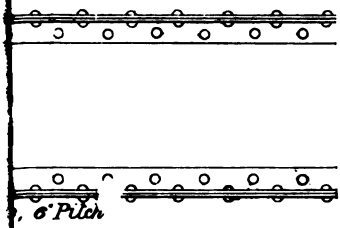
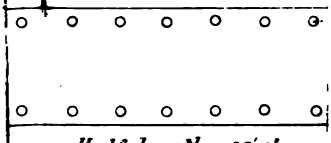


6" Pitch

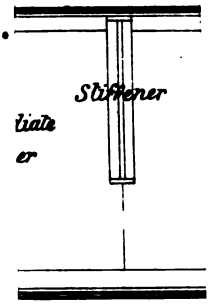
—— Plan



Half length 16'2"

Half effective Span 15'6"

Half clear " 15'0"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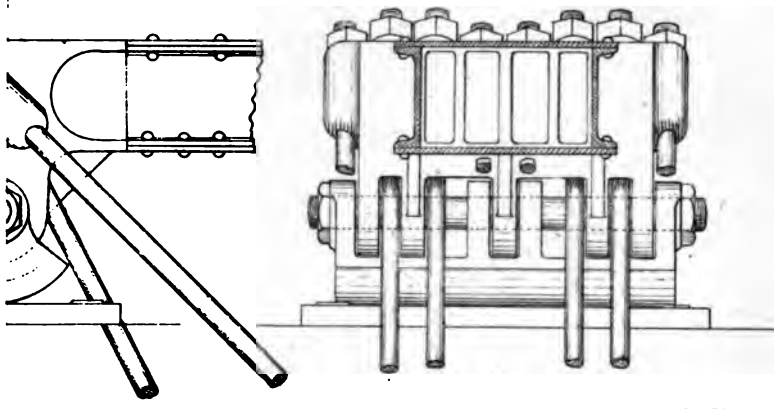
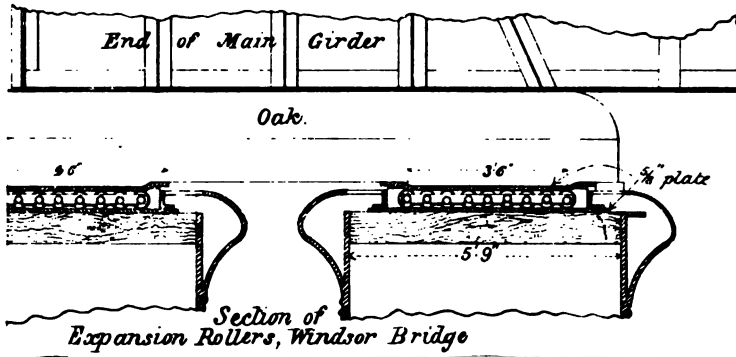
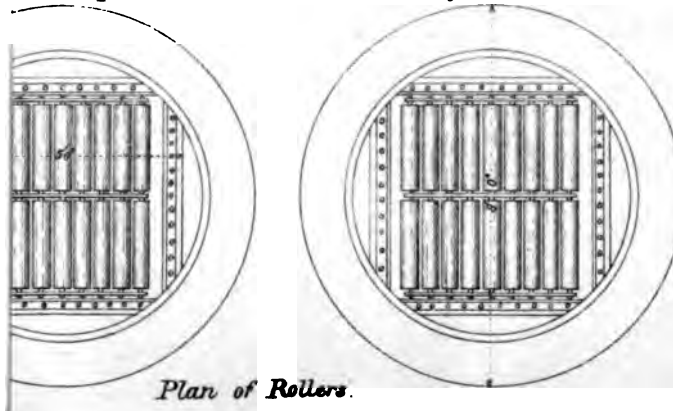


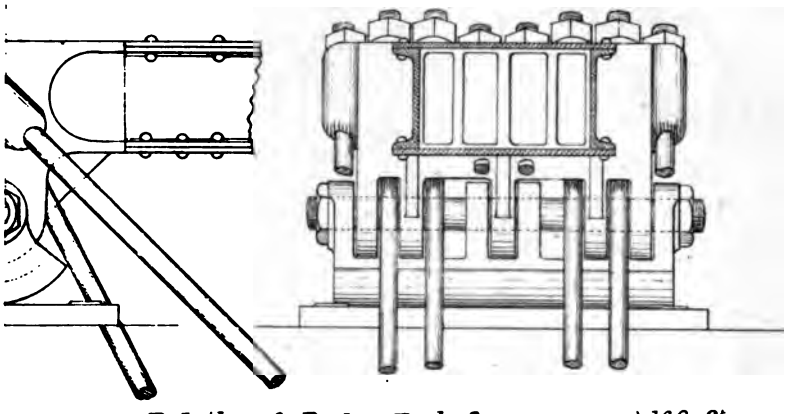
Details of Rocker End, for a span of 160 ft., used in America.



Section of Expansion Rollers, Windsor Bridge



Plan of Rollers.



Details of Rocker End, for a span of 160 ft., used in America.

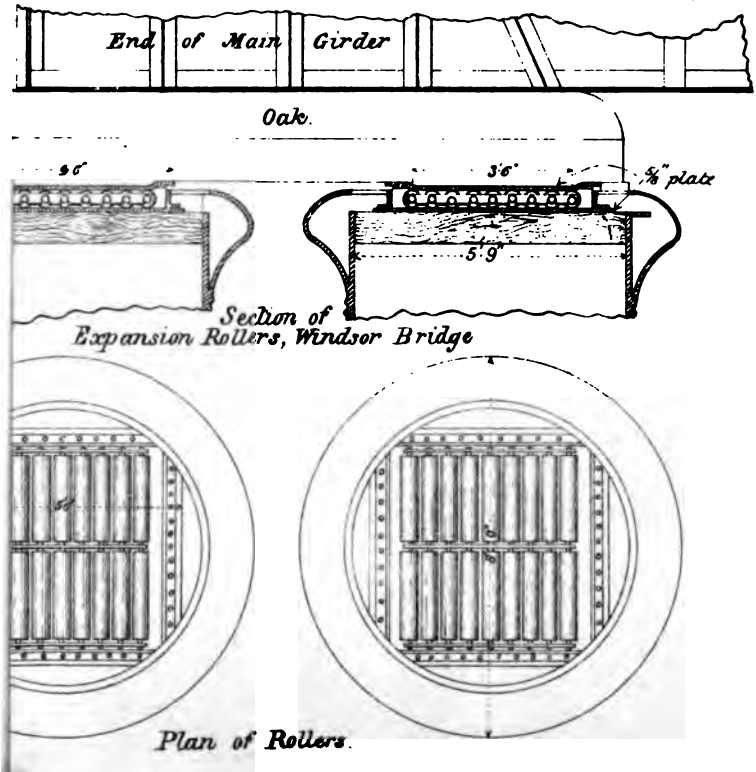
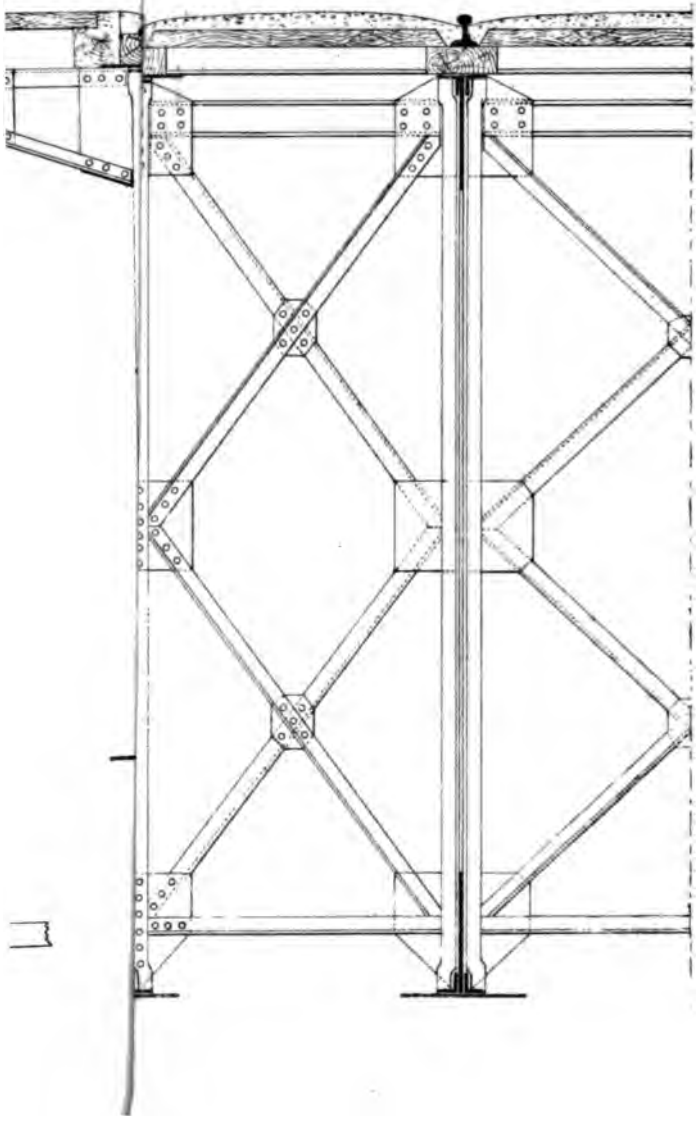


Fig. 4.



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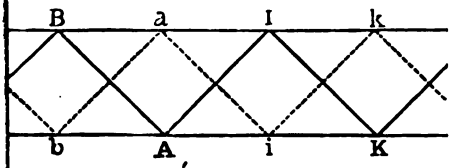


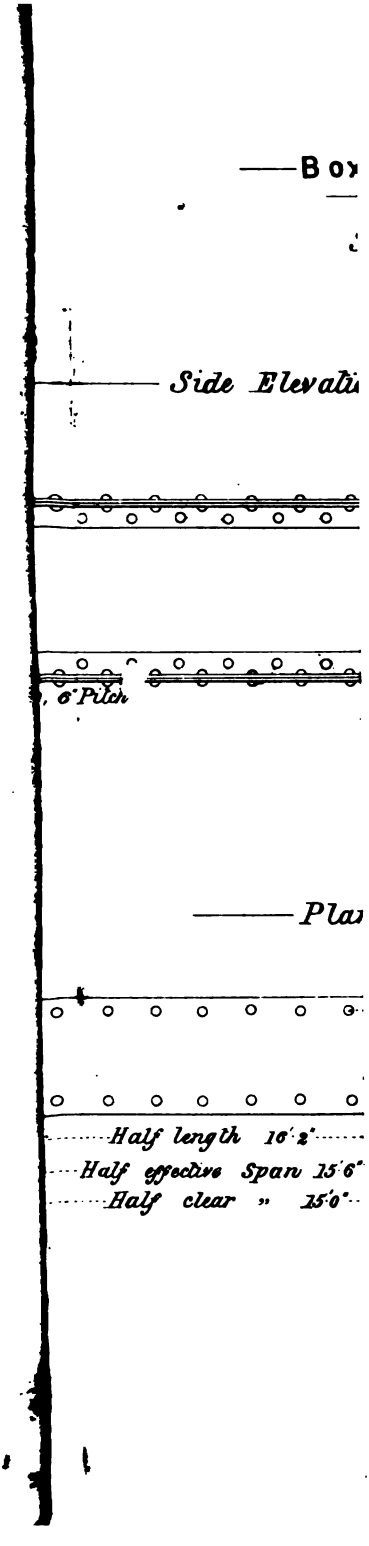
fixed load, i.e., weight of structure. Com

ntinuous black lines; w = weight at eac

on CB	*on* BA	*on* AI	*on* IK
$\frac{1}{6}wn$	$+\frac{1}{6}wn$	$-\frac{1}{6}wn$	$+\frac{1}{6}$
$\frac{2}{6}$ "	$+\frac{2}{6}$ "	$-\frac{2}{6}$ "	$+\frac{2}{6}$
$\frac{3}{6}$ "	$-\frac{3}{6}$ "	$-\frac{3}{6}$ "	$+\frac{3}{6}$
$\frac{2}{6}$ "	$-\frac{2}{6}$ "	$+\frac{2}{6}$ "	$-\frac{2}{6}$
$\frac{1}{6}$ "	$-\frac{1}{6}$ "	$+\frac{1}{6}$ "	$-\frac{1}{6}$
$\frac{3}{6}$ "	$-\frac{3}{6}$ "	$-\frac{3}{6}$ "	$+\frac{3}{6}$

es—broken lines; w = weight at each p

on cb	*on* ba	*on* ai	*on* ik
$\frac{1}{12}wn$	$-\frac{1}{12}wn$	$+\frac{1}{12}wn$	$-\frac{1}{12}$
$\frac{3}{12}$ "	$-\frac{3}{12}$ "	$+\frac{3}{12}$ "	$-\frac{3}{12}$
$\frac{7}{12}$ "	$-\frac{5}{12}$ "	$+\frac{5}{12}$ "	$-\frac{5}{12}$
$\frac{5}{12}$ "	$+\frac{5}{12}$ "	$-\frac{5}{12}$ "	$-\frac{7}{12}$
$\frac{3}{12}$ "	$+\frac{3}{12}$ "	$-\frac{3}{12}$ "	$+\frac{3}{12}$ "
$\frac{1}{12}$ "	$+\frac{1}{12}$ "	$-\frac{1}{12}$ "	$+\frac{1}{12}$ "
$\frac{12}{12}$ "	0	0	$-\frac{12}{12}$ "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III.

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as

	on hH
W	$+\frac{36}{12}Wn$
	0

	on MN	*on* NH
	$+\frac{15}{6}Wn$	0	$+\frac{15}{6}wn+\frac{15}{6}Wn$
h	$+\frac{15}{6}Wn$	$-\frac{15}{6}wn-\frac{15}{6}Wn$	0

	on mn	*on* nh	*on* hH
	$-\frac{25}{12}Wn$	$+\frac{24}{12}wn+\frac{25}{12}Wn$	0	$+\frac{36}{12}w+\frac{36}{12}W$
n	$-\frac{25}{12}Wn$	$+\frac{24}{12}wn-\frac{1}{12}Wn$	$-\frac{36}{12}wn-\frac{36}{12}Wn$	0

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III.

ions

as

	on hH
W	$+\frac{36}{12}Wn$
	0

		on MN	*on* NH
	$+\frac{9}{6}Wn$	0	$+\frac{15}{6}wn+\frac{15}{6}Wn$
h	$+\frac{9}{6}Wn$	$-\frac{15}{6}wn-\frac{15}{6}Wn$	0

		on mn	*on* nh	*on* hH
	$-\frac{24}{12}Wn$	$+\frac{24}{12}wn+\frac{25}{12}Wn$	0	$+\frac{36}{12}w+\frac{36}{12}W$
	$-\frac{24}{12}Wn$	$+\frac{24}{12}wn-\frac{1}{12}Wn$	$-\frac{36}{12}wn-\frac{36}{12}Wn$	0

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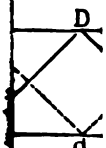

BOOMS

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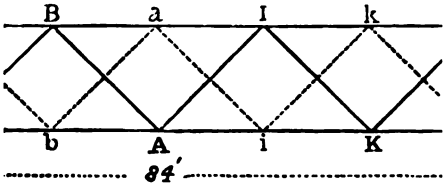
LOADED ON BOTTOM



fixed load, i.e., weight of structure. Com

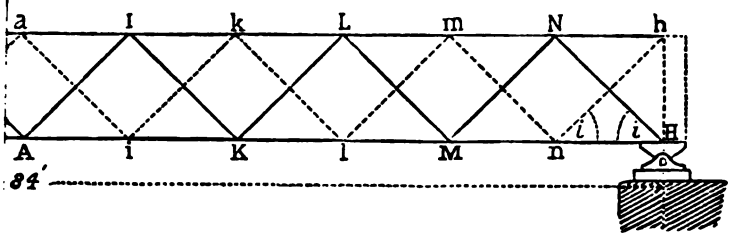
ntinuous black lines; w = weight at eac

on CB	*on* BA	*on* AI	*on* IK
$\frac{1}{6}wn$	$+\frac{1}{6}wn$	$-\frac{1}{6}wn$	$+\frac{1}{6}$
$\frac{2}{6}$ "	$+\frac{2}{6}$ "	$-\frac{2}{6}$ "	$+\frac{2}{6}$
$\frac{3}{6}$ "	$-\frac{3}{6}$ "	$-\frac{3}{6}$ "	$+\frac{3}{6}$
$\frac{2}{6}$ "	$-\frac{2}{6}$ "	$+\frac{2}{6}$ "	$-\frac{2}{6}$
$\frac{1}{6}$ "	$-\frac{1}{6}$ "	$+\frac{1}{6}$ "	$-\frac{1}{6}$
$\frac{3}{6}$ "	$-\frac{3}{6}$ "	$-\frac{3}{6}$ "	$+\frac{3}{6}$

es—broken lines; w = weight at each p

on cb	*on* ba	*on* ai	*on* ik
$\frac{1}{12}wn$	$-\frac{1}{12}wn$	$+\frac{1}{12}wn$	$-\frac{1}{12}$
$\frac{3}{12}$ "	$-\frac{3}{12}$ "	$+\frac{3}{12}$ "	$-\frac{3}{12}$
$\frac{7}{12}$ "	$-\frac{5}{12}$ "	$+\frac{5}{12}$ "	$-\frac{5}{12}$ "
$\frac{5}{12}$ "	$+\frac{5}{12}$ "	$-\frac{5}{12}$ "	$-\frac{7}{12}$ "
$\frac{3}{12}$ "	$+\frac{3}{12}$ "	$-\frac{3}{12}$ "	$+\frac{3}{12}$ "
$\frac{1}{12}$ "	$+\frac{1}{12}$ "	$-\frac{1}{12}$ "	$+\frac{1}{12}$ "
$\frac{12}{12}$ "	0	0	$-\frac{12}{12}$ "

ED ON BOTTOM FLANGE.

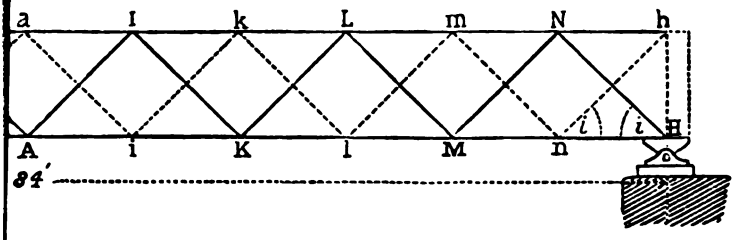


ɔ be provided for. Compressions +. Tensions —.

triangles, it is under the stress produced by each set of braces.

	Upper flange.		
gF	$+\frac{36}{12}$	$(w + W)\ m$	
Fe	$+\left(\frac{36}{12}+\frac{15}{3}\right)$	,,	,,
eD	$+\left(\frac{15}{3}+\frac{84}{12}\right)$	,,	,,
Dc	$+\left(\frac{84}{12}+\frac{24}{3}\right)$	,,	,,
cB	$+\left(\frac{24}{3}+\frac{108}{12}\right)$	,,	,,
Ba	$+\left(\frac{108}{12}+\frac{27}{3}\right)$	,,	,,
aI	$+\left(\frac{27}{3}+\frac{108}{12}\right)$	,,	,,
Ik	$+\left(\frac{108}{12}+\frac{24}{3}\right)$	,,	,,
kL	$+\left(\frac{24}{3}+\frac{84}{12}\right)$	,,	,,
Lm	$+\left(\frac{84}{12}+\frac{15}{3}\right)$	,,	,,
mN	$+\left(\frac{15}{3}+\frac{36}{12}\right)$	,,	,,
Nh	$+\frac{36}{12}$	,,	,,

ED ON BOTTOM FLANGE.

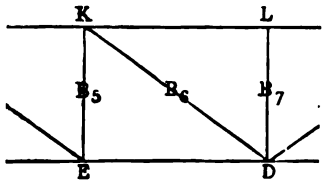


be provided for. Compressions +. Tensions —.

triangles, it is under the stress produced by each set of braces.

	Upper flange.		
gF	$+\frac{36}{12}$	$(w + W)$	m
Fe	$+\left(\frac{36}{12}+\frac{15}{3}\right)$	"	"
eD	$+\left(\frac{15}{3}+\frac{84}{12}\right)$	"	"
Dc	$+\left(\frac{84}{12}+\frac{24}{3}\right)$	"	"
cB	$+\left(\frac{24}{3}+\frac{108}{12}\right)$	"	"
Ba	$+\left(\frac{108}{12}+\frac{27}{3}\right)$	"	"
aI	$+\left(\frac{27}{3}+\frac{108}{12}\right)$	"	"
Ik	$+\left(\frac{108}{12}+\frac{24}{3}\right)$	"	"
kL	$+\left(\frac{24}{3}+\frac{84}{12}\right)$	"	"
Lm	$+\left(\frac{84}{12}+\frac{15}{3}\right)$	"	"
mN	$+\left(\frac{15}{3}+\frac{36}{12}\right)$	"	"
Nh	$+\frac{36}{12}$	"	"

ıBLE FOR E:



tressions +. Tensions − ; u

on B_5	on B_6
$+\frac{1}{6}w$	$-\frac{1}{6}wn$
$+\frac{2}{6}$,,	$-\frac{2}{6}$,,
$+\frac{3}{6}$,,	$-\frac{3}{6}$,,
$-\frac{2}{6}$,,	$+\frac{2}{6}$,,
$-\frac{1}{6}$,,	$+\frac{1}{6}$,,

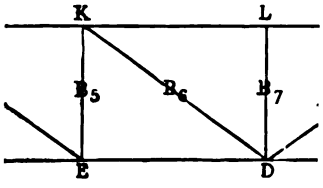
plied on the bottom boom, i

pressions +. Tensions − ;

on B_5	on B_6
Nil	Nil
$-\frac{1}{6}w_1$	$+\frac{1}{6}w_1n$
$+\frac{4}{6}$,,	$+\frac{2}{6}$,,
$+\frac{3}{6}$,,	$-\frac{3}{6}$,,
$+\frac{2}{6}$,,	$-\frac{2}{6}$,,
$+\frac{1}{6}$,,	$-\frac{1}{6}$,,
Nil	Nil

pplied on the top boom, in
sses on the braces in any c

ıBLE FOR E



ressions +. Tensions − ; *u*

on B_5	*on* B_6
$+\frac{1}{6}\ w$	$-\frac{1}{6}\ wn$
$+\frac{2}{6}$,,	$-\frac{2}{6}$,,
$+\frac{3}{6}$,,	$-\frac{3}{6}$,,
$-\frac{2}{6}$,,	$+\frac{2}{6}$,,
$-\frac{1}{6}$,,	$+\frac{1}{6}$,,

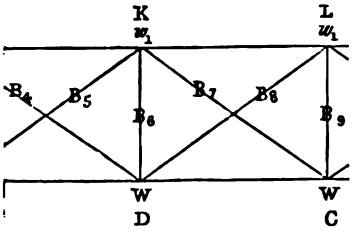
plied on the bottom boom, i

pressions +. Tensions − ;

on B_5	*on* B_6
Nil	Nil
$-\frac{1}{6}\ w_1$	$+\frac{1}{6}\ w_1 n$
$+\frac{4}{6}$,,	$+\frac{2}{6}$,,
$+\frac{3}{6}$,,	$-\frac{3}{6}$,,
$+\frac{2}{6}$,,	$-\frac{2}{6}$,,
$+\frac{1}{6}$,,	$-\frac{1}{6}$,,
Nil	Nil

pplied on the top boom, in
sses on the braces in any c

LE FOR EXA



sions +. Tensions −; W = l

	on B_6	on B_7	
	+ $\frac{1}{5}$ W	− $\frac{1}{5}$ Wn	
	+ $\frac{2}{5}$,,	− $\frac{2}{5}$,,	
	Nil	Nil	−
Wn	+ $\frac{1}{5}$,,	Nil	−
,,	+ $\frac{4}{5}$,,	− $\frac{3}{5}$,,	−

ssions +. Tensions −; w_1=los

	on B_6	on B_7	
	Nil	Nil	
w_1n	+ $\frac{1}{5}$ w_1	Nil	−
l	+ w_1	Nil	−
	+ $\frac{2}{5}$,,	− $\frac{2}{5}$ w_1n	
	+ $\frac{1}{5}$,,	− $\frac{1}{5}$,,	
	Nil	Nil	
,,	+ $\frac{9}{5}$,,	− $\frac{3}{5}$,,	−

Fig. 2.
Loads on top taken as equal.

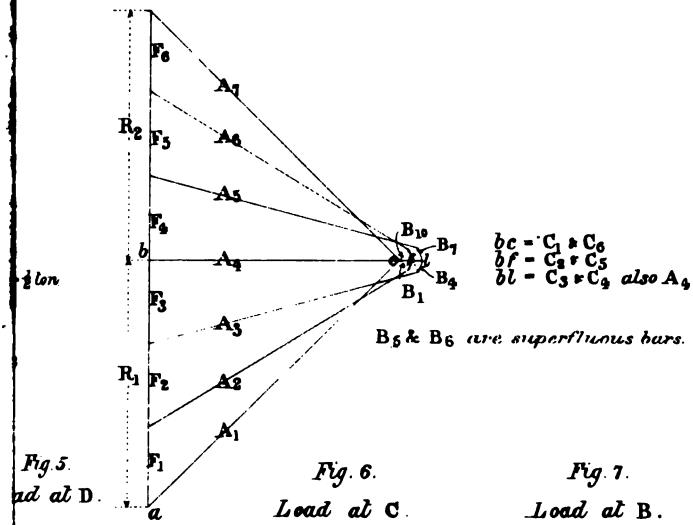


Fig. 5.
ad at D.

Fig. 6.
Load at C.

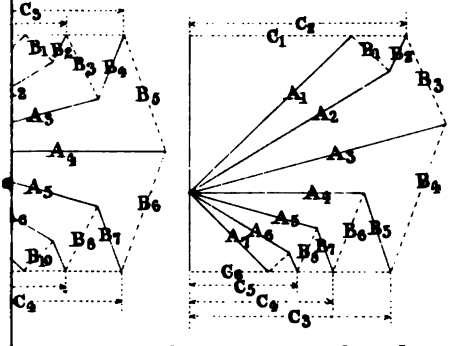
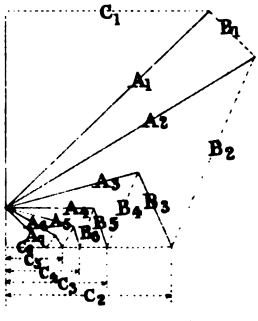
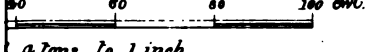


Fig. 7.
Load at B.

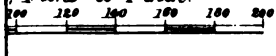


Note. Figs. 6 & 7 are obviously Figs. 4 & 3 reversed and in practice it would be unnecessary to draw them.

d, 2 tons to 1 inch.



, 4 tons to 1 inch.

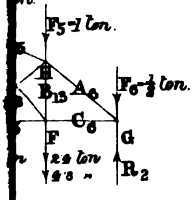


on account of the small size of the diagrams.

on A_1	on C_1	on C_2	on C_3	on C_4	on C_5	on C_6
+ 5	50·0	— 53·0	— 55·1	— 55·1	— 53·0	— 50·0
+ 1	8·3	— 11·2	— 15·0	— 21·0	— 33·3	— 39·8
+ 2	16·7	— 23·9	— 27·6	— 41·6	— 43·7	— 32·0
+ 4	24·3	— 33·0	— 44·2	— 44·2	— 33·0	— 24·3
+ 5	32·0	— 43·7	— 41·6	— 27·6	— 23·9	— 16·7
+ 2	39·8	— 33·3	— 21·0	— 15·0	— 11·2	— 8·3
+ 21	171·1	— 198·1	— 204·5	— 204·5	— 198·1	— 171·1

on A	on C_1	on C_2	on C_3	on C_4	on C_5	on C_6
+ 1	16·6	— 22·4	— 30·0	— 42·0	— 66·6	— 79·6
+ 1	33·4	— 47·8	— 55·2	— 83·2	— 87·4	— 64·0
+ 1	48·6	— 66·0	— 88·4	— 88·4	— 66·0	— 48·6
+ 1	64·0	— 87·4	— 83·2	— 55·2	— 47·8	— 33·4
+ 1	79·6	— 66·6	— 42·0	— 30·0	— 22·4	— 16·6
+ 3	242·2	— 290·2	— 298·8	— 298·3	— 290·2	— 242·2

on A	on C_1	on C_2	on C_3	on C_4	on C_5	on C_6
+ 2	— 171·1	— 198·1	— 204·5	— 204·5	— 198·1	— 171·1
+ 3	— 242·2	— 290·2	— 298·8	— 298·8	— 290·2	— 242·2
+ 5	— 413·3	— 488·3	— 503·3	— 503·3	— 488·3	— 413·3



g. 5.
at D

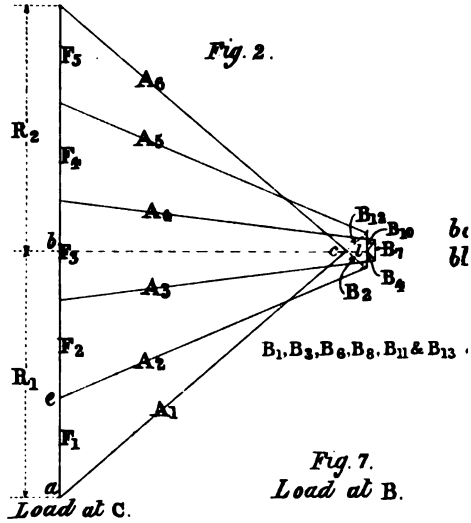
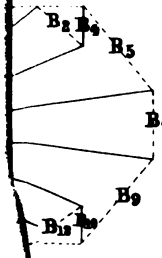
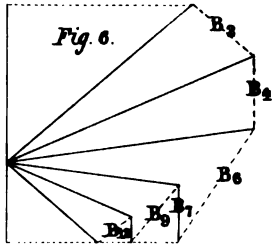


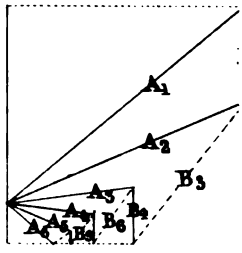
Fig. 2.

Load at C.

Fig. 7.
Load at B.



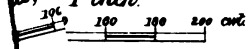




Note. Figs. 6 & 7 are obviously Figs. 2 & 3 reversed and in practice it would be unnecessary to draw them.

, 2, 1 inch.

d, 4 1 inch.



ut on e small size of the diagrams.

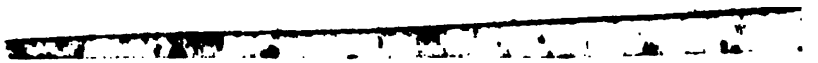
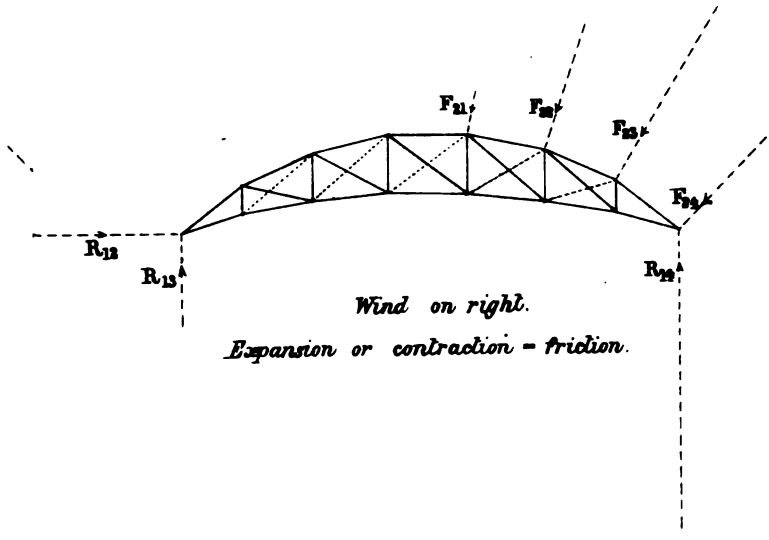


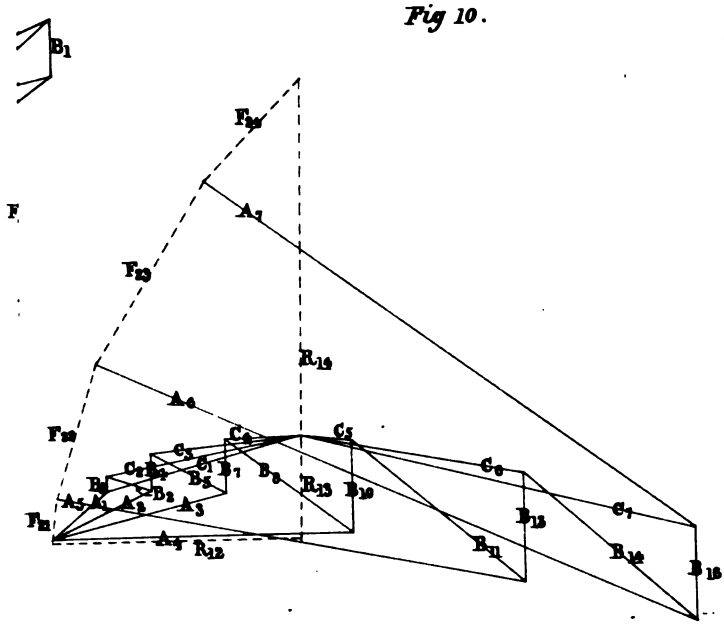
Fig. 5.



Wind on right.

Expansion or contraction = friction.

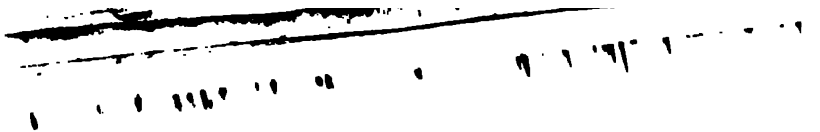
Fig 10.



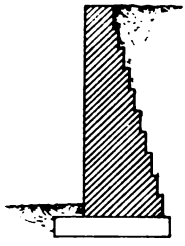
on A_4	on B_{13}	on C_1	on C_2	on C_3	on C_4	on C_5	on C_6
54·2	Nil	— 58·0	— 58·0	— 64·0	— 64·0	— 58·0	— 58·0
25·8	20·0	— 9·6	— 9·6	— 13·0	— 17·3	— 25·4	— 37·3
50·0	Nil	— 19·0	— 19·0	— 25·3	— 34·0	— 37·5	— 37·5
52·3	Nil	— 29·0	— 29·0	— 33·0	— 33·0	— 29·0	— 29·0
34·6	Nil	— 37·5	— 37·5	— 34·0	— 25·3	— 19·0	— 19·0
17·4	Nil	— 37·3	— 25·4	— 17·3	— 13·0	— 9·6	— 9·6
234·0	20·0	— 190·4	— 178·5	— 186·6	— 186·6	— 178·5	— 190·4

on A_4	on B_{13}	on C_1	on C_2	on C_3	on C_4	on C_5	on C_6
51·(	40·0	— 19·2	— 19·2	— 26·0	— 34·6	— 50·8	— 74·6
100·(	Nil	— 38·0	— 33·0	— 50·6	— 68·0	— 75·0	— 75·0
104·(	Nil	— 58·0	— 58·0	— 66·0	— 66·0	— 58·0	— 58·0
69·4	Nil	— 75·0	— 75·0	— 68·0	— 50·6	— 38·0	— 38·0
34·(	Nil	— 74·6	— 50·8	— 34·6	— 26·0	— 19·2	— 19·2
359·(	40·0	— 264·8	— 241·0	— 245·2	— 245·2	— 241·0	— 264·8

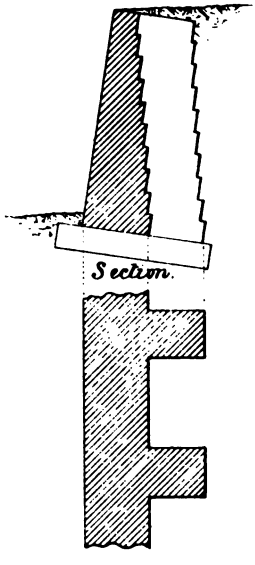
on A_4	on B_{13}	on C_1	on C_2	on C_3	on C_4	on C_5	on C_6
234·(	20·0	— 190·4	— 178·5	— 186·6	— 186·6	— 178·5	— 190·4
359·6	40·0	— 264·8	— 241·0	— 245·2	— 245·2	— 241·0	— 264·8
693·6	60·0	— 455·2	— 419·5	— 431·8	— 431·8	— 419·5	— 455·2



No. 3.



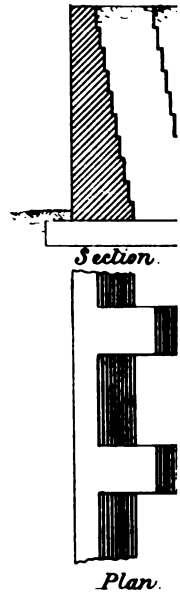
No. 8.



Section.

Horizontal Section

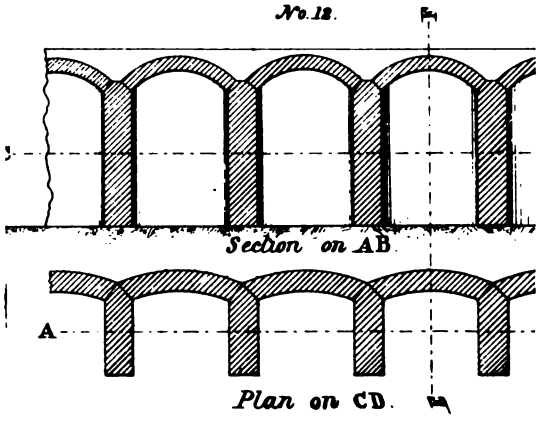
No. 9.



Section.

Plan.

No. 12.

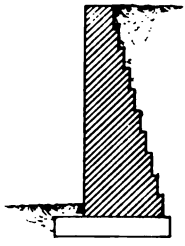


Section on AB.

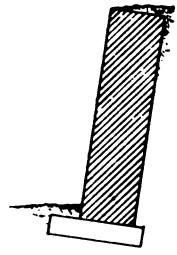
A

Plan on CD.

No. 3.



No. 4



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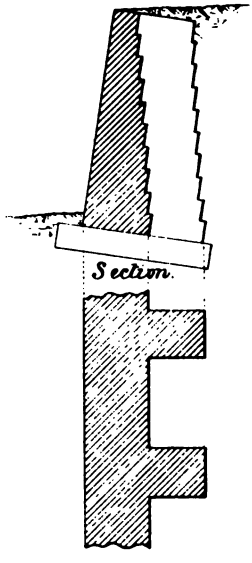
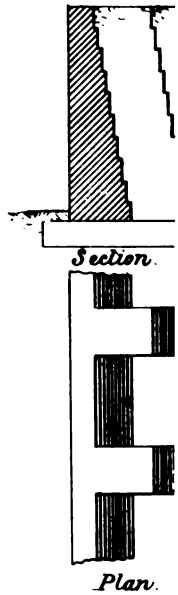


Horizontal Section

No. 9.



Plan.

No. 12.

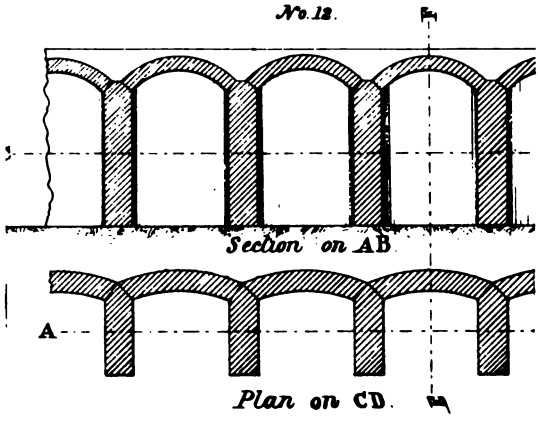
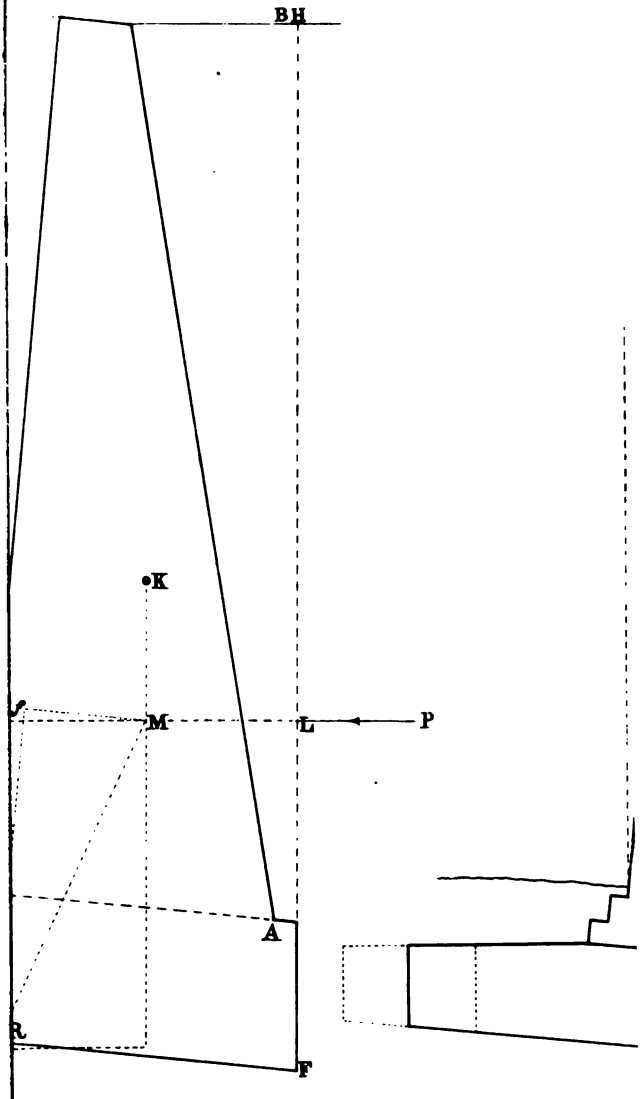
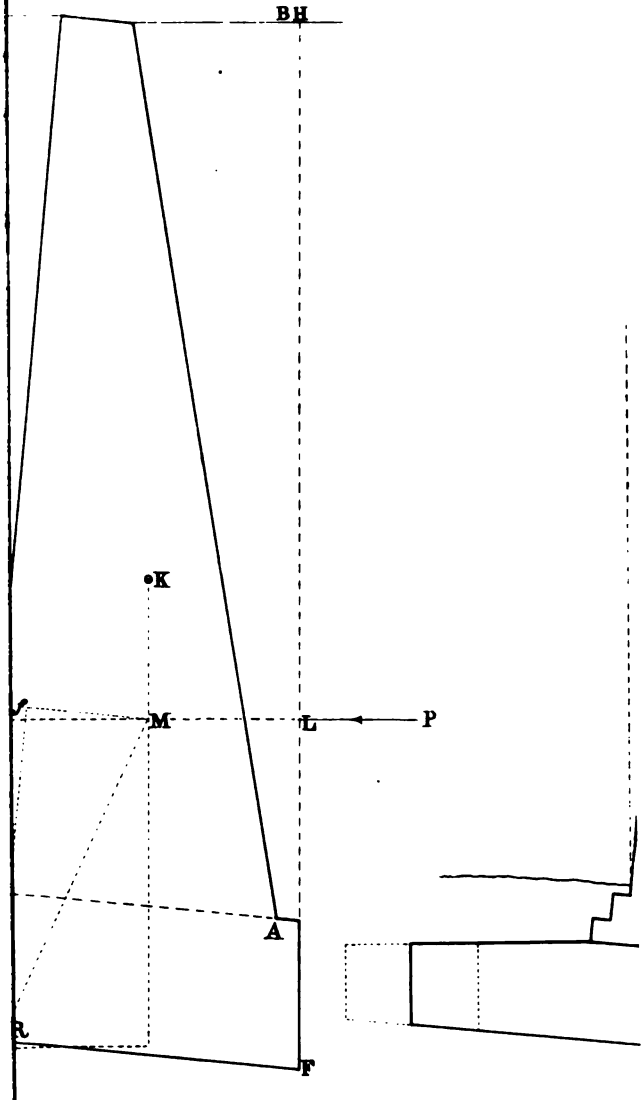


Fig 2.



– 4 feet to 1 inch
g.1 – 4000 lbs. to 1 inch
2 – 8000 " 1 "

Fig. 2.



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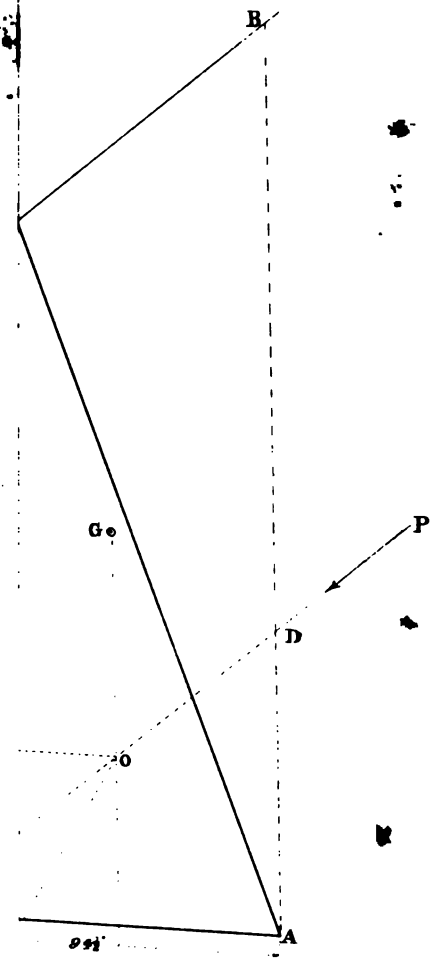
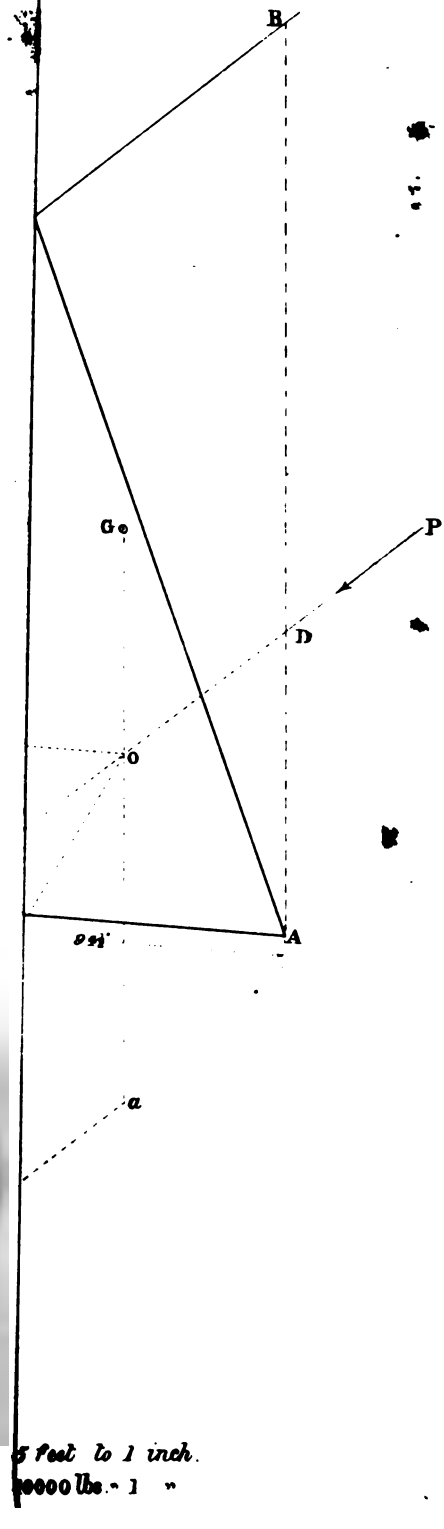


a

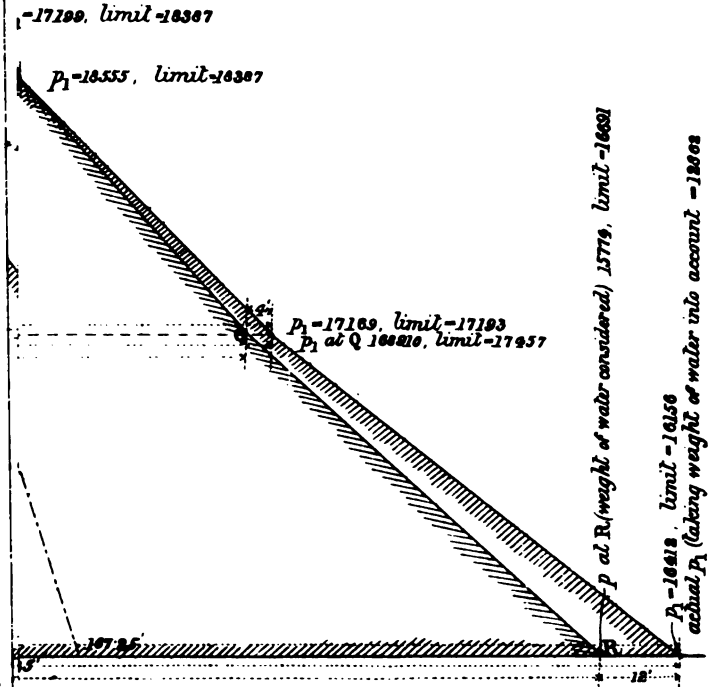
5 feet to 1 inch.
10000 lbs. - 1 "



5 feet to 1 inch.
10000 lbs. = 1 "

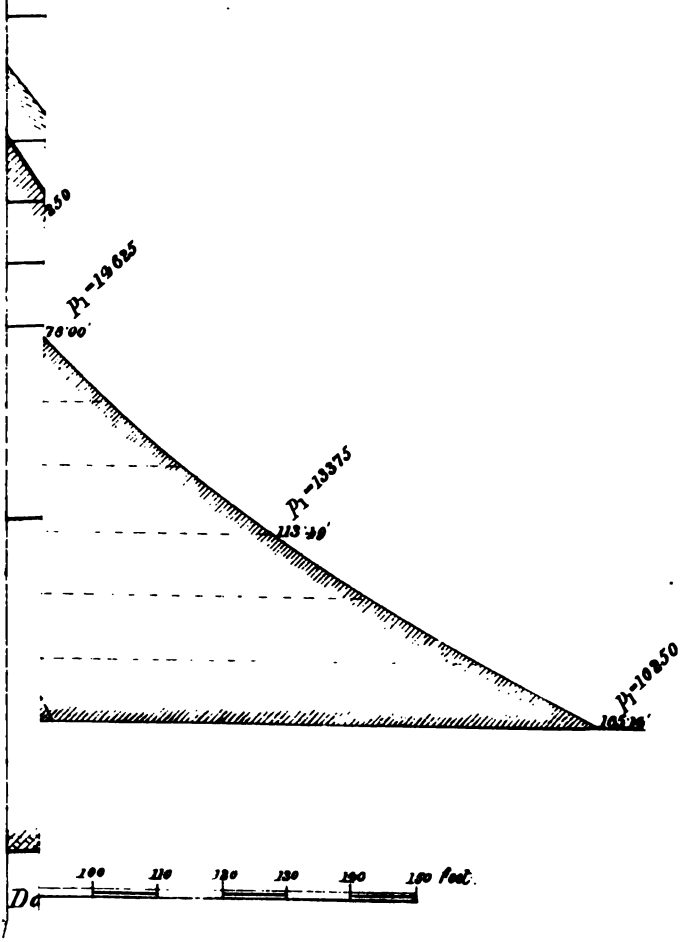
Reservoir Walls. Plate I.

ight of Water = 62·5 lbs per cub. ft.
" Masonry = 162·5 " "



Reservoir Walls. Plate II.

sor Rankine's Section



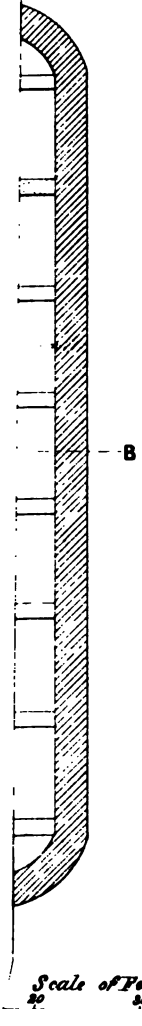
RESERVOIR.

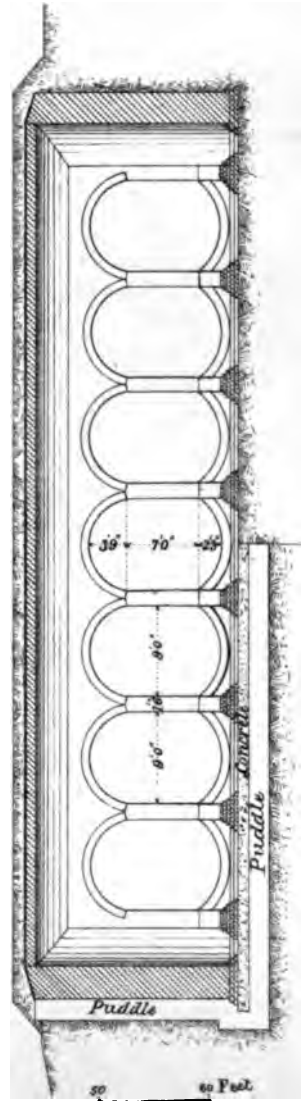
Plate III

Counterforts are sometimes inserted as shewn to resist the thrust of the main arches.

Puddle will be required on gravel, sand, or on open jointed rock but not on water tight strata.

The thickness of the walls must be calculated, as they are not to scale in this drawing

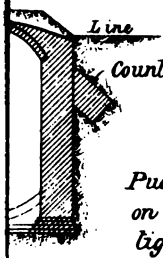




SECTION ON CD

RESERVOIR.

Plate III



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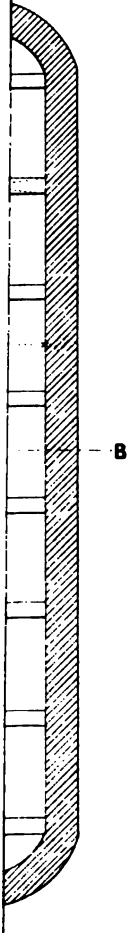
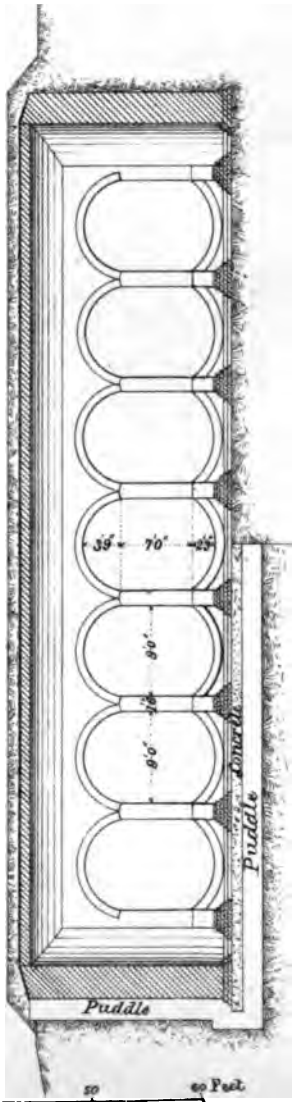




SECTION ON CD

Scale of Feet.
20 20 40 20 40 Feet

Fig. 4

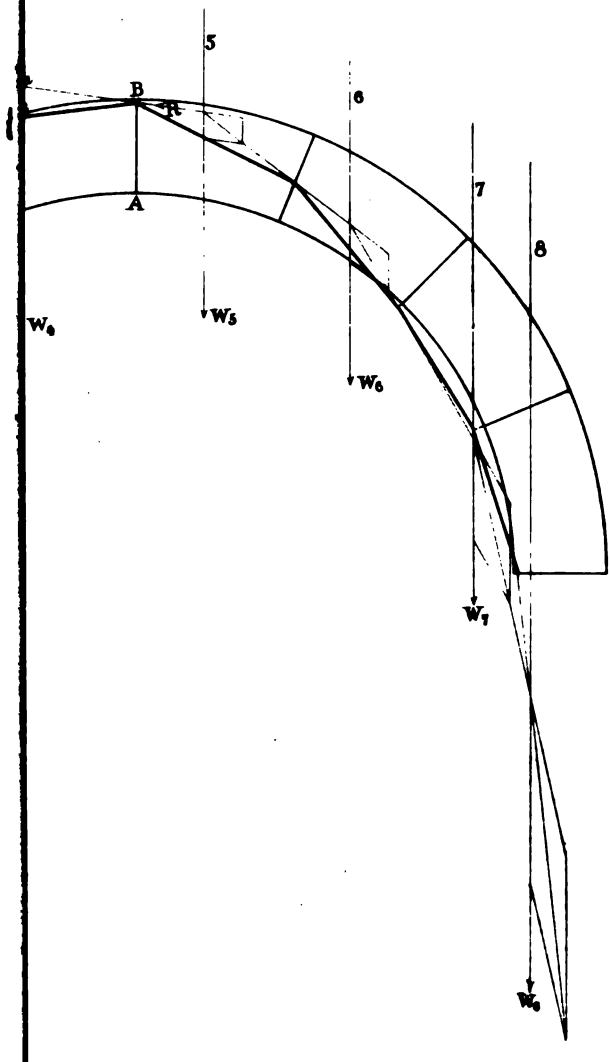
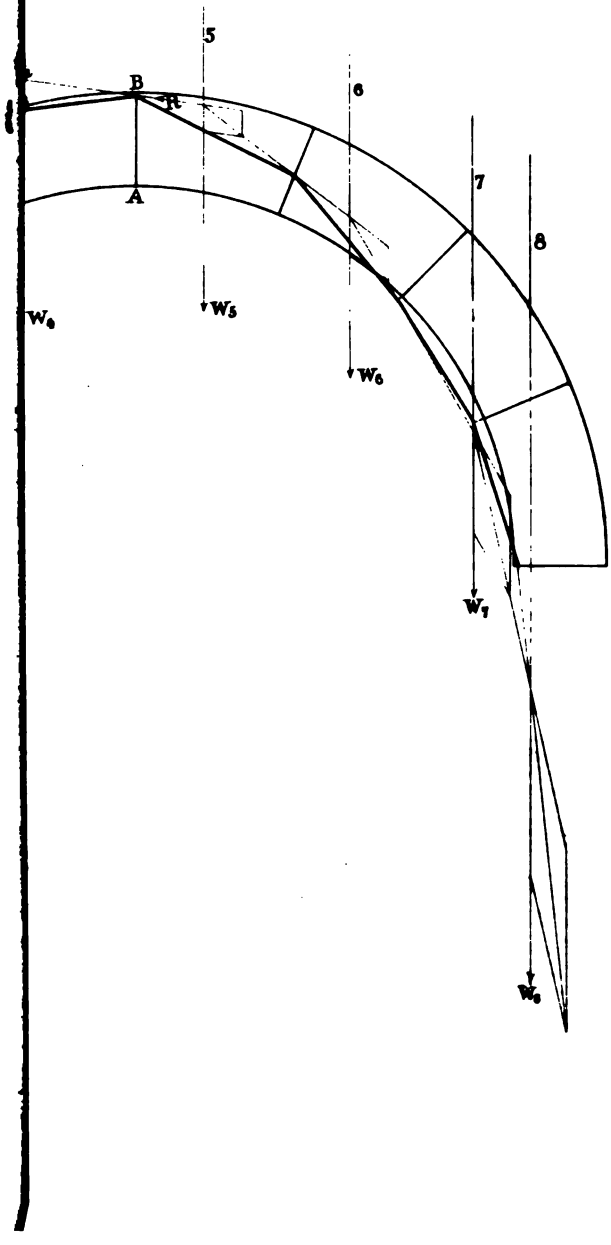


Fig. 4



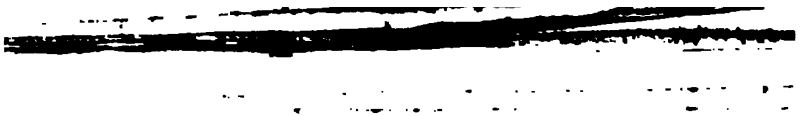
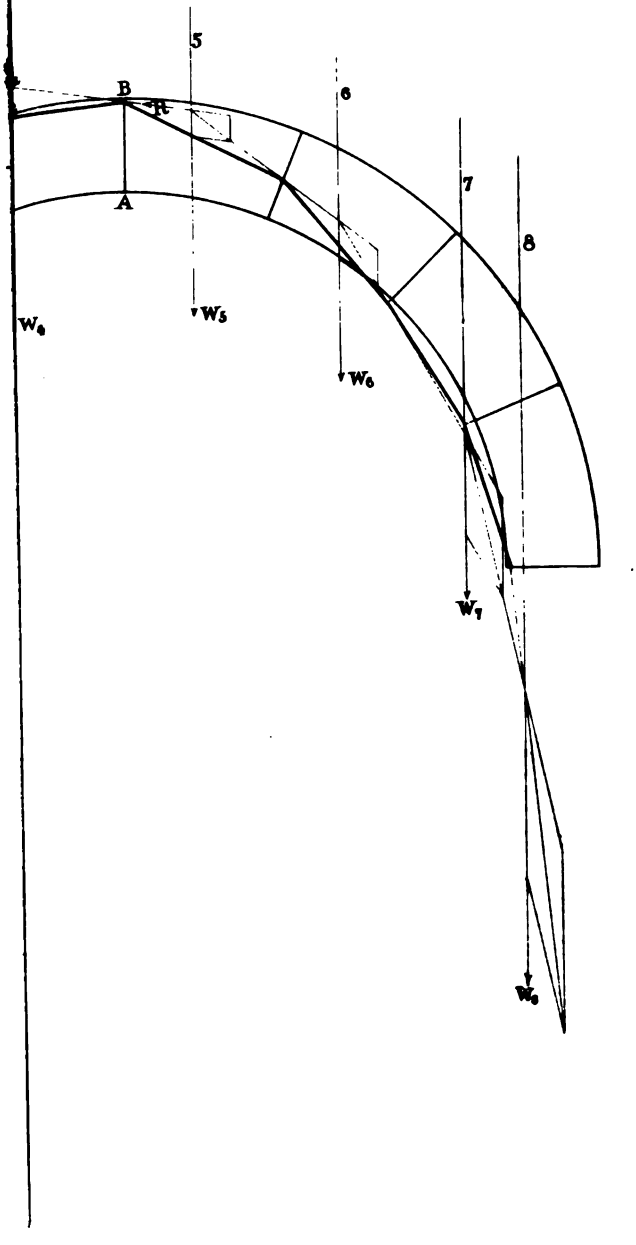
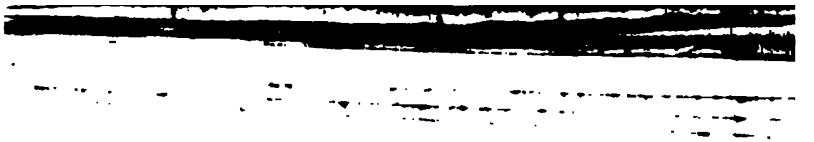


Fig. 4





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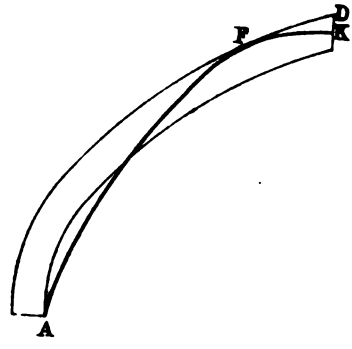

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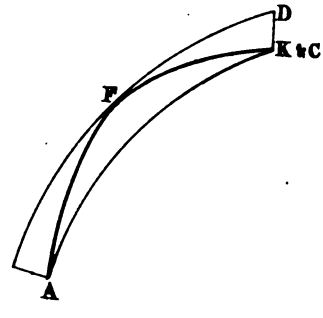


Arches. Plate 4.

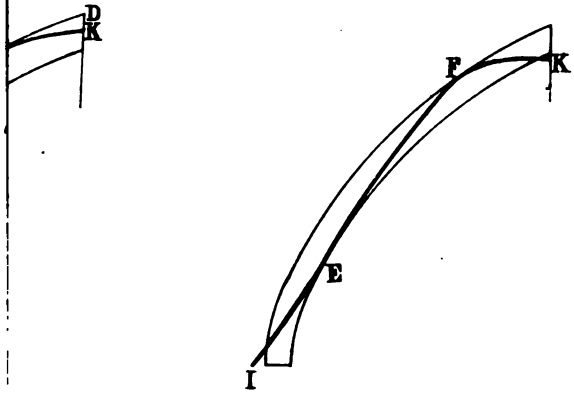
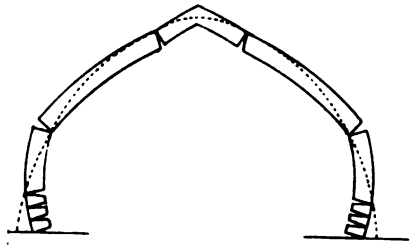


Fig. 4.



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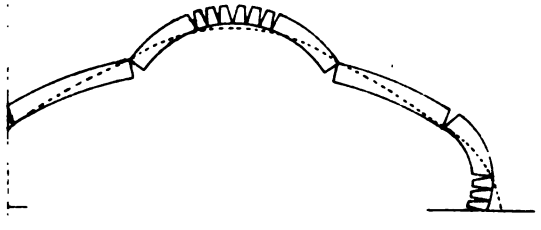

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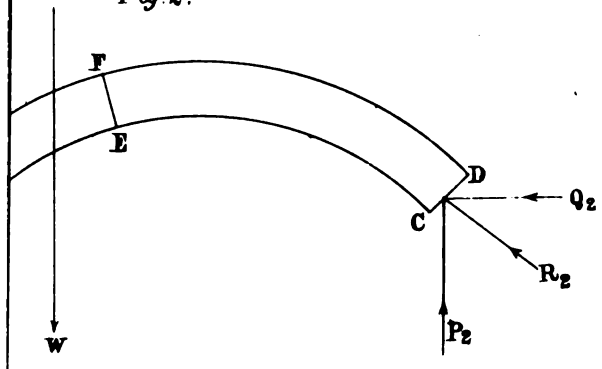
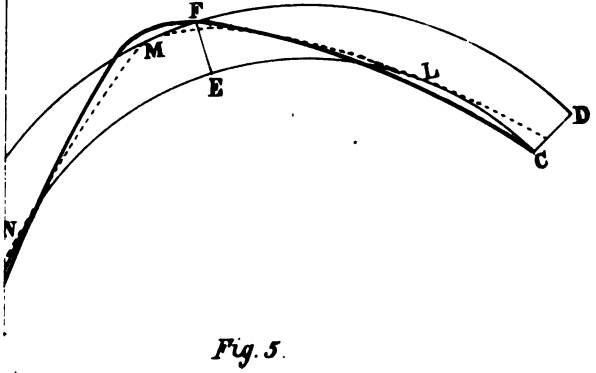


Fig. 3.



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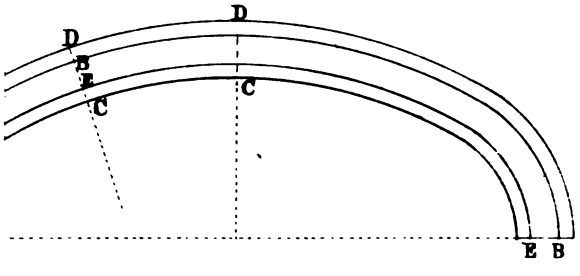

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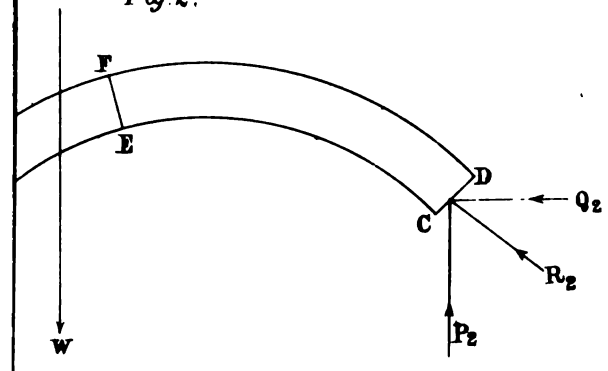

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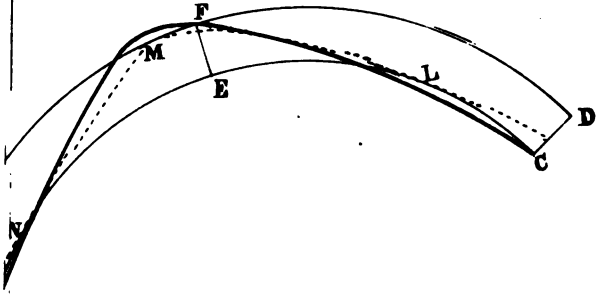

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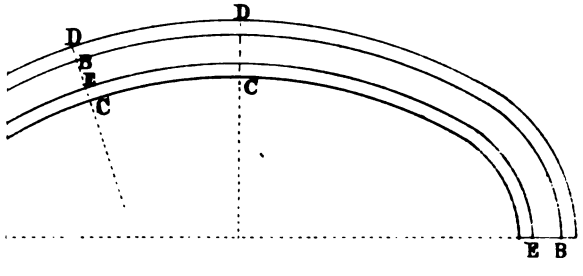
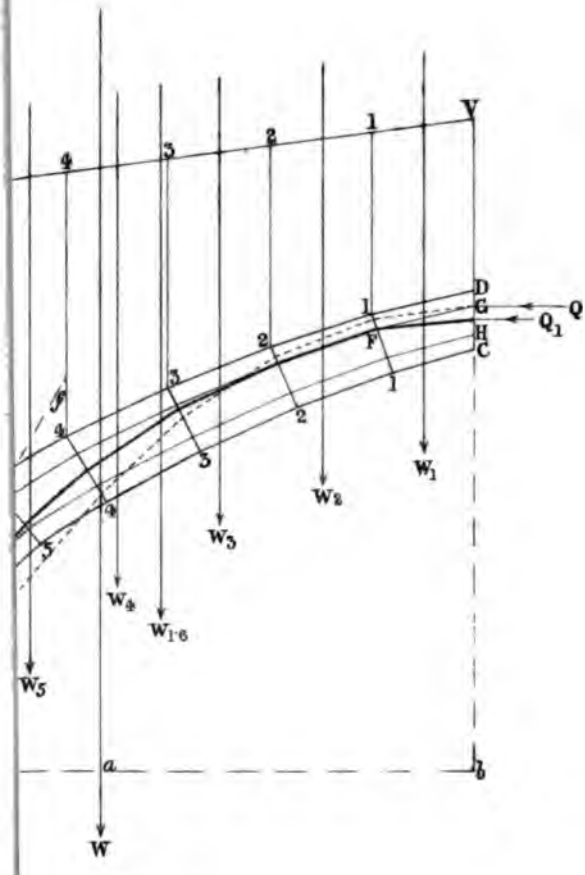
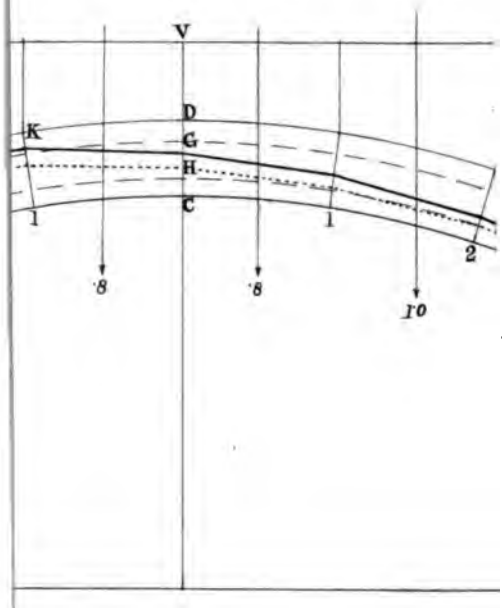


Fig. 1.

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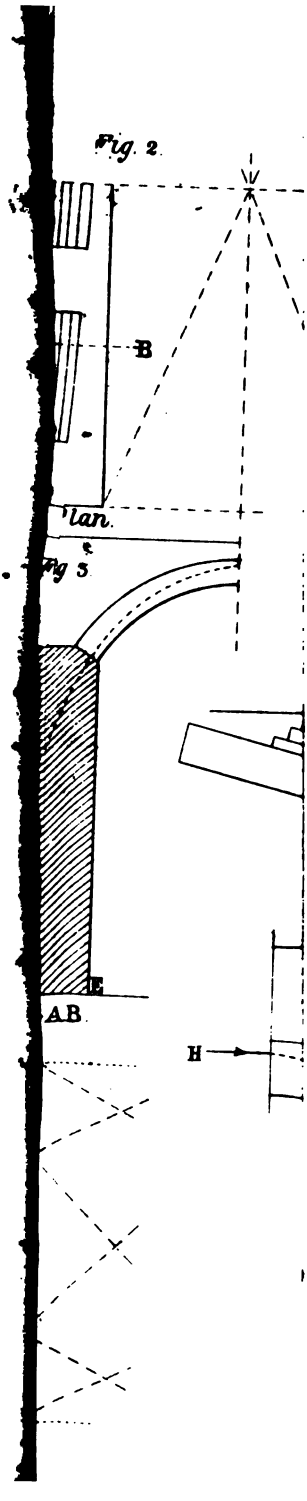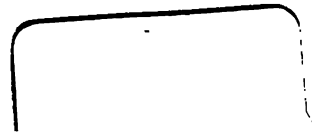
Fig. 2.
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